J.P.Morgan/Reuters

RiskMetrics™—Technical Document

Fourth Edition, 1996

New York
December 17, 1996

- J.P. Morgan and Reuters have teamed up to enhance RiskMetrics™. Morgan will continue to be responsible for enhancing the methods outlined in this document, while Reuters will control the production and distribution of the RiskMetrics™ data sets.
- Expanded sections on methodology outline enhanced analytical solutions for dealing with nonlinear options risks and introduce methods on how to account for non-normal distributions.
- Enclosed diskette contains many examples used in this document. It allows readers to experiment with our risk measurement techniques.
- All publications and daily data sets are available free of charge on J.P. Morgan's Web page on the Internet at **http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html**. This page is accessible directly or through third party services such as CompuServe®, America Online™, or Prodigy®.

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This *Technical Document* provides a detailed description of RiskMetrics™, a set of techniques and data to measure market risks in portfolios of fixed income instruments, equities, foreign exchange, commodities, and their derivatives issued in over 30 countries. This edition has been expanded significantly from the previous release issued in May 1995.

We make this methodology and the corresponding RiskMetrics™ data sets available for three reasons:

1. We are interested in promoting greater transparency of market risks. Transparency is the key to effective risk management.

2. Our aim has been to establish a benchmark for market risk measurement. The absence of a common point of reference for market risks makes it difficult to compare different approaches to and measures of market risks. Risks are comparable only when they are measured with the same yardstick.

3. We intend to provide our clients with sound advice, including advice on managing their market risks. We describe the RiskMetrics™ methodology as an aid to clients in understanding and evaluating that advice.

Both J.P. Morgan and Reuters are committed to further the development of RiskMetrics™ as a fully transparent set of risk measurement methods. We look forward to continued feedback on how to maintain the quality that has made RiskMetrics™ the benchmark for measuring market risk.

RiskMetrics™ is based on, but differs significantly from, the risk measurement methodology developed by J.P. Morgan for the measurement, management, and control of market risks in its trading, arbitrage, and own investment account activities. **We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks**. RiskMetrics™ is nothing more than a high-quality tool for the professional risk manager involved in the financial markets and is not a guarantee of specific results.

RiskMetrics™—Technical Document
Fourth Edition (December 1996)

This book

This is the reference document for RiskMetrics™. It covers all aspects of RiskMetrics and supersedes all previous editions of the *Technical Document*. It is meant to serve as a reference to the methodology of statistical estimation of market risk, as well as detailed documentation of the analytics that generate the data sets that are published daily on our Internet Web sites.

This document reviews

1. The conceptual framework underlying the methodologies for estimating market risks.
2. The statistics of financial market returns.
3. How to model financial instrument exposures to a variety of market risk factors.
4. The data sets of statistical measures that we estimate and distribute daily over the Internet and shortly, the Reuters Web.

Measurement and management of market risks continues to be as much a craft as it is a science. It has evolved rapidly over the last 15 years and has continued to evolve since we launched RiskMetrics in October 1994. Dozens of professionals at J.P. Morgan have contributed to the development of this market risk management technology and the latest document contains entries or contributions from a significant number of our market risk professionals.

We have received numerous constructive comments and criticisms from professionals at Central Banks and regulatory bodies in many countries, from our competitors at other financial institutions, from a large number specialists in academia and last, but not least, from our clients. Without their feedback, help, and encouragement to pursue our strategy of open disclosure of methodology and free access to data, we would not have been as successful in advancing this technology as much as we have over the last two years.

What is RiskMetrics?

RiskMetrics is a set of tools that enable participants in the financial markets to estimate their exposure to market risk under what has been called the “Value-at-Risk framework”. RiskMetrics has three basic components:

- A set of market risk measurement methodologies outlined in this document.
- Data sets of volatility and correlation data used in the computation of market risk.
- Software systems developed by J.P.Morgan, subsidiaries of Reuters, and third party vendors that implement the methodologies described herein.

With the help of this document and the associated line of products, users should be in a position to estimate market risks in portfolios of foreign exchange, fixed income, equity and commodity products.

J.P. Morgan and Reuters team up on RiskMetrics

In June 1996, J.P. Morgan signed an agreement with Reuters to cooperate on the building of a new and more powerful version of RiskMetrics. Since the launch of RiskMetrics in October 1994, we have received numerous requests to add new products, instruments, and markets to the daily volatility and correlation data sets. We have also perceived the need in the market for a more flexible VaR data tool than the standard matrices that are currently distributed over the Internet. The new partnership with Reuters, which will be based on the precept that both firms will focus on their respective strengths, will help us achieve these objectives.

Methodology

J.P. Morgan will continue to develop the RiskMetrics set of VaR methodologies and publish them in the quarterly *RiskMetrics Monitor* and in the annual *RiskMetrics—Technical Document*.

RiskMetrics data sets

Reuters will take over the responsibility for data sourcing as well as production and delivery of the risk data sets. The current RiskMetrics data sets will continue to be available on the Internet free of charge and will be further improved as a benchmark tool designed to broaden the understanding of the principles of market risk measurement.

When J.P. Morgan first launched RiskMetrics in October 1994, the objective was to go for broad market coverage initially, and follow up with more granularity in terms of the markets and instruments covered. This over time, would reduce the need for proxies and would provide additional data to measure more accurately the risk associated with non-linear instruments.

The partnership will address these new markets and products and will also introduce a new customizable service, which will be available over the Reuters Web service. The customizable RiskMetrics approach will give risk managers the ability to scale data to meet the needs of their individual trading profiles. Its capabilities will range from providing customized covariance matrices needed to run VaR calculations, to supplying data for historical simulation and stress-testing scenarios.

More details on these plans will be discussed in later editions of the *RiskMetrics Monitor.*

Systems

Both J.P. Morgan and Reuters, through its Sailfish subsidiary, have developed client-site RiskMetrics VaR applications. These products, together with the expanding suite of third party applications will continue to provide RiskMetrics implementations.

What is new in this fourth edition?

In terms of content, the Fourth Edition of the *Technical Document* incorporates the changes and refinements to the methodology that were initially outlined in the 1995–1996 editions of the *RiskMetrics Monitor*:

- **Expanded framework:** We have worked extensively on refining the analytical framework for analyzing options risk without having to perform relatively time consuming simulations and have outlined the basis for an improved methodology which incorporates better information on the tails of distributions related to financial asset price returns; we've also developed a data synchronization algorithm to refine our volatility and correlation estimates for products which do not trade in the same time zone;

- **New markets:** We expanded the daily data sets to include estimated volatilities and correlations of additional foreign exchange, fixed income and equity markets, particularly in South East Asia and Latin America.

- **Fine-tuned methodology:** We have modified the approach in a number of ways. First, we've changed our definition of price volatility which is now based on a total return concept; we've also revised some of the algorithms used in our mapping routines and are in the process of redefining the techniques used in estimating equity portfolio risk.

- **RiskMetrics products:** While we have continued to expand the list of third parties providing RiskMetrics products and support, this is no longer included with this document. Given the rapid pace of change in the availability of risk management software products, readers are advised to consult our Internet web site for the latest available list of products. This list, which now includes FourFifteen™, J.P. Morgan's own VaR calculator and report generating software, continues to grow, attesting to the broad acceptance RiskMetrics has achieved.

- **New tools to use the RiskMetrics data sets:** We have published an Excel add-in function which enables users to import volatilities and correlations directly into a spreadsheet. This tool is available from our Internet web site.

The structure of the document has changed only slightly. As before, its size warrants the following note: One need not read and understand the entire document in order to benefit from RiskMetrics. The document is organized in parts that address subjects of particular interest to many readers.

Part I: Risk Measurement Framework

This part is for the general practitioner. It provides a practical framework on how to think about market risks, how to apply that thinking in practice, and how to interpret the results. It reviews the different approaches to risk estimation, shows how the calculations work on simple examples and discusses how the results can be used in limit management, performance evaluation, and capital allocation.

Part II: Statistics of Financial Market Returns

This part requires an understanding and interest in statistical analysis. It reviews the assumptions behind the statistics used to describe financial market returns and how distributions of future returns can be estimated.

Part III: Risk Modeling of Financial Instruments

This part is required reading for implementation of a market risk measurement system. It reviews how positions in any asset class can be described in a standardized fashion (foreign exchange, interest rates, equities, and commodities). Special attention is given to derivatives positions. The purpose is to demystify derivatives in order to show that their market risks can be measured in the same fashion as their underlying.

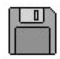
Part IV: RiskMetrics Data Sets

This part should be of interest to users of the RiskMetrics data sets. First it describes the sources of all daily price and rate data. It then discusses the attributes of each volatility and correlation series in the RiskMetrics data sets. And last, it provides detailed format descriptions required to decipher the data sets that can be downloaded from public or commercial sources.

Appendices

This part reviews some of the more technical issues surrounding methodology and regulatory requirements for market risk capital in banks and demonstrates the use of RiskMetrics with the example diskette provided with this document. Finally, Appendix H shows you how to access the RiskMetrics data sets from the Internet.

RiskMetrics examples diskette



This diskette is located inside the back cover. It contains an Excel workbook that includes some of the examples shown in this document. Such examples are identified by the icon shown here.

Future plans

We expect to update this *Technical Document* annually as we adapt our market risk standards to further improve the techniques and data to meet the changing needs of our clients.

RiskMetrics is a now an integral part of J.P. Morgan's Risk Management Services group which provides advisory services to a wide variety of the firm's clients. We continue to welcome any suggestions to enhance the methodology and adapt it further to the needs of the market. All suggestions, requests and inquiries should be directed to the authors of this publication or to your local RiskMetrics contacts listed on the back cover.

Acknowledgments

The authors would like to thank the numerous individuals who participated in the writing and editing of this document, particularly Chris Finger and Chris Athaide from J.P. Morgan's risk management research group, and Elizabeth Frederick and John Matero from our risk advisory practice. Finally, this document could not have been produced without the contributions of our consulting editor, Tatiana Kolubayev. We apologize for any omissions to this list.

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Part I

Risk Measurement Framework

Chapter 1. Introduction

Chapter 1. Introduction

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This chapter serves as an introduction to the RiskMetrics product. RiskMetrics is a set of methodologies and data for measuring market risk. By market risk, we mean the potential for changes in value of a position resulting from changes in market prices.

We define risk as the degree of uncertainty of future net returns. This uncertainty takes many forms, which is why most participants in the financial markets are subject to a variety of risks. A common classification of risks is based on the source of the underlying uncertainty:

- Credit risk estimates the potential loss because of the inability of a counterparty to meet its obligations.
- Operational risk results from errors that can be made in instructing payments or settling transactions.
- Liquidity risk is reflected in the inability of a firm to fund its illiquid assets.
- Market risk, the subject of the methodology described in this document, involves the uncertainty of future earnings resulting from changes in market conditions, (e.g., prices of assets, interest rates). Over the last few years measures of market risk have become synonymous with the term Value-at-Risk.

