Applications of Systematic Indices in the Investment Process

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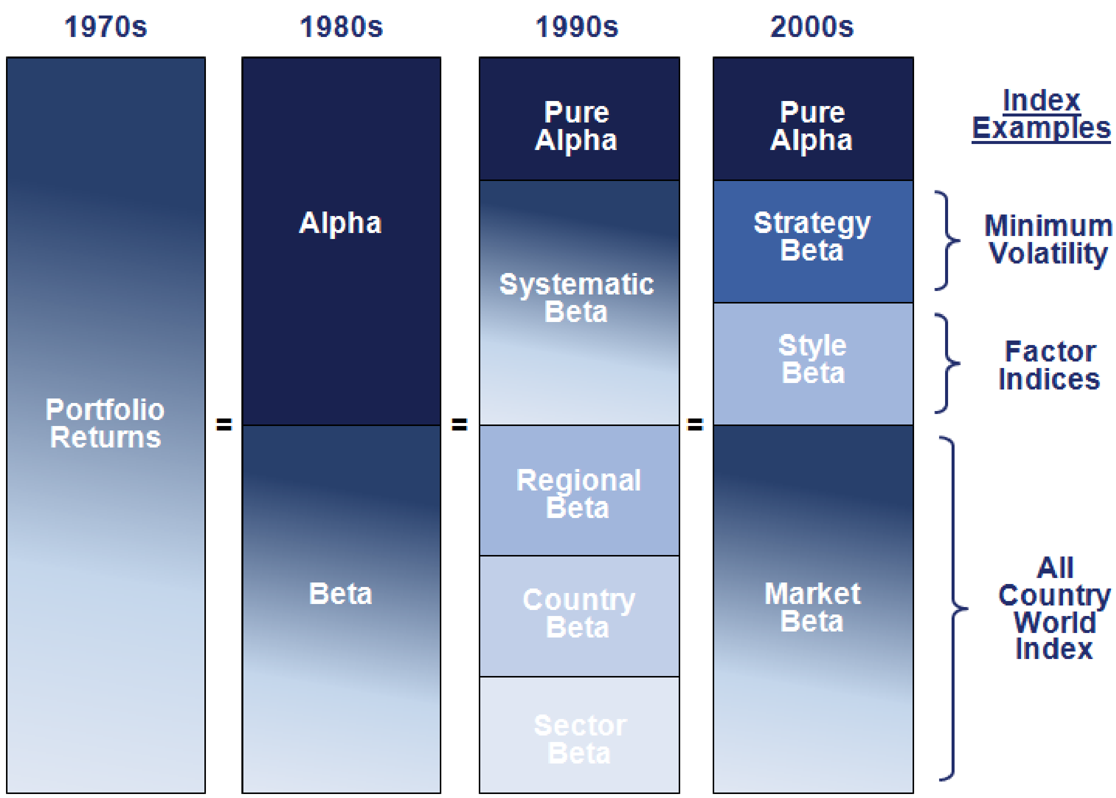
1. Introduction

Traditionally, beta has been defined as the return from broad asset class exposure, while alpha represented additional return from active portfolio management. More recently, investors have started to recognize that many sources of return that were considered added value (alpha) actually represent systematic risk premia (beta) (Anson, 2008, Berger, Kabiller, and Crowell, 2008). As a result, we have seen a proliferation of new indices aiming to capture various sources of systematic return. In parallel with the development of return-based indices, we have also witnessed the launch of several risk-based indices, which use volatility and correlation estimates to determine the index constituent weights (Demey, Maillard, and Roncalli, 2010). A schematic representation of the various sources of return and examples of indices capturing different types of beta are shown in Exhibit 1.

Exhibit 1: Understanding the Different Sources of Portfolio Return



Source: MSCI

Systematic risk factors should be explicitly recognized and integrated in the investment process, as they account for a significant percentage of long-term portfolio return (Ang, Goetzmann, and Schaefer, 2009).

In general, systematic factors can serve two roles in the asset allocation process. First, in strategic asset allocation, adding long-term strategic exposure to selected risk factors or investment strategies could potentially lower the volatility and improve the performance of the overall portfolio. Second, in tactical asset allocation, systematic factors could facilitate the implementation of short-term investment views or the hedging of unwanted risk exposures.

In this paper, we discuss applications of systematic indices. We use this term to refer to indices that capture systematic risk factors or replicate systematic investment strategies.

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Section 2 of the paper presents the key risk factors driving global equity portfolio returns and examines their historical track record. Section 3 highlights various approaches to capture these systematic risk factors through indices and describes their interconnections. Section 4 discusses applications of systematic indices in strategic asset allocation, while section 5 discusses how these indices can serve as the basis for investment tools and hedging instruments in tactical asset allocation. Section 6 describes the mechanics of adding factor exposure to a portfolio and derives the analytical conditions that imply performance improvements. Section 7 concludes and highlights potential investment implications.

This paper focuses mainly on systematic indices targeting equity risk factors and discusses their applications in the investment process. Systematic indices capturing risk factors or trading strategies from other asset classes such as fixed income and currencies can also serve similar roles in the asset allocation process (Bender, Briand, Nielsen, and Stefek, 2010).

2. Equity Factors and their Historical Track Record

The main factors driving global equity portfolio returns fall into three categories, namely, countries, industries, and styles. These factors play a vital role in explaining portfolio returns and capture key dimensions of many equity investment strategies. Style factors in particular encapsulate important fundamental or market trading characteristics of equity securities, for example, volatility, momentum, size, value, growth, leverage, liquidity, etc.