RiskMetrics has three basic components:

- The first is a set of methodologies outlining how risk managers can compute Value-at-Risk on a portfolio of financial instruments. These methodologies are explained in this *Technical Document*, which is an annual publication, and in the *RiskMetrics Monitor*, the quarterly update to the *Technical Document*.
- The second is data that we distribute to enable market participants to carry out the methodologies set forth in this document.
- The third is Value-at-Risk calculation and reporting software designed by J.P. Morgan, Reuters, and third party developers. These systems apply the methodologies set forth in this document and will not be discussed in this publication.

This chapter is organized as follows:

- Section 1.1 presents the definition of Value-at-Risk (VaR) and some simple examples of how RiskMetrics offers the inputs necessary to compute VaR. The purpose of this section is to offer a basic approach to VaR calculations.
- Section 1.2 describes more detailed examples of VaR calculations for a more thorough understanding of how RiskMetrics and VaR calculations fit together. In Section 1.2.2 we provide an example of how to compute VaR on a portfolio containing options (nonlinear risk) using two different methodologies.
- Section 1.3 presents the contents of RiskMetrics at both the general and detailed level. This section provides a step-by-step analysis of the production of RiskMetrics volatility and correlation files as well as the methods that are necessary to compute VaR. For easy reference we provide section numbers within each step so that interested readers can learn more about that particular subject.

Reading this chapter requires a basic understanding of statistics. For assistance, readers can refer to the glossary at the end of this document.

1.1 An introduction to Value-at-Risk and RiskMetrics

Value-at-Risk is a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon. VaR answers the question: how much can I lose with *x*% probability over a given time horizon. For example, if you think that there is a 95% chance that the DEM/USD exchange rate will not fall by more than 1% of its current value over the next day, you can calculate the maximum potential loss on, say, a USD 100 million DEM/USD position by using the methodology and data provided by RiskMetrics. The following examples describe how to compute VaR using standard deviations and correlations of financial returns (provided by RiskMetrics) under the assumption that these returns are normally distributed. (RiskMetrics provides alternative methodological choices to address the inacurracies resulting from this simplifying assumption).

- **Example 1:** You are a USD-based corporation and hold a DEM 140 million FX position. What is your VaR over a 1-day horizon given that there is a 5% chance that the realized loss will be greater than what VaR projected? The choice of the 5% probability is discretionary and differs across institutions using the VaR framework.

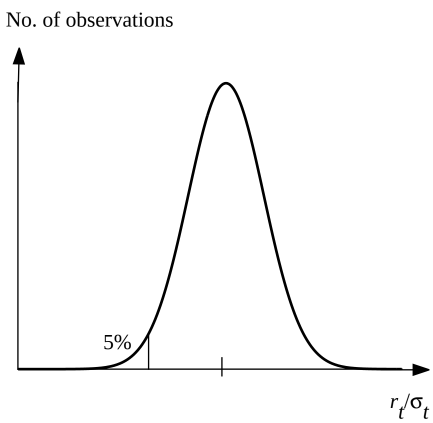
What is your exposure?

The first step in the calculation is to compute your exposure to market risk (i.e., mark-to-market your position). As a USD-based investor, your exposure is equal to the market value of the position in your base currency. If the foreign exchange rate is 1.40 DEM/USD, the market value of the position is USD 100 million.

What is your risk?

Moving from exposure to risk requires an estimate of how much the exchange rate can potentially move. The standard deviation of the return on the DEM/USD exchange rate, measured historically can provide an indication of the size of rate movements. In this example, we calculated the DEM/USD daily standard deviation to be 0.565%. Now, under the standard RiskMetrics assumption that standardized returns ((r_t/σ_t) on DEM/USD are normally distributed given the value of this standard deviation, VaR is given by 1.65 times the standard deviation (that is, 1.65σ) or 0.932% (see Chart 1.1). This means that the DEM/USD exchange rate is not expected to drop more than 0.932%, 95% of the time. **RiskMetrics provides users with the VaR statistics 1.65σ.**

Chart 1.1
VaR statistics



In USD, the VaR of the position[1] is equal to the market value of the position times the estimated volatility or:

FX Risk: \$100 million × 0.932% = \$932,000

What this number means is that 95% of the time, you will not lose more than \$932,000 over the next 24 hours.

[1] This is a simple approximation.

• **Example 2:** Let's complicate matters somewhat. You are a USD-based corporation and hold a DEM 140 million position in the 10-year German government bond. What is your VaR over a 1-day horizon period, again, given that there is a 5% chance of understating the realized loss?

What is your exposure?

The only difference versus the previous example is that you now have both interest rate risk on the bond and FX risk resulting from the DEM exposure. The exposure is still USD 100 million but it is now at risk to two market risk factors.

What is your risk?

If you use an estimate of 10-year German bond standard deviation of 0.605%, you can calculate:

Interest rate risk: $100 million $\times$ 1.65 $\times$ 0.605% = $999,000
FX Risk: $100 million $\times$ 1.65 $\times$ 0.565% = $932,000

Now, the total risk of the bond is not simply the sum of the interest rate and FX risk because the correlation[2] between the return on the DEM/USD exchange rate the return on the 10-year German bond is relevant. In this case, we estimated the correlation between the returns on the DEM/USD exchange rate and the 10-year German government bond to be –0.27. Using a formula common in standard portfolio theory, the total risk of the position is given by:

[1.1] $$\mathrm{VaR} = \sqrt{\sigma^2_{\text{Interest rate}} + \sigma^2_{\text{FX}} + (2 \times \rho_{\text{Interest rate, FX}} \times \sigma_{\text{Interest rate}} \times \sigma_{\text{FX}})}$$

$$\begin{aligned}\mathrm{VaR} &= \sqrt{(0.999)^2 + (0.932)^2 + (2 \times -0.27 \times 0.999 \times 0.932)} \\ &= \$\ 1.168 \text{ million}\end{aligned}$$

To compute VaR in this example, RiskMetrics provides users with the VaR of interest rate component (i.e., 1.65 $\times$ 0.605), the VaR of the foreign exchange position (i.e., 1.65 $\times$ 0.565) and the correlation between the two return series, –0.27.

1.2 A more advanced approach to Value-at-Risk using RiskMetrics

Value-at-Risk is a number that represents the potential change in a portfolio's future value. How this change is defined depends on (1) the horizon over which the portfolio's change in value is measured and (2) the "degree of confidence" chosen by the risk manager.

VaR calculations can be performed without using standard deviation or correlation forecasts. These are simply **one** set of inputs that can be used to calculate VaR, and that RiskMetrics provides for that purpose. The principal reason for preferring to work with standard deviations (volatility) is the strong evidence that the volatility of financial returns is predictable. Therefore, if volatility is predictable, it makes sense to make forecasts of it to predict future values of the return distribution.

[2] Correlation is a measure of how two series move together. For example, a correlation of 1 implies that two series move perfectly together in the same direction.

Suppose we want to compute the Value-at-Risk of a portfolio over a 1-day horizon with a 5% chance that the actual loss in the portfolio's value is greater than the VaR estimate. The Value-at-Risk calculation consists of the following steps.

1. Mark-to-market the current portfolio. Denote this value by V_0.
2. Define the future value of the portfolio, V_1, as $V_1 = V_0 e^r$ where[3] r represents the return on the portfolio over the horizon. For a 1-day horizon, this step is unnecessary as RiskMetrics assumes a 0 return.
3. Make a forecast of the 1-day return on the portfolio and denote this value by $\hat{r}$, such that there is a 5% chance that the actual return will be less than $\hat{r}$. Alternatively expressed,

$$\text{Probability } (r < \hat{r}) = 5\%.$$

4. Define the portfolio's future "worst case" value $\hat{V}_1$, as $\hat{V}_1 = V_0 e^{\hat{r}}$. The Value-at-Risk estimate is simply $V_0 - \hat{V}_1$.

   Notice that the VaR estimate can be written as $V_0\left(1 - e^{\hat{r}}\right)$. In the case that $\hat{r}$ is sufficiently small, $e^{\hat{r}} \approx 1 + \hat{r}$ so that VaR is approximately equal to $V_0\hat{r}$. is approximately equal to $V_0\hat{r}$. The purpose of a risk measurement system such as RiskMetrics is to offer a means to compute $\hat{r}$.

Within this more general framework we use a simple example to demonstrate how the RiskMetrics methodologies and data enable users to compute VaR. Assume the forecast horizon over which VaR is measured is one day and the level of "confidence" in the forecast to 5%. Following the steps outlined above, the calculation would proceed as follows:

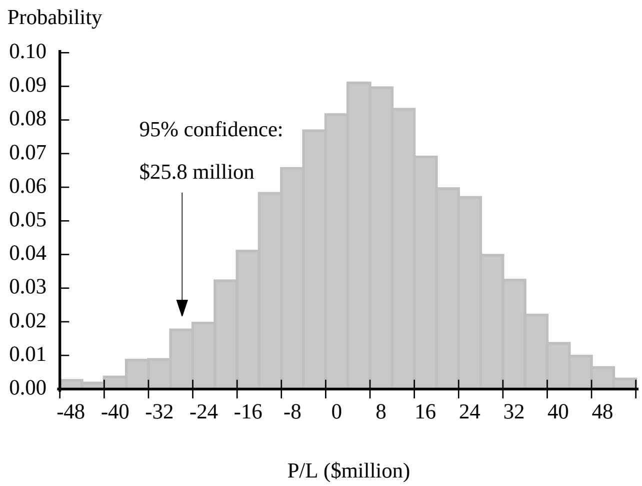
1. Consider a portfolio whose current marked-to-market value, V_0, is USD 500 million.
2. To carry out the VaR calculation we require 1-day forecasts of the mean $\mu_{1|0}$. Within the RiskMetrics framework, we assume that the mean return over a 1-day horizon period is equal to 0.
3. We also need the standard deviation, $\sigma_{1|0}$, of the returns in this portfolio. Assuming that the return on this portfolio is distributed conditionally normal, $\hat{r} = -1.65\sigma_{1|0} + \mu_{1|0}$. The RiskMetrics data set provides the term $1.65\sigma_{1|0}$. Hence, setting $\mu_{1|0} = 0$ and $\sigma_{1|0} = 0.0321$, we get $V_1 = \text{USD } 474.2 \text{ million}$.[4]
4. This yields a Value-at-Risk of USD 25.8 million (given by $V_0 - \hat{V}_1$).

The histogram in Chart 1.2 presents future changes in value of the portfolio. VaR reduces risk to just one number, i.e., a loss associated with a given probability. It is often useful for risk managers to focus on the total distribution of potential gains and losses and we will discuss why this is so later in this document. (See Section 6.3).

[3] Where e is approximately 2.27183

[4] This number is computed from $\left(e^{-1.65\sigma}\right)V_0$

Chart 1.2
Simulated portfolio changes

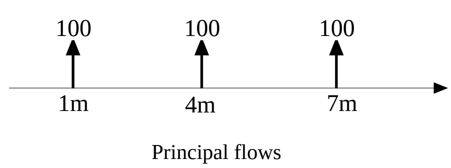


1.2.1 Using RiskMetrics to compute VaR on a portfolio of cash flows

Calculating VaR usually involves more steps than the basic ones outlined in the examples above. Even before calculating VaR, you need to estimate to which risk factors a particular portfolio is exposed. The preferred methodology for doing this is to decompose financial instruments into their basic cash flow components. The RiskMetrics methodology and data allow users to compute the VaR on portfolios consisting of a variety of cash flows. We use a simple example (a portfolio consisting of three cash flows) to demonstrate how to compute VaR.