Exhibit 2: Barra Global Equity Model (GEM2) Style Factor Summary Statistics

A. January 1997 to June 2002 (66 months)

Factor Name	Average Squared t-statistic	Percent Observations with |t|>2	(weekly) Factor Kurtosis	Annualized Factor Return	Annualized Factor Volatility	Factor Sharpe Ratio	Correlation with ESTU
Volatility	119.46	82.6	4.92	-5.98	8.21	-0.73	0.785
Momentum	48.12	78.5	6.34	7.62	4.46	1.71	0.109
Size	43.65	79.9	4.73	2.37	2.97	0.80	0.291
Value	11.76	57.6	4.55	7.65	1.96	3.89	-0.156
Growth	9.75	50.7	4.49	-0.09	1.61	-0.05	0.407
Non-linear Size	8.86	49.7	4.31	1.61	1.89	0.85	0.151
Liquidity	10.18	53.8	4.00	4.99	1.63	3.06	0.506
Leverage	4.73	33.0	5.55	-1.18	1.13	-1.04	-0.029
Average	32.06	60.72	4.86	2.12	2.98	1.06	0.26

B. July 2002 to June 2008 (72 months)

Factor Name	Average Squared t-statistic	Percent Observations with |t|>2	(weekly) Factor Kurtosis	Annualized Factor Return	Annualized Factor Volatility	Factor Sharpe Ratio	Correlation with ESTU
Volatility	89.50	84.8	4.07	0.36	4.98	0.07	0.836
Momentum	35.37	76.4	5.71	4.62	2.65	1.75	-0.113
Size	26.45	65.0	5.33	0.71	1.67	0.42	0.411
Value	11.89	53.1	10.74	3.86	1.37	2.83	0.082
Growth	4.45	30.4	4.86	1.00	0.80	1.24	0.039
Non-linear Size	6.85	44.0	5.34	1.25	1.11	1.13	0.171
Liquidity	8.46	46.3	4.34	0.35	1.25	0.28	0.462
Leverage	4.30	33.7	8.59	-0.09	0.84	-0.11	0.016
Average	23.41	54.21	6.12	1.51	1.83	0.95	0.24

Source: MSCI

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Exhibit 2 summarizes the historical track record of the style risk factors from the Barra Global Equity Model GEM2 (Menchero, Morozov, and Shepard, 2008). The summary statistics presented in this table provide valuable insights into the characteristics of equity style factors and their potential role in asset allocation.

In this exhibit, the factors are listed approximately in order of explanatory power and statistical significance, highlighted by the first two columns in the table. Volatility is by far the most important and most volatile factor, followed by Momentum, Size, and Value during the observed period. On the other hand, Growth, Leverage, Liquidity, and Non-Linear Size have relatively lower volatility and lower explanatory power during the observed period. The third column in the table (factor kurtosis) shows that all equity style factors exhibited "fat tails" and serves as a reminder that empirical style factor returns do not follow the normal distribution.

The last four columns in Exhibit 2 are particularly important for understanding the potential role of equity style factors in asset allocation as they present key performance statistics: return, volatility, risk-adjusted performance, and correlation with the broad equity market (estimation universe, or ESTU for short).

These statistics show that Momentum and Value enjoyed strong absolute and risk-adjusted performance across both sub-periods. In addition, these two factors exhibited very low correlation with equities during both sub-periods. As a result, gaining strategic exposure to Momentum and Value could have potentially lowered the risk and improved the performance of a typical long-only equity or multi-asset class portfolio during the observed period. On the other hand, the Volatility factor had negative return, high volatility, and high correlation with equities. These characteristics suggest that acquiring negative exposure to the Volatility factor by tilting the portfolio toward low volatility stocks could have resulted in additional performance and diversification gains and made the Volatility factor a prime candidate for hedging through tactical asset allocation during the observed period.

In this section of the paper, we presented the key risk factors driving global equity portfolio returns and examined their historical performance. In the next section, we highlight various approaches that can be used to capture these factors through indices and describe their interconnections.

3. Systematic Indices and Their Interconnections

Historically, it was common for investors to implement factor-based strategies by using individual securities to construct portfolios with desirable factor exposures and characteristics. More recently, we have seen an explosion of new instruments, such as exchange traded funds, exchange traded notes, total return swaps, structured products, etc., that enable investors to capture different types of beta in a more efficient fashion. A common characteristic shared by many of these new investment instruments is that they rely on an underlying index that represents the particular risk factor or investment strategy they wish to replicate.

At first inspection, many systematic indices may appear unrelated. However, under closer scrutiny, it becomes apparent that these indices are highly interconnected because they have the same primary objective (capturing systematic beta) and only differ in the underlying investment beliefs and assumptions supporting the particular methodology used to construct the index. Exhibit 3 describes a general framework for understanding the interconnections between different risk and return based indices. This framework shows that many new index methodologies that appear unique or disconnected can be embedded in and understood as special cases of mean-variance portfolio construction (Melas, 2010).