Step 1. Each financial position in a portfolio is expressed as one or more cash flows that are marked-to-market at current market rates. For example, consider an instrument that gives rise to three USD 100 cash flows each occurring in 1, 4, and 7 months' time as shown in Chart 1.3.

Chart 1.3
Actual cash flows



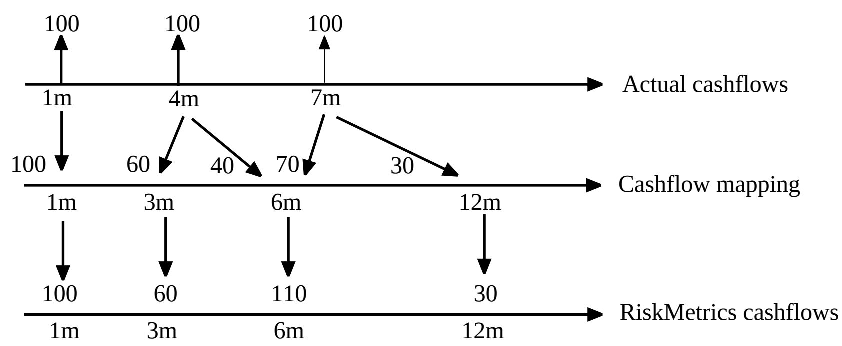
Step 2. When necessary, the actual cash flows are converted to RiskMetrics cash flows by mapping (redistributing) them onto a standard grid of maturity vertices, known as RiskMetrics vertices, which are fixed at the following intervals:

1m 3m 6m 12m 2yr 3yr 4yr 5yr 7yr 9yr 10yr 15yr 20yr 30yr

The purpose of the mapping is to standardize the cash flow intervals of the instrument such that we can use the volatilities and correlations that are routinely computed for the given vertices in the RiskMetrics data sets. (It would be impossible to provide volatility and correlation estimates on every possible maturity so RiskMetrics provides a mapping methodology which distributes cash flows to a workable set of standard maturities). The methodology for mapping cash flows is detailed in Chapter 6.

To map the cash flows, we use the RiskMetrics vertices closest to the actual vertices and redistribute the actual cash flows as shown in Chart 1.4.

Chart 1.4
Mapping actual cash flows onto RiskMetrics vertices



The RiskMetrics cash flow map is used to work backwards to calculate the return for each of the actual cash flows from the cash flow at the associated RiskMetrics vertex, or vertices.

For each actual cash flow, an analytical expression is used to express the relative change in value of the actual cash flow in terms of an underlying return on a particular instrument. Continuing with Chart 1.4, we can write the return on the actual 4-month cash flow in terms of the combined returns on the 3-month (60%) and 6-month (40%) RiskMetrics cash flows:

[1.2] $$r_{4m} = 0.60 r_{3m} + 0.40 r_{6m}$$

where

r_{4m} = return on the actual 4-month cash flow

r_{3m} = return on the 3-month RiskMetrics cash flow

r_{6m} = return on the 6-month RiskMetrics cash flow

Similarly, the return on the 7-month cash flow can be written as

[1.3] $$r_{7m} = 0.70 r_{6m} + 0.30 r_{12m}$$

Note that the return on the actual 1-month cash flow is equal to the return on the 1-month instrument.

Step 3. VaR is calculated at the 5th percentile of the distribution of portfolio return, and for a specified time horizon. In the example above, the distribution of the portfolio return, r_p, is written as:

[1.4] $$r_p = 0.33 r_{1m} + 0.20 r_{3m} + 0.37 r_{6m} + 0.10 r_{12m}$$

where, for example the portfolio weight 0.33 is the result of 100 divided by the total portfolio value 300.

Now, to compute VaR at the 95th percent confidence level we need the fifth percentile of the portfolio return distribution. Under the assumption that r_p is distributed conditionally normal, the fifth percentile is $-1.65\sigma_p$ where σ_p is the standard deviation of the portfolio return distribution. Applying Eq. [1.1] to a portfolio containing more than two instruments requires using simple matrix algebra. We can thus express this VaR calculation as follows:

[1.5] $$VaR = \sqrt{\vec{V} R \vec{V}^T}$$

where $\vec{V}$ is a vector of VaR estimates per instrument,

$$\vec{V} = [\,(0.33 \cdot 1.65\sigma_{1m}),\ (0.20 \cdot 1.65\sigma_{3m}),\ (0.37 \cdot 1.65\sigma_{6m}),\ (0.10 \cdot 1.65\sigma_{12m})\,]\ ,$$

and R is the correlation matrix

[1.6] $$R = \begin{bmatrix} 1 & \rho_{3m,\,1m} & \rho_{6m,\,1m} & \rho_{12m,\,1m} \\ \rho_{1m,\,3m} & 1 & \rho_{6m,\,3m} & \rho_{12m,\,3m} \\ \rho_{1m,\,6m} & \rho_{3m,\,6m} & 1 & \rho_{12m,\,6m} \\ \rho_{1m,\,12m} & \rho_{3m,\,12m} & \rho_{6m,\,12m} & 1 \end{bmatrix}$$

where, for example, $\rho_{1m,\,3m}$ is the correlation estimate between 1-month and 3-month returns.

Note that RiskMetrics provides the vector of information

$$\vec{V} = [\,(1.65\sigma_{1m}),\ (1.65\sigma_{3m}),\ (1.65\sigma_{6m}),\ (1.65\sigma_{12m})\,]$$

as well as the correlation matrix R. What the user has to provide are the actual portfolio weights.

1.2.2 Measuring the risk of nonlinear positions

When the relationship between position value and market rates is nonlinear, then we cannot estimate changes in value by multiplying "estimated changes in rates" by "sensitivity of the position to changing rates;" the latter is not constant (i.e., the definition of a nonlinear position). In our previous examples, we could easily estimate the risk of a fixed income or foreign exchange product by assuming a linear relationship between the value of an instrument and the value of its underlying. This is not a reasonable assumption when dealing with nonlinear products such as options.

RiskMetrics offers two methodologies, an **analytical approximatio**n and a **structured Monte Carlo simulation** to compute the VaR of nonlinear positions:

1. The first method approximates the nonlinear relationship via a mathematical expression that relates the return on the position to the return on the underlying rates. This is done by using what is known as a Taylor series expansion.

   This approach no longer necessarily assumes that the change in value of the instrument is approximated by its delta alone (the first derivative of the option's value with respect to the underlying variable) but that a second order term using the option's gamma (the second derivative of the option's value with respect to the underlying price) must be introduced to measure the curvature of changes in value around the current value. In practice, other "greeks" such as vega (volatility), rho (interest rate) and theta (time to maturity) can also be used to improve the accuracy of the approximation. In Section 1.2.2.1, we present two types of analytical methods for computing VaR—the delta and delta-gamma approximation.

2. The second alternative, structured Monte Carlo simulation, involves creating a large number of possible rate scenarios and revaluing the instrument under each of these scenarios. VaR is then defined as the 5th percentile of the distribution of value changes. Due to the required revaluations, this approach is computationally more intensive than the first approach.

The two methods differ not in terms of how market movements are forecast (since both use the RiskMetrics volatility and correlation estimates) but in how the value of portfolios changes as a result of market movements. The analytical approach approximates changes in value, while the structured Monte Carlo fully revalues portfolios under various scenarios.

Let us illustrate these two methods using a practical example. We will consider throughout this section a portfolio comprised of two assets:

Asset 1: a future cash flow stream of DEM 1 million to be received in one year's time. The current 1-year DEM rate is 10% so the current market value of the instrument is DEM 909,091.

Asset 2: an at-the-money (ATM) DEM put/USD call option with contract size of DEM 1 million and expiration date one month in the future. The premium of the option is 0.0105 and the spot exchange rate at which the contract was concluded is 1.538 DEM/USD. We assume the implied volatility at which the option is priced is 14%.

The value of this portfolio depends on the USD/DEM exchange rate and the one-year DEM bond price. Technically, the value of the option also changes with USD interest rates and the implied volatility, but we will not consider these effects. Our risk horizon for the example will be five days. We take as the daily volatilities of these two assets $\sigma_{FX} = 0.42\%$ and $\sigma_B = 0.08\%$ and as the correlation between the two $\rho = -0.17$.

Both alternatives will focus on price risk exclusively and therefore ignore the risk associated with volatility (vega), interest rate (rho) and time decay (theta risk).

1.2.2.1 Analytical method

There are various ways to analytically approximate nonlinear VaR. This section reviews the two alternatives which we discussed previously.

Delta approximation

The standard VaR approach can be used to come up with first order approximations of portfolios that contain options. (This is essentially the same simplification that fixed income traders use when they focus exclusively on the duration of their portfolio). The simplest such approximation is to estimate changes in the option value via a linear model, which is commonly known as the "delta approximation." Delta is the first derivative of the option price with respect to the spot exchange rate. The value of δ for the option in this example is –0.4919.

In the analytical method, we must first write down the return on the portfolio whose VaR we are trying to calculate. The return on this portfolio consisting of a cash flow in one year and a put on the DEM/call on the USD is written as follows:

[1.7] $$r_p = r_{1y} + r_{\frac{DEM}{USD}} + \delta r_{\frac{DEM}{USD}}$$

where

r_{1p} = the price return on the 1-year German interest rates

$r_{\frac{DEM}{USD}}$ = the return on the DEM/USD exchange rate

δ = the delta of the option

Under the assumption that the portfolio return is normally distributed, VaR at the 95% confidence level is given by

[1.8] $$\text{VaR} = 1.65\sqrt{\sigma_{1y}^2 + (1+\delta)^2\sigma_{\frac{DEM}{USD}}^2 + 2(1+\delta)\rho_{1y,\frac{DEM}{USD}}\sigma_{1y}\sigma_{\frac{DEM}{USD}}}$$

Using our volatilities and correlations forecasts for DEM/USD and the 1-year DEM rate (scaled up to the weekly horizon using the square root of time rule), the weekly VaR for the portfolio using the delta equivalent approach can be approximated by:

	Market value in USD	VaR(1w)
1-yr DEM cash flow	$591,086	$1,745
FX position - FX hedge	$300,331	$4,654
		Diversified VaR
		$4,684

Delta-gamma approximation
The delta approximation is reasonably accurate when the exchange rate does not change significantly, but less so in the more extreme cases. This is because the delta is a linear approximation of a non linear relationship between the value of the exchange rate and the price of the option as shown in Chart 1.5. We may be able to improve this approximation by including the gamma term, which accounts for nonlinear (i.e. squared returns) effects of changes in the spot rate (this attempts to replicate the convex option price to FX rate relationship as shown in Chart 1.5). The expression for the portfolio return is now

[1.9] $$r_p = r_{1y} + r_{\frac{DEM}{USD}} + \delta r_{\frac{DEM}{USD}} + 0.5 \cdot \Gamma P_{\frac{DEM}{USD}}\left(r_{\frac{DEM}{USD}}\right)^2$$

where

$P_{\frac{DEM}{USD}}$ = the value of the DEM/USD exchange rate when the VaR forecast is made

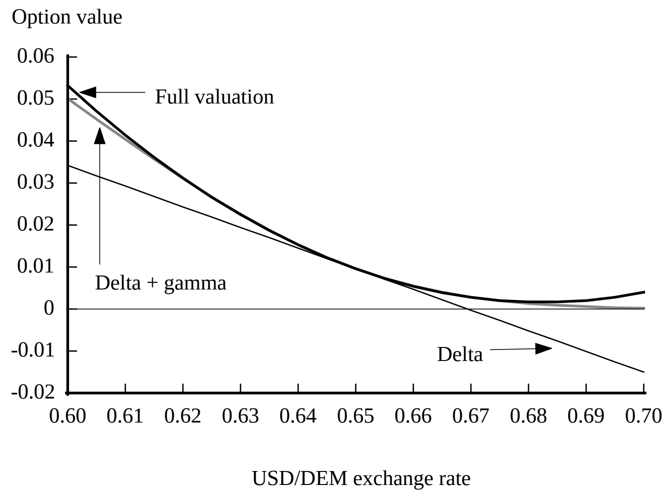
Γ = the gamma of the option.