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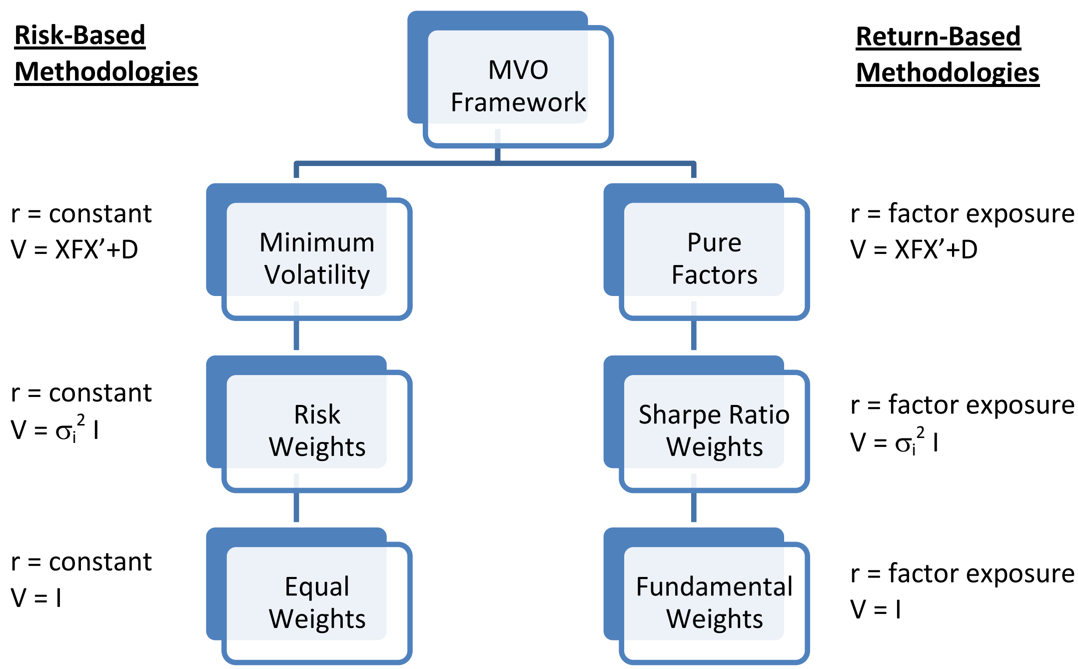
For example, the common underlying assumption across many risk based methodologies, such as the minimum-variance portfolio, risk-weighted portfolio, and equally weighted portfolio, is that expected returns are constant. In addition to constant expected returns, the equally weighted portfolio assumes constant volatilities and correlations. Under these three assumptions, we can show analytically that the equally weighted portfolio[1] is the solution to the general mean-variance portfolio optimization problem. Also, we can show analytically that if we relax the assumption of constant volatilities but continue to assume constant expected returns and correlations, we can maximize return per unit of risk by constructing a risk-weighted portfolio[2]. Finally, by assuming we can forecast volatilities and correlations but have no views on expected returns, we can achieve maximum diversification benefits by holding the minimum-variance portfolio.

Exhibit 3: Framework for Analyzing Systematic Risk- and Return-Based Indices



Source: MSCI

Similarly, many popular systematic return-based portfolio construction methods can also be embedded into the mean-variance framework presented in Exhibit 3. For example, under this framework, so-called "fundamentally" weighted portfolios (Arnott, Hsu, and Moore, 2005) are merely special cases of simple factor portfolios that assume constant volatilities and correlations and proxy expected returns with an accounting measure of company size[3] (sales, earnings, dividends, etc). More generally, we can use the mean-variance framework to construct simple factor portfolios that proxy any systematic factor (for example, momentum, liquidity, leverage, etc.) by simply setting expected returns equal to each asset's exposure to the target factor and assuming constant volatilities and correlations.

[1] Equally weighted portfolios typically suffer from lower liquidity, reduced capacity, and higher turnover compared to capitalization-weighted portfolios, making them more difficult to implement in practice.
[2] A risk-weighted portfolio is weighted by the inverse of the historical volatility of each security.
[3] As accounting measures of size tend to be correlated with market capitalization, fundamentally weighted portfolios have relatively low tracking error and retain some of the attractive liquidity and capacity characteristics of market-capitalization-weighted indices.

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Simple factor portfolios like the ones described above provide a first approximation to the underlying risk factors as they focus exclusively on expected return (factor exposure) and ignore expected volatilities and correlations. An incrementally more sophisticated approach for constructing factor portfolios is to take into account both expected returns and expected volatilities and weight assets according to their expected Sharpe ratios. Finally, by setting expected returns equal to factor exposures and taking into account both volatilities and correlations, the general mean-variance optimization framework can be used to construct pure factor portfolios. These portfolios replicate a particular systematic factor, while at the same time eliminating exposure to all other factors and minimizing portfolio risk. Melas, Suryanaryanan, and Cavaglia (2010) investigated different approaches for constructing pure factor portfolios in practice and highlighted passive and active investment management applications of pure factor portfolios.

To better understand the similarities and differences between alternative systematic index construction methods, we examine more closely two alternative approaches for capturing the value risk premium, highlighted in Kang and Melas (2009).

A simple way of capturing the value risk premium is to screen stocks based on certain valuation ratios. This approach typically results in high exposure to the value factor; however, it could also lead to significant industry concentration, as well as significant exposure to other sources of systematic risk. For example, as of July 2009, the traditional MSCI Europe Value Index, which is based on a country neutral screening methodology, overweights the Financials sector by 16.4% and underweights the Consumer Staples sector by 8.6% relative to the standard MSCI Europe Index.

The recently developed MSCI Value Factor Indices represent an alternative way of capturing the pure value risk premium, while being neutral in both country and industry allocation relative to the market. This is achieved through an optimization process that targets high exposure to the value factor while controlling exposure to country, industry, and other style risk factors.

Due to their high concentration in certain industries, the performance of traditional style indices may be driven by unintentional industry bets during periods when these industries perform very differently from the overall market. This is illustrated by comparing the performance of the MSCI Europe Value Index ("Value index") and the MSCI Europe Value Factor Index ("Value factor index") relative to the standard MSCI Europe Index.

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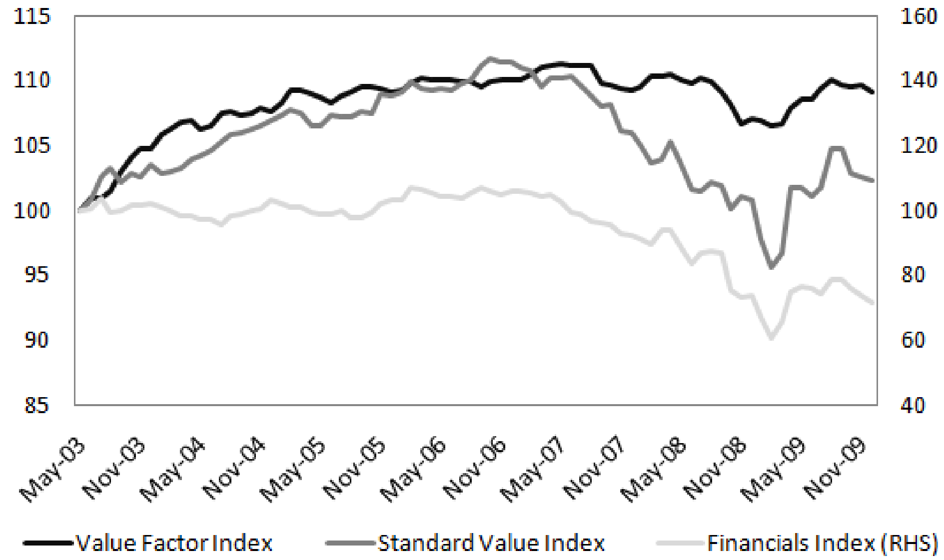
Exhibit 4 shows that from 2003 to 2007, the Value index performed in line with the Value factor index. However, as the subprime debt crisis began to spread, the financial sector started to significantly underperform the market from the second half of 2007, which dragged down the performance of the Value index that had a sizable overweight in Financials. The impact of the financial sector on the Value index is further illustrated by the recovery of the sector in 2009, when the relative performance of the Value index improved in tandem with the financial sector.

Exhibit 4: Performance of Different Indices Relative to the MSCI Europe Index



Source: MSCI

In comparison, the performance of the Value factor index relative to the market was not impacted by the underperformance of financials during the crisis, thanks to its industry-neutral approach. In the market turmoil between July 2007 and February 2009, the Value factor index outperformed the Value index by around 10%. This example demonstrates that the Value factor index may be a more natural choice than the traditional Value index for investors who wish to capture the value premium without any active views on countries or industries. On the other hand, traditional value indices remain appropriate benchmarks for investors who view active exposure to countries, industries, or other style factors as a natural consequence of a value investment process.

In this section of the paper, we highlighted various approaches to capture systematic risk factors through indices and described their interconnections. In the next section, we discuss applications of systematic indices in the strategic asset allocation process.

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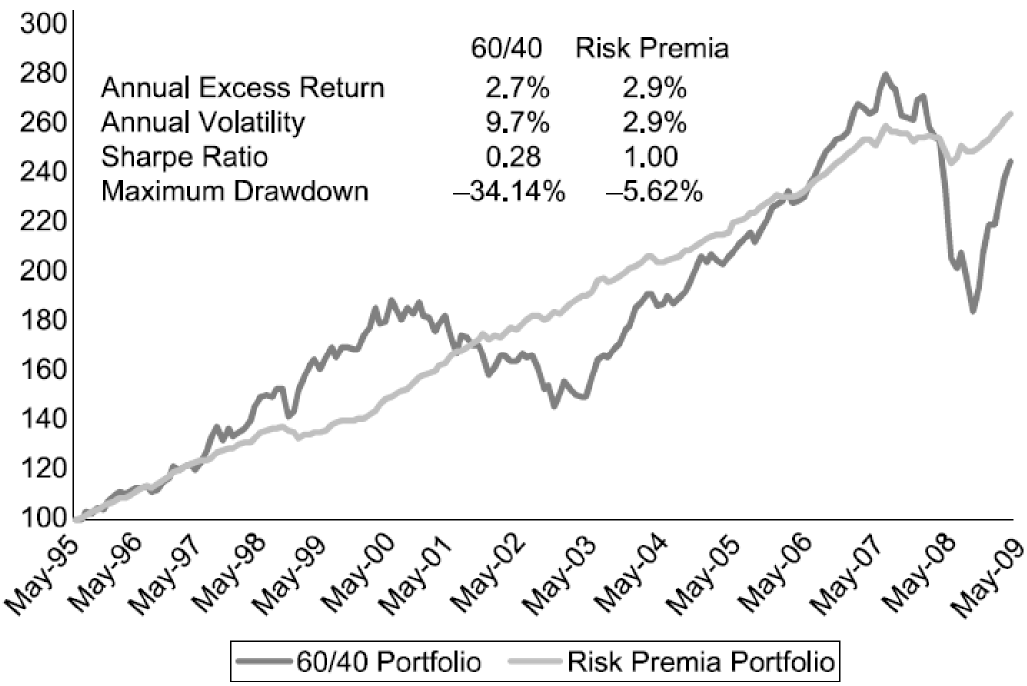
4. Using Systematic Indices in Strategic Asset Allocation

Making strategic allocations to selected risk factors using instruments such as the ones described in the previous section could potentially lower the volatility or improve the performance of a multi-asset class portfolio.

Bender, Briand, Nielsen, and Stefek (2010) highlight the potential benefits of building a diversified portfolio of risk premia from different asset classes and compare the historical performance of this portfolio with a balanced portfolio consisting of 60% global equities and 40% fixed income. Exhibit 5, taken from this study, shows that an equally weighted portfolio of systematic risk factors from different asset classes has historically produced returns similar to a 60/40 equity/bond portfolio, but with much lower volatility.