In this example, Γ = DEM/USD 15.14.

Now, the gamma term (the fourth term in Eq. [1.9]) introduces skewness into the distribution of r_p (i.e., the distribution is no longer symmetrical around its mean). Therefore, since this violates one of the assumptions of normality (symmetry) we can no longer calculate the 95th percentile VaR as 1.65 times the standard deviation of r_p. Instead we must find the appropriate multiple (the counterpart to –1.65) that incorporates the skewness effect. We compute the 5th percentile of r_p 's distribution (Eq. [1.9]) by computing its first four moments, i.e., r_p 's mean, variance, skewness and kurtosis. We then find distribution whose first four moments match those of r_p 's. (See Section 6.3 for details.)

Applying this methodology to this approach we find the VaR for this portfolio to be USD 3,708. Note that in this example, incorporating gamma reduces VaR relative to the delta only approximation (from USD 5006 to USD 3708).

Chart 1.5
Value of put option on USD/DEM
strike = 0.65 USD/DEM. Value in USD/DEM.



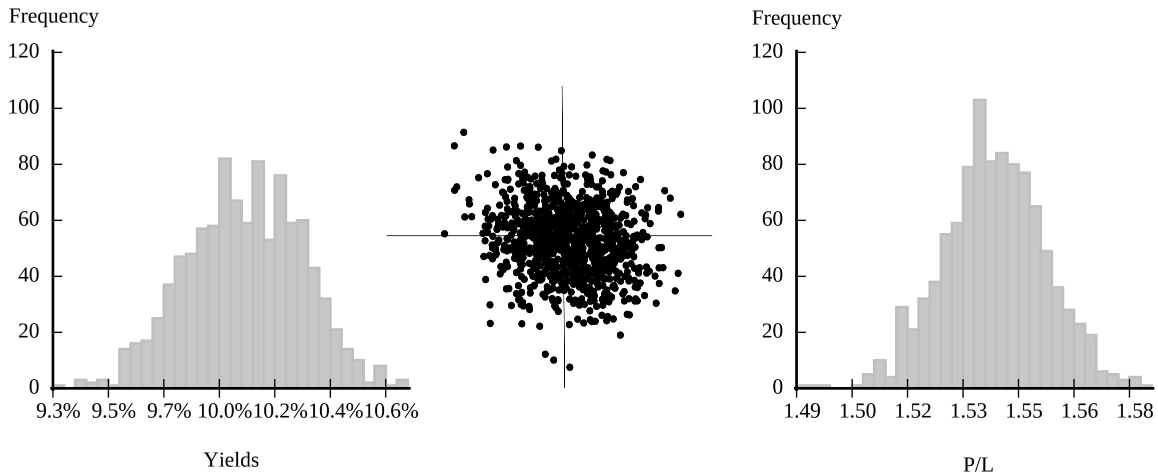
1.2.2.2 Structured Monte-Carlo Simulation
Given the limitations of analytical VaR for portfolios whose P/L distributions may not be symmetrical let alone normally distributed, another possible route is to use a model which instead of estimating changes in value by the product of a rate change (σ) and a sensitivity (δ, Γ), focuses on revaluing positions at changed rate levels. This approach is based on a full valuation precept where all instruments are marked to market under a large number of scenarios driven by the volatility and correlation estimates.

The Monte Carlo methodology consists of three major steps:

1. **Scenario generation —**Using the volatility and correlation estimates for the underlying assets in our portfolio, we produce a large number of future price scenarios in accordance with the lognormal models described previously. The methodology for generating scenarios from volatility and correlation estimates is described in Appendix E.
2. **Portfolio valuation** — For each scenario, we compute a portfolio value.
3. **Summary —** We report the results of the simulation, either as a portfolio distribution or as a particular risk measure.

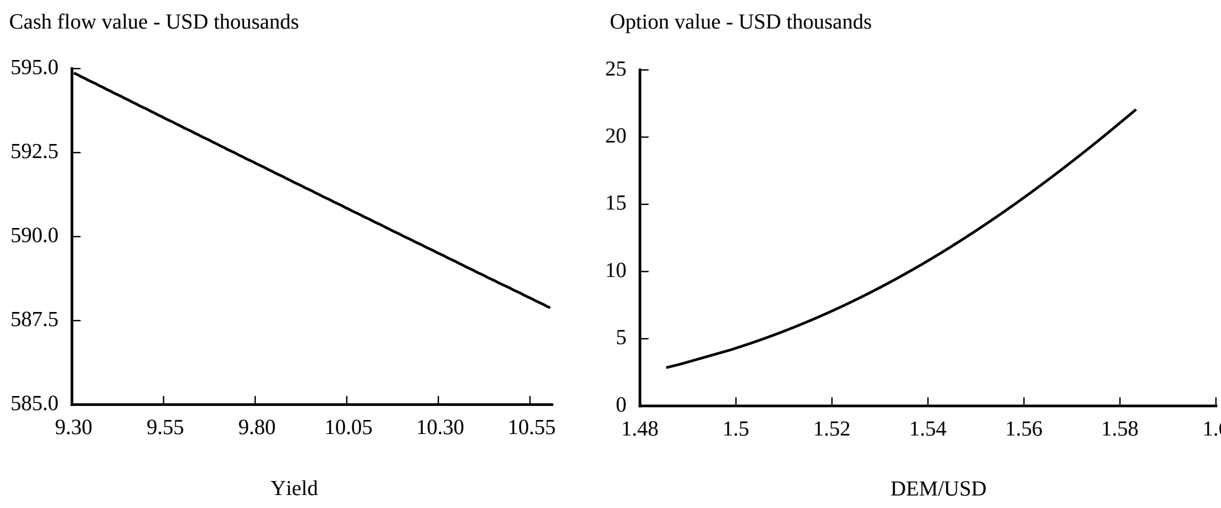
Using our volatility and correlation estimates, we can apply our simulation technique to our example portfolio. We can generate a large number of scenarios (1000 in this example case) of DEM 1-year and DEM/USD exchange rates at the 1-week horizon. Chart 1.6 shows the actual distributions for both instruments as well as the scattergram indicating the degree of correlation (–0.17) between the two rate series.

Chart 1.6
Histogram and scattergram of rate distributions
2-yr DEM rate and DEM/USD rate



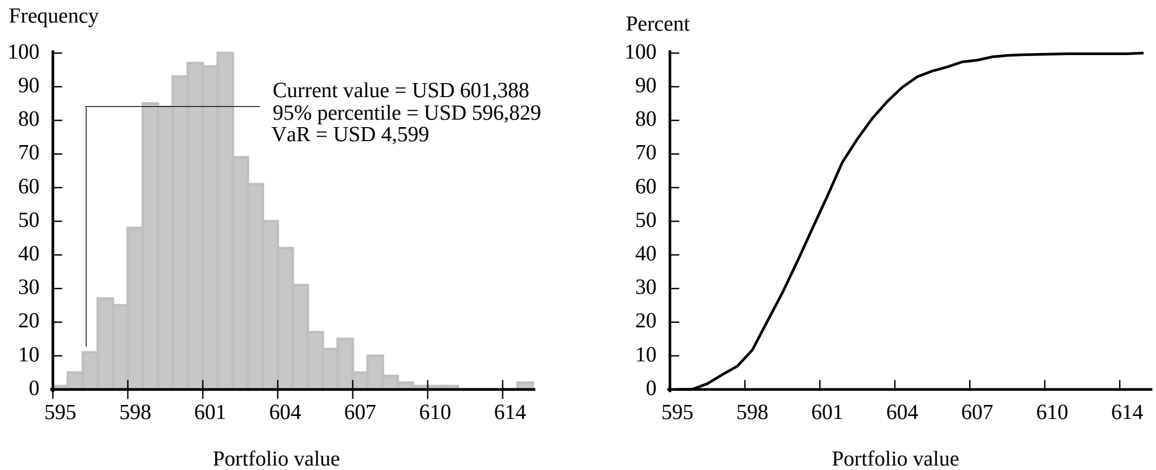
With the set of interest and foreign exchange rates obtained under simulation, we can revalue both of the instruments in our portfolio. Their respective payouts are shown in Chart 1.7.

Chart 1.7
Valuation of instruments in sample portfolio
Value of the cash flow stream *Value of the FX option*



The final task is to analyze the distribution of values and select the VaR using the appropriate percentile. Chart 1.8 shows the value of the components of the portfolio at the end of the horizon period.

Chart 1.8
Representation of VaR
Histogram of portfolio values



The charts above provide a visual indication as to why the delta approximation is usually not suitable for portfolios that contain options. The distribution of returns in portfolios that include options is typically skewed. The standard delta equivalent VaR approach expects symmetry around the mean and applies a basic normal distribution approach (i.e., the 95% percentile equates to a 1.65 standard deviation move). In this case, the lack of symmetry in the distribution does not allow us to apply the normal approximation. Furthermore, the distribution's skewness results in a VaR number that is basically position dependent (i.e., the risk is different whether you are long or short the option).

1.3 What RiskMetrics provides

As discussed previously, RiskMetrics has three basic components which are detailed below.

1.3.1 An overview

With RiskMetrics J.P. Morgan and Reuters provide

1. A set of methodologies for statistical market risk measures that are based on, but differ significantly from, the methodology developed and used within J.P. Morgan. This approach was developed so as to enable other financial institutions, corporate treasuries, and investors to estimate their market risks in a consistent and reasonable fashion. Methodology defines how positions are to be mapped and how potential market movements are estimated and is detailed in the following chapters.

2. Daily recomputed data sets which are comprehensive sets of consistently estimated instrument level VaRs (i.e., 1.65 standard deviations) and correlations across a large number of asset classes and instruments. We currently distribute three different data sets over the Internet: one for short term trading risks, the second for intermediate term investment risks and the third for regulatory reporting. These are made available to the market free of charge.