Exhibit 5: Cumulative Return of Traditional and Factor-Based Asset Allocation



Source: MSCI. Note: The Risk Premia Portfolio in Exhibit 5 represents the historical performance of an equally weighted monthly rebalanced combination of the following risk factors and systematic strategies: Equity Value, Equity Size, Equity Momentum, Term Spread, Credit Spread, High Yield Spread, Currency Carry, Currency Value, Currency Momentum, Merger Arbitrage, and Convertible Arbitrage.

Similar benefits could be achieved by adding factor overlays to a core equity portfolio. In Section 2 we saw that historically the Value and Momentum factors enjoyed positive return and exhibited low correlation with equities, while the Volatility factor had negative return and high correlation with equities. These patterns suggest that adding positive exposure to the Value and Momentum factors and negative exposure to the Volatility factor could enhance the risk adjusted performance of a global equity portfolio.

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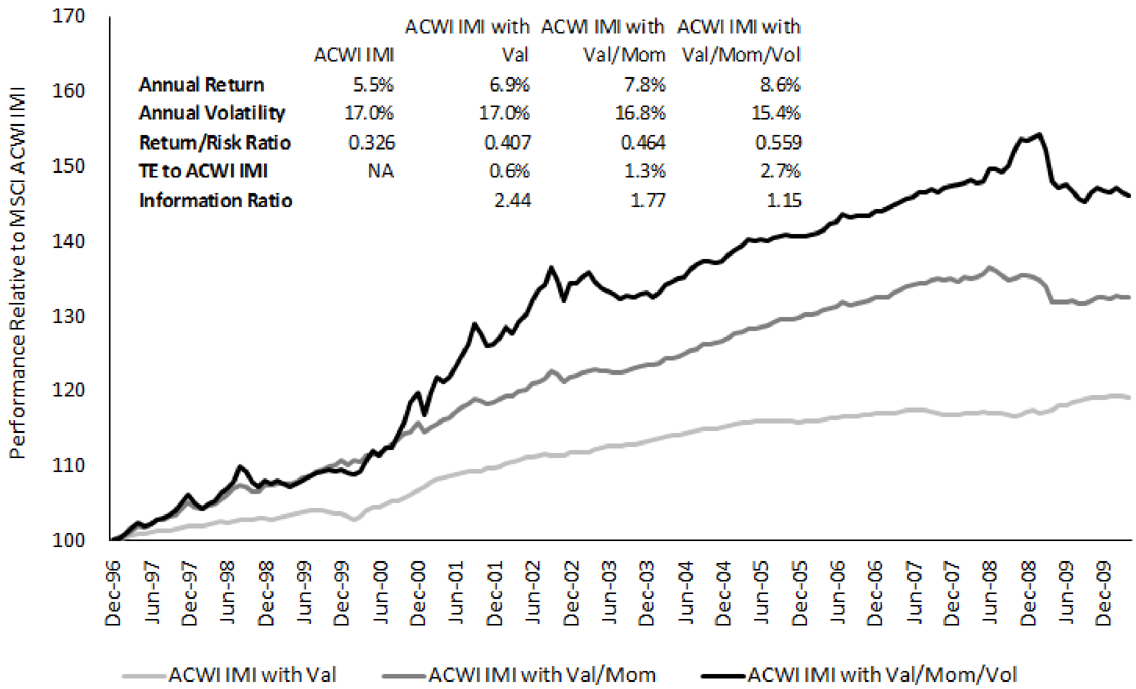
Exhibit 6 shows the simulated relative performance of a broad global equity portfolio (MSCI All Country World Investable Market Index, or ACWI IMI for short) combined with factor overlays targeting the Value, Momentum, and Volatility factors (exposure equivalent to 0.25 standard deviations for Value and Momentum and -0.25 for Volatility). Adding one factor at a time results in incremental return enhancement and risk reduction. The strategy combining all three factors and MSCI ACWI IMI resulted in annualized return of 8.6% and volatility of 15.4% compared to 5.5% and 17.0%, respectively, for MSCI ACWI IMI.

Exhibit 6: Global Equity Portfolio with Value, Momentum, and Volatility Overlays



Source: MSCI. Note: "ACWI IMI" is short for the MSCI All Country World Investable Market Index.

Another recent effort to replicate a systematic investment strategy in an index is the development of minimum volatility indices that aim to capture the broad equity risk premium with lower portfolio volatility. For instance, the MSCI World Minimum Volatility Index outperformed the MSCI World Index by 1.7% per annum over the period May 1998 to May 2010, with almost 30% lower realized volatility (12.2% vs. 16.8% realized volatility). As equity risk often dominates traditional multi-asset class portfolios, such significant reduction in equity risk could have a positive impact on the total risk of multi-asset class portfolios (Nielsen and Subramanian, 2008).

So far in this section, we have discussed potential strategic asset allocation applications of systematic indices using security-weighting schemes not based on market capitalization. Recently, there has also been increased interest in indices using country-weighting schemes based on economic size rather than market capitalization, especially given the divergence between economic size and market size of countries with the fastest growing economies. We conclude this section with a brief reference to GDP-weighted indices and their potential applications in strategic asset allocation (Kouzmenko and Nagy, 2010).

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GDP weighting effectively assigns higher weight to emerging markets and lower weight to developed markets. A global GDP-weighted index would tend to overweight countries that have a low market cap to GDP ratio. Advocates of the GDP-weighted asset allocation method argue that as these markets progressively develop, their equities attract inflows that could result in above-average returns. Another potentially attractive characteristic of a GDP-weighted approach to asset allocation is that it proxies a "mean-reversion" strategy by periodically reducing exposure to markets that have performed well recently and increasing exposure to markets that have performed badly.

Exhibit 7 shows that historically a global GDP- weighted index has outperformed a global market-cap- weighted index in both absolute and risk-adjusted terms.

Exhibit 7: Historical Performance of Standard and GDP-Weighted Equity Indices

Index	Period	Annualized return	Annualized risk	Return / Risk	Annualized relative performance
MSCI ACWI	1988-2009	4.7%	15.5%	0.30	
MSCI ACWI GDP	1988-2009	7.4%	16.5%	0.45	2.6%
MSCI World Index	1969-2009	6.3%	14.9%	0.42	
MSCI World Index GDP	1969-2009	7.0%	14.7%	0.48	0.7%
MSCI EM	1988-2009	9.6%	24.4%	0.39	
MSCI EM GDP	1988-2009	14.5%	28.2%	0.52	4.5%

Source: MSCI. Note: "ACWI" is short for the MSCI All Country World Index, and "EM" is short for Emerging Markets.

Exhibit 8 shows the difference between the GDP weight and the market-capitalization weight of selected countries and regions in MSCI ACWI. It is striking to see that a simple concept such as GDP weighting has been so effective at capturing the two major shifts in equity asset allocation in the last 20 years: underweighting Japan in the 1990s and overweighting emerging markets in the 2000s.

Exhibit 8: Difference between GDP and Market-Cap Weights in MSCI ACWI

Source: MSCI

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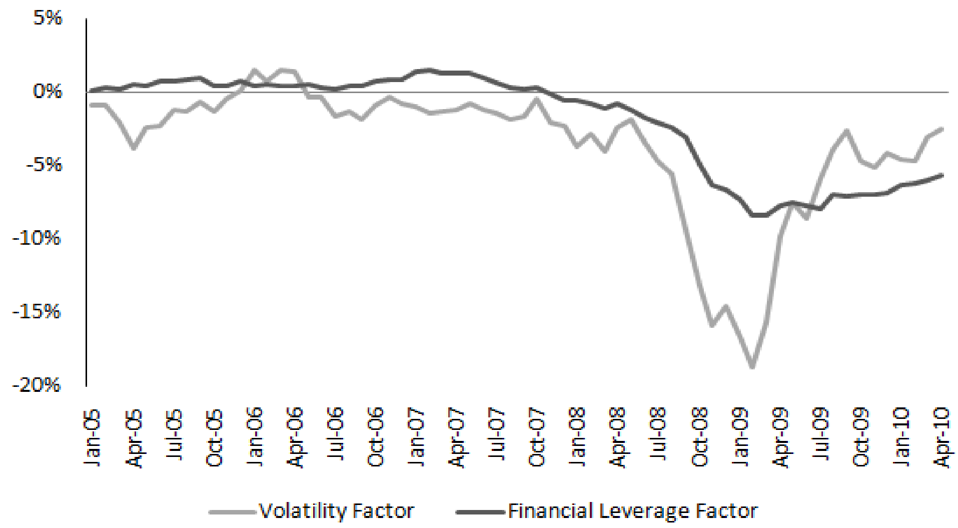
In this section, we reviewed applications of systematic indices in strategic asset allocation. In the next section we discuss how systematic indices could potentially serve as the basis for investment tools and hedging instruments in tactical asset allocation.

5. Using Systematic Indices in Tactical Asset Allocation

Systematic indices could be used on a tactical basis to tilt a portfolio toward certain factors or to remove undesired risk exposures. For instance, unintended or incidental exposure to risk factors such as Volatility and Financial Leverage could have significant impact on the risk and performance of a portfolio, especially during periods of market turmoil. Exhibit 9 shows that these two factors were relatively stable prior to early 2007, but experienced sharp falls during the 2007-2009 market turmoil. Therefore, controlling these exposures could be an important tactical decision, especially during periods of high market volatility.

Exhibit 9: Cumulative Returns of Volatility Factor and Financial Leverage Factor



Source: MSCI

One way to reduce exposure to the Volatility factor in a portfolio would be simply to sell the most volatile stocks. However, this approach could be problematic, as it may conflict with the fundamental process used to select these stocks and lead to costly portfolio rebalancing. An alternative way to reduce exposure to the Volatility factor is to introduce a factor overlay that shorts the Volatility factor. For example, the MSCI Low Volatility Factor Index is designed to replicate a short position in the Volatility risk factor and can potentially be used in such a tactical hedging process. Similarly, the MSCI Low Leverage Factor Index could potentially be used to eliminate exposure to the Financial Leverage factor.

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Exhibit 10 shows two simulated historical examples of applying these hedging strategies to a small capitalization portfolio (Volatility hedge) and a financials portfolio (Leverage hedge) during the recent market turmoil.

Exhibit 10: Using Factor Overlays to Hedge Volatility and Financial Leverage

	MSCI Europe Small Cap Index	Small Cap Index Imperfect Hedge	Small Cap Index Perfect Hedge
Return	-12.1%	-9.37%	-7.67%
Volatility	27.5%	26.1%	27.4%
Sharpe Ratio	-0.439	-0.360	-0.280
Outperformance (to Small Cap)	0.00%	2.68%	4.38%
Tracking error (to Small Cap)	0.00%	2.93%	1.61%
Information Ratio	NA	0.915	2.722

Annualized statistics for period June 2007 to Dec 2009. No transaction cost assumptions included.

	MSCI Europe Financials	Financials with Imperfect Hedge	Financials with Perfect Hedge
Return	-21.2%	-17.84%	-18.42%
Volatility	36.5%	36.2%	35.5%
Sharpe Ratio	-0.582	-0.493	-0.518
Outperformance (to Financials)	0.00%	3.40%	2.82%
Tracking error (to Financials)	0.00%	1.95%	1.62%
Information Ratio	NA	1.748	1.742

Annualized statistics for period June 2007 to Dec 2009. No transaction cost assumptions included.