   In the near future, a more customizable version of RiskMetrics where users will be able to create covariance matrices from a large underlying database according to various numerical methods will be made available over the Reuters Web. This product will not replace the data sets available over the Internet but will provide subscribers to the Reuters services with a more flexible tool.

The four basic classes of instruments that RiskMetrics methodology and data sets cover are represented as follows:

- Fixed income instruments are represented by combinations of amounts of cash flows in a given currency at specified dates in the future. RiskMetrics applies a fixed number of dates (14 vertices) and two types of credit standings: government and non-government. The data sets associated with fixed income are zero coupon instrument VaR statistics, i.e., 1.65σ, and correlations for both government and swap yield curves.

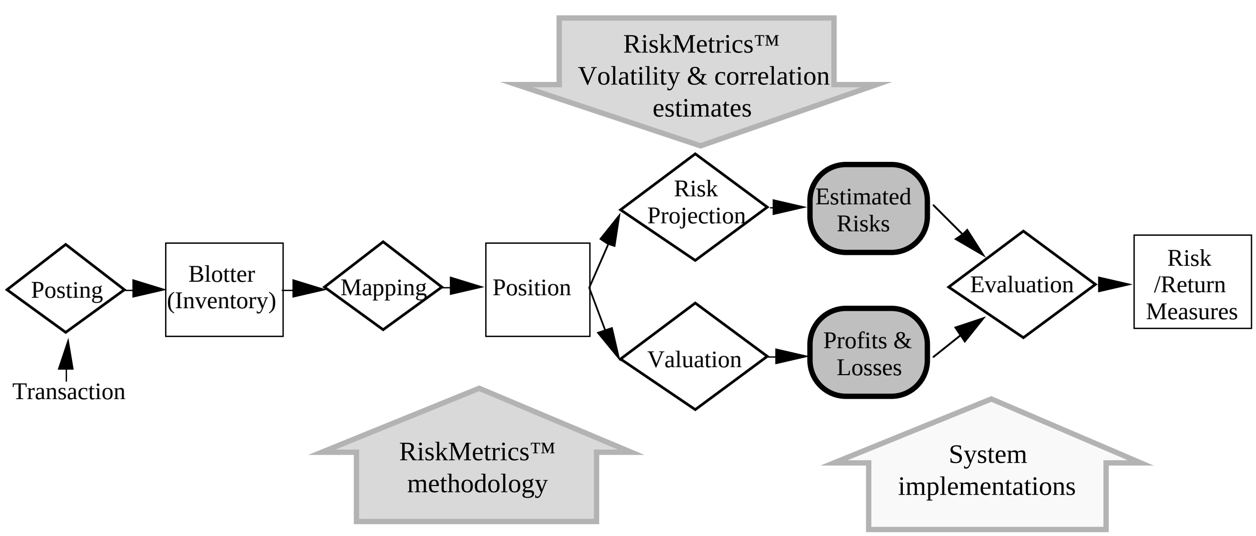
- Foreign exchange transactions are represented by an amount and two currencies. RiskMetrics allows for 30 different currency pairs (as measured against the USD).

- Equity instruments are represented by an amount and currency of an equity basket index in any of 30 different countries. Currently, RiskMetrics does not consider the individual characteristics of a company stock but only the weighted basket of companies as represented by the local index.

- Commodities positions are represented by amounts of selected standardized commodity futures contracts traded on commodity exchanges

3. Software provided by J.P. Morgan, Reuters and third party firms that use the RiskMetrics methodology and data documented herein.

Chart 1.9
Components of RiskMetrics



Since the RiskMetrics methodology and the data sets are in the public domain and freely available, anyone is free to implement systems utilizing these components of RiskMetrics. Third parties have developed risk management systems for a wide range of clients using different methodologies. The following paragraphs provide a taxonomy comparing the different approaches.

1.3.2 Detailed specification

The section below provides a brief overview of how the RiskMetrics datasets are produced and how the parameters we provide can be used in a VaR calculation.

1.3.2.1 Production of volatility and correlation data sets

RiskMetrics provides the following sets of volatility and corresponding correlation data files. One set is for use in estimating VaR with a forecast horizon of one day. The other set is optimized for a VaR forecast horizon of one month. The third set is based on the quantitative criteria set by the Bank for International Settlements on the use of VaR models to estimate the capital required to cover market risks. The process by which these data files are constructed are as follows:

1. Financial prices are recorded from global data sources. (In 1997, RiskMetrics will switch to using Reuters data exclusively). For certain fixed income instruments we construct zero rates. See Chapter 9 for data sources and RiskMetrics building blocks.
2. Fill in missing prices by using the Expectation Maximization algorithm (detailed in Section 8.2). Prices can be missing for a variety of reasons, from technical failures to holiday schedules.
3. Compute daily price returns on all 480 time series (Section 4.1).
4. Compute standard deviations and correlations of financial price returns for a 1-day VaR forecast horizon. This is done by constructing exponentially weighted forecasts. (See Section 5.2). Production of the daily statistics also involves setting the sample daily mean to zero. (See Section 5.3). If data is recorded at different times (Step 1), users may require an adjustment algorithm applied to the correlation estimates. Such an algorithm is explained in Section 8.5. Also, users who need to rebase the datasets to account for a base currency other than the USD should see Section 8.4.
5. Compute standard deviations and correlations of financial price returns for 1-month VaR forecast horizon. This is done by constructing exponentially weighted forecasts (Section 5.3). Production of the monthly statistics also involves setting the sample daily mean to zero.

1.3.2.2 RiskMetrics VaR calculation

1. The first step in the VaR calculation is for the user to define three parameters: (1) VaR forecast horizon—the time over which VaR is calculated, (2) confidence level—the probability that the realized change in portfolio will be less than the VaR prediction, and (3) the base currency.
2. For a given portfolio, once the cash flows have been identified and marked-to-market (Section 6.1) they need to be mapped to the RiskMetrics vertices (Section 6.2).
3. Having mapped all the positions, a decision must be made as to how to compute VaR. If the user is willing to assume that the portfolio return is approximately conditionally normal, then download the appropriate data files (instrument level VaRs and correlations) and compute VaR using the standard RiskMetrics approach (Section 6.3).
4. If the user's portfolio is subject to nonlinear risk to the extent that the assumption of conditional normality is no longer valid, then the user can choose between two methodologies—delta-gamma and structured Monte Carlo. The former is an approximation of the latter. See Section 6.3 for a description of delta-gamma and Chapter 7for an explanation of structured Monte Carlo.

Chapter 2. Historical perspective of VaR

Chapter 2. Historical perspective of VaR

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Measuring the risks associated with being a participant in the financial markets has become the focus of intense study by banks, corporations, investment managers and regulators. Certain risks such as counterparty default have always figured at the top of most banks' concerns. Others such as market risk (the potential loss associated with market behavior) have only gotten into the limelight over the past few years. Why has the interest in market risk measurement and monitoring arisen? The answer lies in the significant changes that the financial markets have undergone over the last two decades.

1. Securitization: Across markets, traded securities have replaced many illiquid instruments, e.g., loans and mortgages have been securitized to permit disintermediation and trading. Global securities markets have expanded and both exchange traded and over-the-counter derivatives have become major components of the markets.

   These developments, along with technological breakthroughs in data processing, have gone hand in hand with changes in management practices—a movement away from management based on accrual accounting toward risk management based on marking-to-market of positions. Increased liquidity and pricing availability along with a new focus on trading led to the implementation of frequent revaluation of positions, the mark-to-market concept.

   As investments became more liquid, the potential for frequent and accurate reporting of investment gains and losses has led an increasing number of firms to manage daily earnings from a mark-to-market perspective. The switch from accrual accounting to mark-to-market often results in higher swings in reported returns, therefore increasing the need for managers to focus on the volatility of the underlying markets. The markets have not suddenly become more volatile, but the focus on risks through mark-to-market has highlighted the potential volatility of earnings.

   Given the move to frequently revalue positions, managers have become more concerned with estimating the potential effect of changes in market conditions on the value of their positions.

2. Performance: Significant efforts have been made to develop methods and systems to measure financial performance. Indices for foreign exchange, fixed income securities, commodities, and equities have become commonplace and are used extensively to monitor returns within and/or across asset classes as well as to allocate funds.

   The somewhat exclusive focus on returns, however, has led to incomplete performance analysis. Return measurement gives no indication of the cost in terms of risk (volatility of returns). Higher returns can only be obtained at the expense of higher risks. While this trade-off is well known, the risk measurement component of the analysis has not received broad attention.

   Investors and trading managers are searching for common standards to measure market risks and to estimate better the risk/return profile of individual assets or asset classes. Notwithstanding the external constraints from the regulatory agencies, the management of financial firms have also been searching for ways to measure market risks, given the potentially damaging effect of miscalculated risks on company earnings. As a result, banks, investment firms, and corporations are now in the process of integrating measures of market risk into their management philosophy. They are designing and implementing market risk monitoring systems that can provide management with timely information on positions and the estimated loss potential of each position.

Over the last few years, there have been significant developments in conceptualizing a common framework for measuring market risk. The industry has produced a wide variety of indices to measure return, but little has been done to standardize the measure of risk. Over the last 15 years many market participants, academics, and regulatory bodies have developed concepts for measuring market risks. Over the last five years, two approaches have evolved as a means to measure market risk. The first approach, which we refer to as the statistical approach, involves forecasting a portfolio's return distribution using probability and statistical models. The second approach is referred to as scenario analysis. This methodology simply revalues a portfolio under different values of market rates and prices. Note that in stress scenario analysis does not necessarily require the use of a probability or statistical model. Instead, the future rates and prices that are used in the revaluation can be arbitrarily chosen. Risk managers should use both approaches—the statistical approach to monitor risks continuously in all risk-taking units and the scenario approach on a case-by-case basis to estimate risks in unique circumstances. **This document explains, in detail, the statistical approach—RiskMetrics—to measure market risk.**

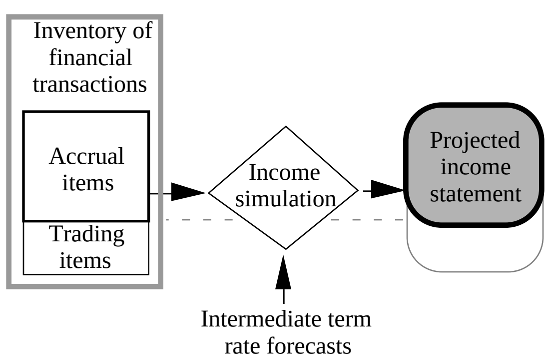
This chapter is organized as follows:

- Section 2.1 reviews how VaR was developed to support the risk management needs of trading activities as opposed to investment books. Though the distinction to date has been an accounting one not an economic one, VaR concepts are now being used across the board.
- Section 2.2 looks at the basic steps of the risk monitoring process.
- Section 2.3 reviews the alternative VaR models currently being used and how RiskMetrics provides end-users with the basic building blocks to test different approaches.