Source: MSCI. Note: The "Imperfect Hedge" examples use MSCI Factor Indices, while the "Perfect Hedge" examples use the theoretical factor returns

6. The Mechanics of Adding Factor Exposure to Portfolios

In the previous sections of this paper, we examined various methods to capture systematic risk factors through indices and discussed applications of these indices in strategic and tactical asset allocation. An important question for investors considering adding factor exposure to their portfolios through systematic indices is under what conditions the addition of factor exposure would lead to superior portfolio performance. In this section we discuss the mechanics of adding factor exposure to a core portfolio and derive the analytical conditions that would imply risk-adjusted performance improvements.

There has been growing debate recently among plan sponsors and other long-term institutional investors about so called "risk parity" strategies (Ruban and Melas, 2010). The main idea behind these strategies is that increasing exposure to fixed income through leverage may potentially lead to lower risk or improved risk-adjusted performance (Sharpe ratio). In this section of the paper, we apply the same concept, but instead of using leverage to increase exposure to fixed income, we use leverage to add exposure to factors. More specifically, we derive the conditions that imply that adding factors to a typical core institutional portfolio through leverage would lead to lower volatility or improved risk-adjusted performance.

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Adding Factors to Lower Volatility

One of the main motivations behind institutional allocations to new asset classes and strategies is to improve performance. However, improving performance, as we will show in the next section, relies on assumptions regarding ex-ante returns, which are generally difficult to estimate reliably. A less ambitious but equally important motivation behind institutional allocations to new asset classes is the desire to lower volatility through diversification.

In practice, typical institutional portfolios have several core investments, for example, government bonds, corporate bonds, public equities, private equities, real estate, hedge funds, infrastructure assets, etc. Combining these core institutional portfolios with factor portfolios could lead to lower volatility through diversification. The combination could be achieved either through rebalancing or through leverage. Rebalancing of the core portfolio would involve selling long-term holdings, which typically include illiquid asset classes. On the other hand, introducing factors through leverage eliminates the need to liquidate core holdings and allows more dynamic management of the factor exposures.

A pertinent question for investors considering adding factors to a core portfolio to lower volatility is under what conditions the combined levered portfolio would indeed have lower volatility compared to the initial unlevered core portfolio. More specifically, how low should the correlation between the core portfolio and the factor portfolio be in order to achieve lower volatility in the combined levered portfolio? In Appendix 1, we derive the following analytical condition that implies lower volatility in the combined portfolio:

$$\rho < -\frac{1}{2}k\theta \qquad (1)$$

In this expression, $k>0$ is the leverage ratio, and θ is the ratio of factor portfolio to core portfolio volatility. This analytical condition provides two important insights. The first insight is that correlation between the factor portfolio and the core portfolio must be negative for the levered portfolio to have lower volatility than the core portfolio. The second insight is that correlation between the factor portfolio and the core portfolio must be more negative for higher leverage and for higher factor volatility. For example, we see that for 20% leverage, and assuming the factor portfolio and the core portfolio have the same volatility, correlation must be lower (more negative) than -0.10. On the other hand, for 50% leverage, and assuming the factor portfolio is twice as volatile as the core portfolio, correlation must be below -0.50 for the strategy to achieve its objective (lower volatility in the levered portfolio).

Adding Factors to Improve Performance

As we mentioned in the previous section, the main motivation behind institutional allocations to new asset classes and strategies is to improve risk-adjusted performance. A widely used measure of risk-adjusted performance is the excess return to volatility ratio, also known as the Sharpe ratio. Combining a core portfolio and a factor overlay could lead to improved risk-adjusted performance, but under what conditions? In Appendix 2, we derive the following analytical condition that implies that adding a factor overlay to a core portfolio through leverage leads to superior risk-adjusted performance:

$$\rho < -\frac{1}{2}\left(k\theta - \frac{S_F}{S_C}\right) \qquad (2)$$

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In the last expression, k represents the leverage ratio, θ is the ratio of factor portfolio to core portfolio volatility, while S_C and S_F represent the Sharpe ratio of the core portfolio and the factor portfolio, respectively. For example, we see that, for 50% leverage and assuming the factor portfolio and the core portfolio have the same volatility and the same Sharpe ratio, correlation has to be lower than 0.25 for the strategy to achieve its objective (improved risk-adjusted performance).

Expressions 1 and 2 have an intuitive interpretation. For a conservative investor who adds factor exposure through leverage to a core portfolio in order to lower volatility, Expression 1 shows that the correlation between the core portfolio and the factor portfolio must be negative and must rise in proportion to leverage and to the volatility of the factor portfolio in order to achieve lower volatility for the combined portfolio. On the other hand, for a more ambitious investor who adds factors aiming to improve risk adjusted performance, Expression 2 shows that the correlation threshold is higher. In other words, moderately negative or even positive correlation between the core portfolio and the factor overlay could still lead to improved risk-adjusted performance, as long as the Sharpe ratio of the factor portfolio is sufficiently high, compared to the Sharpe ratio of the initial core portfolio.

Adding Factors: A Practical Example

In Section 4, we saw that a core equity portfolio combined with factor overlays targeting the Value, Momentum, and Volatility factors historically experienced significant risk reduction and risk-adjusted performance improvement benefits.

In Exhibit 11, we use the analytical expressions derived in the previous two sections to confirm which particular factor overlay would lead to risk reduction or risk-adjusted performance improvement when combined with the core global equity portfolio. The analytical results presented in this table confirm the empirical findings reported in Exhibit 6 and highlight once again the potential benefits of adding factor exposures to a core portfolio.

Exhibit 11: Global Equity Portfolio with Value, Momentum, and Volatility Overlays

	Correlation ρ	Leverage k	Volatility ratio θ	Sharpe ratio of core portfolio	Sharpe ratio of factor portfolio	$\rho < -\frac{1}{2}k\theta$	$\rho < -\frac{1}{2}\left(k\theta - \frac{S_F}{S_C}\right)$
Adding Value	0.06	1.25	0.13	0.12	2.32	ρ > -0.08 cannot lower risk	ρ < 9.96; can improve Sharpe ratio
Adding Momentum	-0.22	1.25	0.30	0.12	0.58	ρ > -0.19 cannot lower risk	ρ < 2.34; can improve Sharpe ratio
Adding Volatility	-0.82	1.25	0.44	0.12	0.27	ρ < -0.27 can lower risk	ρ < 0.91; can improve Sharpe ratio
Adding Val/Mom	-0.20	1.50	0.15	0.12	1.64	ρ < -0.11 can lower risk	ρ < 6.98; can improve Sharpe ratio
Adding Val/Mom/Vol	-0.66	1.75	0.21	0.12	0.97	ρ < -0.18 can lower risk	ρ < 4.04; can improve Sharpe ratio

Source: MSCI

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7. Conclusions and Potential Investment Implications

Systematic risk premia account for a substantial part of long-term portfolio performance. While market cap indices represent effective instruments to capture the market beta, systematic indices are important tools to capture additional systematic style and strategy betas. We reviewed various systematic risk- and return-based indices that are designed to capture certain desirable factor exposures, risk characteristics, or investment strategies.

More importantly, we discussed the practical investment applications of systematic indices. A risk-premia-based approach to strategic asset allocation is increasingly gaining recognition amongst institutional investors (Ang, Goetzmann, and Schaefer, 2009). Systematic indices could be important tools in implementing such risk-based asset allocation, to capture diversified risk exposures in a disciplined, cost-effective fashion. Furthermore, systematic indices also have potentially appealing tactical applications, in hedging unintended exposure to certain risk factors as well as in tilting a portfolio toward certain styles.

Also, we discussed the mechanics of integrating factors into the asset allocation process. Adding factors to a typical institutional portfolio could be motivated by the desire to lower risk or to improve risk-adjusted performance. We derived the analytical conditions that imply lower risk or improved risk-adjusted performance. Our analysis shows that the conservative objective of lowering risk can be achieved only if the correlation between the core portfolio and the factor overlay is negative. On the other hand, adding a factor overlay to a core portfolio could lead to improved risk-adjusted performance even if the correlation between the two portfolios is positive. However, in this case, any ex-ante performance improvement relies on assumptions regarding the Sharpe ratio of the factor overlay relative to the core portfolio.

In summary, systematic indices complement market-cap indices by providing additional tools for institutional investors to implement their strategic and tactical asset allocation. Using these tools to introduce systematic factor exposures could result in potentially significant risk reduction and risk-adjusted performance enhancement benefits. This field is likely to continue to attract research interest and institutional product development.

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Appendix 1: Deriving the Analytical Condition that Implies Lower Volatility

We want to derive the condition that implies that using leverage to add a factor overlay to a core portfolio leads to lower volatility for the levered portfolio. Using σ_L and σ_C to denote the volatilities of the levered portfolio and the initial core portfolio, we can write:

$$\sigma_L^2 < \sigma_C^2 \tag{A1.1}$$

We can expand this condition by expressing the volatility of the levered portfolio as a function of core portfolio volatility σ_C, factor portfolio volatility σ_F, correlation ρ between the core and the factor portfolio, and leverage $k>0$:

$$\sigma_C^2 + k^2\sigma_F^2 + 2k\sigma_C\sigma_F\rho < \sigma_C^2 \tag{A1.2}$$

Using θ to denote the ratio of factor portfolio volatility to core portfolio volatility, we derive the following correlation condition that implies that adding a factor overlay to a core portfolio leads to lower volatility:

$$\rho < -\frac{1}{2}k\theta \tag{A1.3}$$

Appendix 2: Deriving the Analytical Condition that Implies Better Risk-Adjusted Performance

We want to derive the condition that implies that using leverage to add a factor overlay to a core portfolio leads to better risk-adjusted performance. We use the return-to-variance ratio as the measure of risk-adjusted performance. We want to derive the condition that implies that the levered portfolio that incorporates a factor overlay has superior risk-adjusted performance (higher return-to-variance ratio) compared to the initial portfolio:

$$\frac{r_C + kr_F}{\sigma_C^2 + k^2\sigma_F^2 + 2k\sigma_C\sigma_F\rho} > \frac{r_C}{\sigma_C^2} \tag{A2.1}$$

By dividing both the numerator and denominator by σ_C^2, and using θ to denote the ratio of factor portfolio volatility σ_F to core portfolio volatility σ_C, the last condition becomes:

$$\frac{\frac{r_C}{\sigma_C^2} + k\theta^2\frac{r_F}{\sigma_F^2}}{1 + k^2\theta^2 + 2k\theta\rho} > \frac{r_C}{\sigma_C^2} \tag{A2.2}$$

Assuming the ex-ante return of the core portfolio is positive, we have:

$$\frac{1 + k\theta^2\frac{r_F}{\sigma_F^2}\frac{\sigma_C^2}{r_C}}{1 + k^2\theta^2 + 2k\theta\rho} > 1 \tag{A2.3}$$

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From the last expression, we can derive the correlation condition that implies superior risk-adjusted performance for the levered portfolio compared to the initial core portfolio:

$$\rho < -\frac{1}{2}\left(k\theta - \frac{S_F}{S_C}\right) \tag{A2.4}$$

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