2.1 From ALM to VaR

A well established method of looking at market risks in the banking industry is to forecast earnings under predetermined price/rate market conditions (or scenarios). Earnings here are defined as earnings reported in a firm's Financial Statements using generally accepted accounting principles. For many institutions the bulk of activities are reported on an accrual basis, i.e., transactions are booked at historical costs +/- accruals. Only a limited number of trading items are marked to market. Because changes in market rates manifest themselves only slowly when earnings are reported on an accrual basis, the simulation of income has to be done over extended periods, i.e., until most of the transactions on the books mature. Chart 2.1 illustrates this conventional Asset/Liability Management approach.

Chart 2.1
Asset liability management



There are two major drawbacks to this methodology:

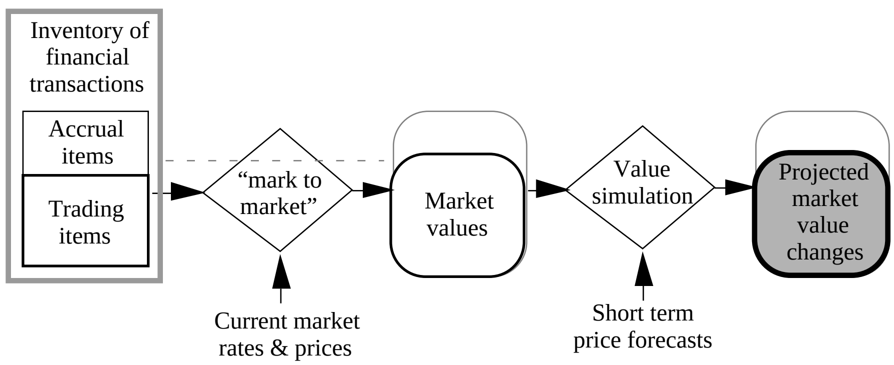
- It requires projecting market rate developments over extended periods into the future.

- It supports the illusion that gains and losses occur at the time they show up in the accrual accounts (i.e., when they are realized following accounting principles). What this means is that return is only defined as net interest earnings, a framework which ignores the change in price component of the return function.

  Every investor would agree that the total return on a bond position is the sum of the interest earned and the change in the value of the bond over a given time horizon. Traditional ALM, as a result of accounting conventions, ignores the change in value of the instrument since positions are not marked to market. This has often lead crafty ALM managers to create positions which look attractive on paper because of high net interest earnings, but which would not perform as well if their change in market value were considered.

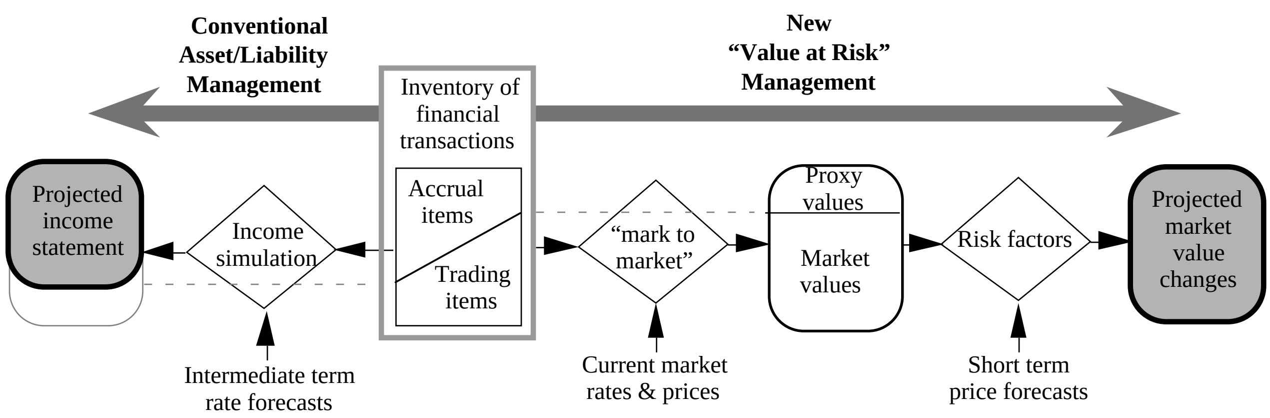
The market risk in trading positions is usually measured differently and managed separately. Trading positions are marked-to-market and the market value is then subjected to projections of changes in short term in rates and prices. This is much less hazardous as rate forecasts are usually limited to short horizons, i.e., the time it should take to close out or hedge the trading position.

Chart 2.2
Value-at-Risk management in trading



The distinction between accrual items and trading items and their separate treatment for market risk management has led to significant complications—particularly when transactions classified as “trading items” under generally accepted accounting principles are used to hedge transactions classified as “accrual items”. In an effort to overcome this difficulty, many firms – particularly those with relatively large trading books have expanded the market risk approach to also include accrual items, at least for internal risk management reporting. This is done by estimating the fair market value of the accrual items and the changes in their fair value under different short term scenarios. Thus we are witnessing the evolution of an alternative to the conventional approach of Asset/Liability Management, the Value-at-Risk approach. It started in pure trading operations, but is now gaining increased following in the financial industry.

Chart 2.3
Comparing ALM to VaR management



The advantages of VaR Management are that it

- Incorporates the mark-to-market approach uniformly.
- Relies on a much shorter horizon forecast of market variables. This improves the risk estimate as short horizon forecasts tend to be more accurate than long horizon forecasts.

Of course, drawbacks exist. One of them is that it may not be trivial to mark certain transactions to market or even understand their behavior under certain rate environments. This is particularly true for instruments such as demand deposits in a retail banking environment for example. Whatever the difficulties, the aim of getting an integrated picture of a firm's exposure to market risks is worth a number of assumptions, some of which may be reasonable representations of reality. In the case of demand deposits, a recent article by Professor Robert Jarrow outlines how power swaps could be modelled to represent a retail bank's core deposit base risks (RISK, February 1996).

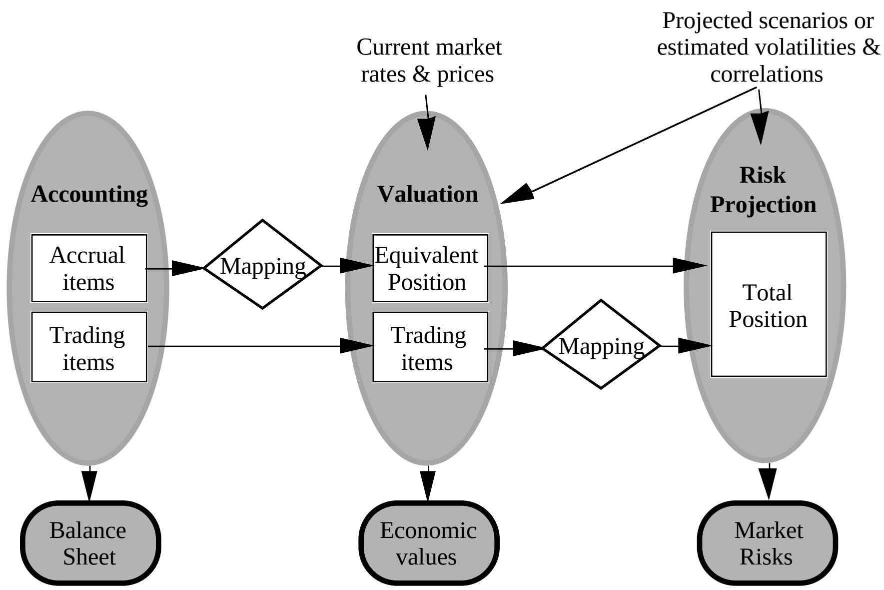
Some critics also argue that marking-to-market all transactions over short time periods creates too much "earnings" or volatility. Looking at risks in this fashion may be misleading. This is the direction of the industry and its accounting regulators however and it will be up to financial analysts to adapt to the new environment. The volatility of earnings will not just appear out of the blue. The changes in accounting practices will ultimately show economic reality as it really is.

Market risk can be absolute or relative. In its absolute form, what is measured is the loss in the value of a position or a portfolio resulting from changes in market conditions. Absolute market risk is what managers of trading operations measure. Corporates who wish to estimate real potential losses from their treasury operations also focus on absolute market risk. Regulators are interested in absolute market risks in relation to a firm's capital. When investment performance is measured against an index, the inherent market risk is relative in the sense that it measures the potential underperformance against a benchmark.

2.2 VaR in the framework of modern financial management

As discussed before there are two steps to VaR measurement. First, all positions need to be marked to market (valuation). Second we need to estimate the future variability of the market value. Chart 2.4 illustrates this point.

Chart 2.4
Two steps beyond accounting



2.2.1 Valuation

Trading items are valued at their current prices/rates as quoted in liquid secondary markets. To value transactions for which, in the absence of a liquid secondary market, no market value exists, we first map them into equivalent positions, or decompose them into parts for which secondary market prices exist. The most basic such "part" is a single cash flow with a given maturity and currency of the payor. Most transactions can be described as a combination of such cash flows and thus can be valued approximately as the sum of market values of their component cash flows.

Only non-marketable items that contain options cannot be valued in this simple manner. For their valuation we also need expected volatilities and correlations of the prices and rates that affect their value, and we need an options pricing model. Volatilities describe potential movements in rates with a given probability; correlations describe the interdependencies between different rates and prices. Thus, for some valuations, we require volatilities and correlations.

2.2.2 Risk estimation

Here we estimate value changes as a consequence of expected changes in prices and rates. The potential changes in prices are defined by either specific scenarios or a set of volatility and correlation estimates. If the value of a position depends on a single rate, then the potential change in value is a function of the rates in the scenarios or volatility of that rate. If the value of a position depends on multiple rates, then the potential change in its value is a function of the combination of rates in each scenario or of each volatility and of each correlation between all pairs of rates.

Generating equivalent positions on an aggregate basis facilitates the simulation. As will be shown later, the simulation can be done algebraically (using statistics and matrix algebra), or exhaustively by computing estimated value changes for many combinations of rate changes.

In the RiskMetrics framework, forecasts of volatilities and correlations play a central role. They are required for valuations in the case of derivatives, the critical inputs for risk estimation.

2.3 Alternative approaches to risk estimation

More than one VaR model is currently being used and most practitioners have selected an approach based on their specific needs, the types of positions they hold, their willingness to trade off accuracy for speed (or vice versa), and other considerations.

The different models used today differ on basically two fronts:

- How the changes in the values of financial instruments are estimated as a result of market movements.
- How the potential market movements are estimated.

What makes the variety of models currently employed is the fact that the choices made on the two fronts mentioned above can be mixed and matched in different ways.

2.3.1 Estimating changes in value

There are basically two approaches to estimating how the value of a portfolio changes as a result of market movements: analytical methods and simulation methods.

2.3.1.1 Analytical methods

One such method is the analytical sensitivity approach based on the following equation:

$$\text{estimated value change} = f\,(\text{position sensitivity, estimated rate/price change})$$

where the position sensitivity factor establishes the relationship between the value of the instrument and of the underlying rate or price, and determines the accuracy of the risk approximation.

In its simplest form, the analytical sensitivity approach looks like this:

$$\text{estimated value change} = \text{position sensitivity} \times \text{estimated rate change}$$

For example, the value change of a fixed income instrument can be estimated by using the instrument's duration. Although this linear approximation simplifies the convex price/yield relationship of a bond, it is extensively used in practice because duration often accounts for the most significant percentage of the risk profile of a fixed income instrument. Similar simplifications can be made for options where the estimated change in value is approximated by the option's delta.

The initial versions of RiskMetrics basically used an analytical VaR approach that assumed that market risk could be estimated by using a simple first-order approximation such as the one outlined above. We have since extended the analytical approach to account for nonlinear relationships between market value and rate changes (e.g., options), which requires accounting for gamma risk in addition to delta risk. The more refined version of the analytical approach looks like this:

$$\begin{aligned}\text{estimated value change} &= (\text{position sensitivity 1} \times \text{estimated rate change}) \\ &\quad + 1/2\,(\text{position sensitivity 2}) \times (\text{estimated rate change})^2 + \ldots\end{aligned}$$

In the case of an option, the first-order position sensitivity is the delta, while the second-order term is the gamma. Higher order effects can also be estimated using an analytical approach, but the math typically gets more complex.

The analytical approach requires that positions be summarized in some fashion so that the estimated rate changes can be applied. This process of aggregating positions is called mapping and is described in Chapter 6.

The advantages of analytical models is that they are computationally efficient and enable users to estimate risk in a timely fashion.

2.3.1.2 Simulation methods

The second set of approaches, typically referred to as Full Valuation models rely on revaluing a portfolio of instruments under different scenarios. How these scenarios are generated varies across models, from basic historical simulation to distributions of returns generated from a set of volatility and correlation estimates such as RiskMetrics. Some models include user-defined scenarios that are based off of major market events and which are aimed at estimating risk in crisis conditions. This process is often referred to a stress testing.

Full Valuation models typically provide a richer set of risk measures since users are able to focus on the entire distribution of returns instead of a single VaR number. Their main drawback is the fact that the full valuation of large portfolios under a significant number of scenarios is computationally intensive and takes time. It may not be the preferred approach when the goal is to provide senior management with a timely snapshot of risks across a large organization.

2.3.2 Estimating market movements

The second discriminant between VaR approaches is how market movements are estimated. There is much more variety here and the following list is not an exhaustive list of current practice.

RiskMetrics

RiskMetrics uses historical time series analysis to derive estimates of volatilities and correlations on a large set of financial instruments. It assumes that the distribution of past returns can be modelled to provide us with a reasonable forecast of future returns over different horizons.

While RiskMetrics assumes conditional normality of returns, we have refined the estimation process to incorporate the fact that most markets show kurtosis and leptokurtosis. We will be publishing factors to adjust for this effect once the RiskMetrics customizable data engine becomes available on the Reuters Web.

These volatility and correlation estimates can be used as inputs to:

- Analytical VaR models

- Full valuation models. In Appendix E we outline how the RiskMetrics volatility and correlation data sets can be used to drive simulations of future returns.

Historical Simulation

The historical simulation approach, which is usually applied under a full valuation model, makes no explicit assumptions about the distribution of asset returns. Under historical simulation, portfolios are valued under a number of different historical time windows which are user defined. These lookback periods typically range from 6 months to 2 years.

Once the RiskMetrics customizable data engine becomes available on the ReutersWeb, users will be able to access the underlying historical data needed to perform this type of simulation.

Monte Carlo Simulation

While historical simulation quantifies risk by replicating one specific historical path of market evolution, stochastic simulation approaches attempt to generate many more paths of market returns. These returns are generated using a defined stochastic process (for example, assume that interest rates follow a random walk) and statistical parameters that drive the process (for example, the mean and variance of the random variable).The RiskMetrics data sets can be used as inputs to this process.

In addition, the following VaR models add refinements to the results generated by the approaches listed above.

Implied volatilities

Some practitioners will also look to the market to provide them with an indication of future potential return distributions. Implied volatility as extracted from a particular option pricing model is the market's forecast of future volatility. Implied volatilities are often used in comparison to history to refine the risk analysis.

Implied volatilities are not currently used to drive global VaR models as this would require observable options prices on all instruments that compose a portfolio. Unfortunately, the universe of consistently observable options prices is not yet large enough; generally only exchange traded options are reliable sources of prices. In particular, the number of implied correlations that can be derived from traded options prices is insignificant compared to the number of correlations required to estimate risks in portfolios containing many asset types.

User-defined scenarios

Most risk management models add user-defined rate and price movements to the standard VaR number, if only to test the effect of what could happen if historical patterns do not repeat themselves. Some scenarios are subjectively chosen while others recreate past crises events. The latter is referred to as stress testing and is an integral part of a well designed risk management process.

Selecting the appropriate measurement method is not, however, straightforward. Judgment in the choice of methodologies will always be important. Cost benefit trade-offs are different for each user, depending on his position in the markets, the number and types of instruments traded, and the technology available. Different choices can be made even at different levels of an organization, depending on the objectives. While trading desks of a bank may require precise risk estimation involving simulation on relatively small portfolios, senior management may opt for an analytical approach that is cost efficient and timely. It is important for senior management to know whether the risk of the institution is $10 million or $50 million. It is irrelevant for them to make the distinction between $10 million and $11 million. Achieving this level of accuracy at the senior management level is not only irrelevant, but can also be unachievable operationally, or at a cost which is not consistent with shareholder value.

Since its introduction, RiskMetrics has become the umbrella name for a series of VaR methodologies, from the simple analytical estimation based on the precept that all instruments are linear (the so-called delta approximation) to the structured Monte Carlo simulation.

Not all participants with exposure to the financial and commodities markets will have the resources to perform extensive simulations. That is why we have strived in this update of the *RiskMetrics— Technical Document* to refine analytical approximations of risk for non-linear instruments (the delta-gamma approximations). During 1997, the availability of historical rates and prices under the RiskMetrics customizable data engine will make historical simulation an option for users of our products.

Table 2.1
Two discriminating factors to review VaR models

		How to estimate the change in the value of instruments		
			Analytical	**Full Valuation**
How to estimate rate and price changes	**Full VaR model**	RiskMetrics	Covariance matrices applied to standard instrument maps.	Covariance matrices used to define scenarios for structured Monte Carlo.
		Historical simulation	Not applicable.	Portfolios revalued under historical return distributions (lookback period varies.
		Monte Carlo	Not applicable.	Statistical parameters determine stochastic processes. Sources of data vary (can include RiskMetrics covariance matrices).
	Partial VaR model	Implied volatilities	Covariance matrices applied to standard instrument maps.	Covariance matrices used to define scenarios for structured Monte Carlo.
		User defined	Sensitivity analysis on single instruments.	Limited number of scenarios.

Chapter 3. Applying the risk measures

Chapter 3. Applying the risk measures

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The measures of market risk outlined in the preceding sections can have a variety of applications. We will highlight just a few:

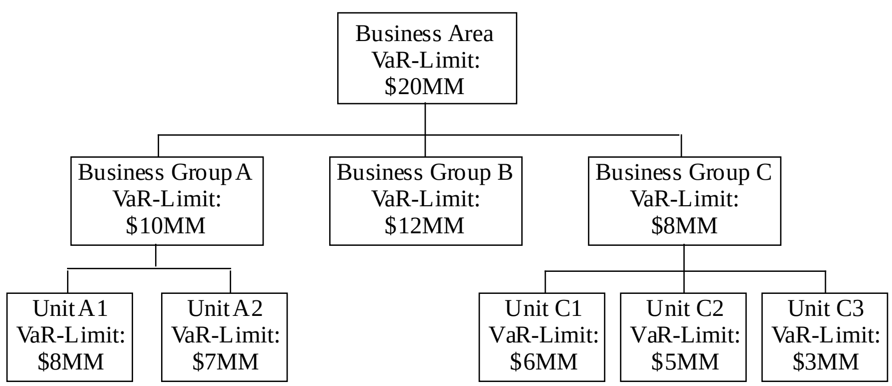
- To measure and compare market risks.
- To check the valuation/risk models.
- To evaluate the performance of risk takers on a return/risk basis.
- To estimate capital levels required to support risk taking.

3.1 Market risk limits

Position limits have traditionally been expressed in nominal terms, futures equivalents or other denominators unrelated to the amount of risk effectively incurred. The manager of a USD bond portfolio will be told for example that he cannot hold more than 100 million USD worth of U.S. Treasury bonds. In most cases, the measure contains some risk constraint expressed in a particular maturity or duration equivalent (if the 100 million limit is in 2-year equivalents, the manager will not be able to invest 100 million in 30-year bonds). Setting limits in terms of Value-at-Risk has significant advantages: position benchmarks become a function of risk and positions in different markets while products can be compared through this common measure. A common denominator rids the standard limits manuals of a multitude of measures which are different for every asset class. Limits become meaningful for management as they represent a reasonable estimate of how much could be lost.

A further advantage of Value-at-Risk limits comes from the fact that VaR measures incorporate portfolio or risk diversification effects. This leads to hierarchical limit structures in which the risk limit at higher levels can be lower than the sum of risk limits of units reporting to it.

Chart 3.1
Hierarchical VaR limit structure

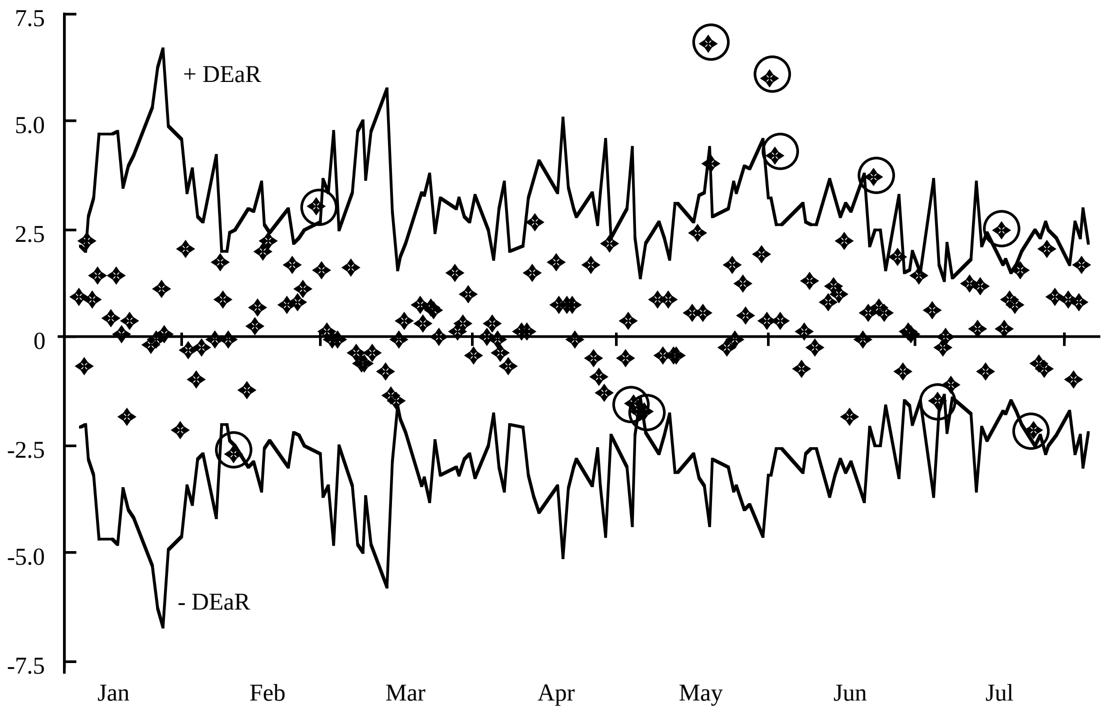


Setting limits in terms of risk helps business managers to allocate risk to those areas which they feel offer the most potential, or in which their firms' expertise is greatest. This motivates managers of multiple risk activities to favor risk reducing diversification strategies.

3.2 Calibrating valuation and risk models

An effective method to check the validity of the underlying valuation and risk models is to compare DEaR estimates with realized daily profits and losses over time. Chart 3.2 illustrates the concept. The stars show the daily P&L of a global trading business during the first 7 months of 1993, the two lines show the Daily Earnings at Risk, plus and minus.

Chart 3.2
Ex post validation of risk models: DEaR vs. actual daily P&L



By definition, the cone delimited by the +/–DEaR lines should contain 90% of all the stars, because DEaR is defined as the maximum amount of expected profit or losses 90% of the time. If there are substantially more than 10% of the stars outside the DEaR-cone, then the underlying models underestimate the risks. If there are no stars outside the DEaR cone and not even close to the lines, then the underlying models overestimate the risks.

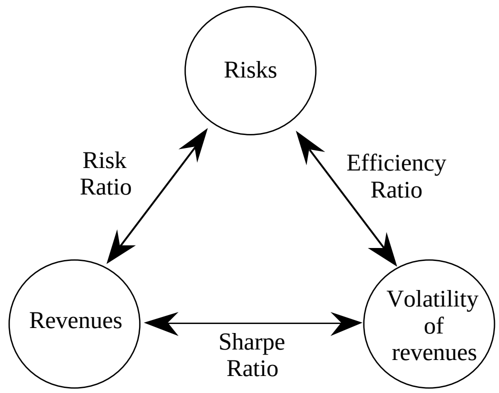
This type of chart is only a reasonable reflection of the risk statistics if the daily profit and losses are derived solely from overnight risk taking and not intraday trading and other activities. Often this is not the case. Then instead of the daily P&L you should plot what is often referred to as the "no-action-P&L"; it describes the hypothetical P&L on the position that would have been incurred if the previous day's closing position had been kept for the next 24 hours and then revalued. This data is often difficult to collect.

3.3 Performance evaluation

To date, trading and position taking talent have been rewarded to a significant extent on the basis of total returns. Given the high rewards bestowed on outstanding trading talent this may bias the trading professionals towards taking excessive risks.It is often referred to as giving traders a free option on the capital of your firm. The interest of the firm or capital provider may be getting out of line with the interest of the risk taking individual unless the risks are properly measured and returns are adjusted for the amount of risk effectively taken.

To do this correctly one needs a standard measure of risks. Ideally risk taking should be evaluated on the basis of three interlinked measures: revenues, volatility of revenues, and risks. This is illustrated by Chart 3.3:

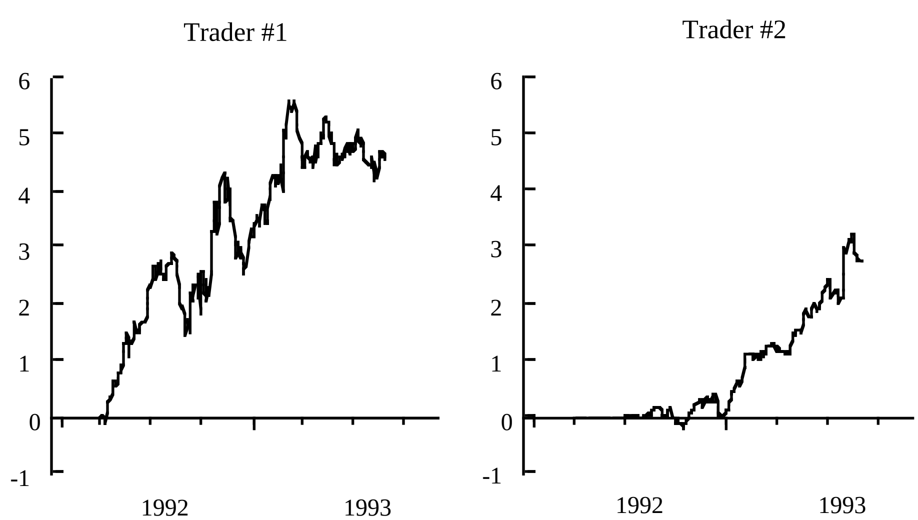
Chart 3.3
Performance evaluation triangle



Including estimated (ex ante) and realized (ex post) volatility of profits adds an extra dimension to performance evaluation. The ratio of P&L over risk (risk ratio) and of P&L over volatility (Sharpe ratio) can be combined into what we define as a trader's efficiency ratio (estimated risk/realized volatility) that measures an individual's capacity to translate estimated risk into low realized volatility of revenues.

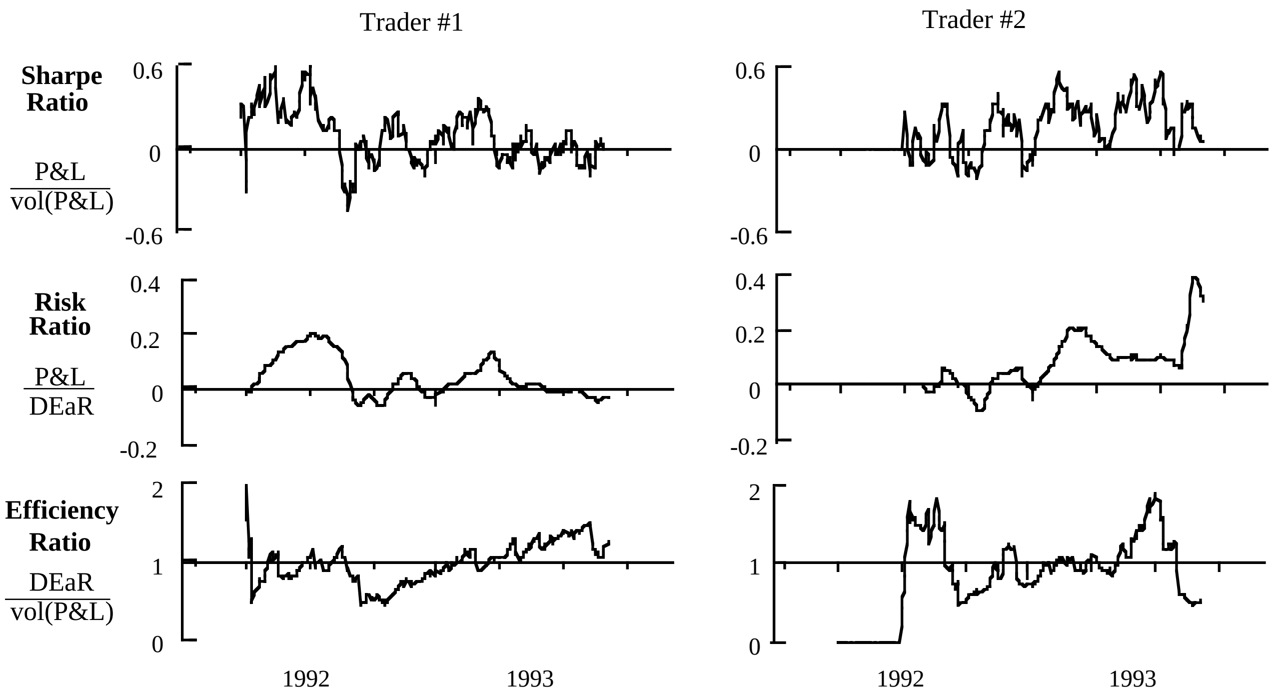
Consider an example to illustrate the issue. Assume you have to evaluate Trader #1 relative to Trader #2 and the only information on hand is the history of their respective cumulative trading revenues (i.e., trading profits). This information allows you to compare their profits and volatility of their profits, but says nothing about their risks.

Chart 3.4
Example: comparison of cumulative trading revenues
cumulative revenues



With risk information you can compare the traders more effectively. Chart 3.5 shows, for the two traders the risk ratio, sharpe ratio, and efficiency ratio over time.

Chart 3.5
Example: applying the evaluation triangle



Note, you have no information on the type of market these traders operate in or the size of positions they have taken. Nevertheless Chart 3.5 provides interesting comparative information which lead to a richer evaluation.

3.4 Regulatory reporting, capital requirement

Financial institutions such as banks and investment firms will soon have to meet capital requirements to cover the market risks that they incur as a result of their normal operations. Currently the driving forces developing international standards for market risk based capital requirements are the European Community which issued a binding Capital Adequacy Directive (EC-CAD) and the Basel Committee on Banking Supervision at the Bank for International Settlements (Basel Committee) which has recently come out with revised proposals on the use of banks internal models. (See Appendix F for more information.)

3.4.1 Capital Adequacy Directive

The European Union's EEC 93/6 directive mandates banks and investment firms to set capital aside to cover market risks. In a nutshell the EC-CAD computes the capital requirement as a sum of capital requirements on positions of different types in different markets. It does not take into account the risk reducing effect of diversification. As a result, the strict application of the current recommendations will lead to financial institutions, particularly the ones which are active internationally in many different markets, to overestimate their market risks and consequently be required to maintain very high capital levels. While there may be some prudential advantages to this, it is not an efficient allocation of financial resources and could lead certain activities to be moved outside the jurisdiction of the financial regulatory authorities.

3.4.2 Basel Committee Proposal

In January 1996, the Basel Committee on Banking Supervision of the BIS issued a revised consultative proposal on an "Internal Model-Based Approach to Market Risk Capital Requirements" that represents a big step forward in recognizing the new quantitative risk estimation techniques used by the banking industry. These proposals recognize that current practice among many financial institutions has superseded the original guidelines in terms of sophistication, and that banks should be given the flexibility to use more advanced methodologies. This so-called "internal models" approach addresses a number of issues that were raised when banks commented on the original proposal dated April 1993.

Table 3.1 compares the methodologies for estimating market risks as recently proposed by the Basel Committee with the RiskMetrics methodology covered in this document. This comparison focuses exclusively on the so-called quantitative factors that the BIS guidelines will require banks to use. It does not address the qualitative ones related to the risk management process and which are beyond the scope of this document.

While the methodologies outlined in the BIS proposals have come a long way in overcoming important objections to the first set of proposals, there are still a number of issues that will be debated further. In order to facilitate the discussion between regulators and regulated, we have published since mid-1995 in parallel with the existing volatility and correlation data sets, a RiskMetrics Regulatory Data Set. The distribution of this regulatory data set is not an endorsement of the Basel committee proposals and the following paragraphs which explain how the data set can be used do not constitute J.P. Morgan's official position on the content and scope of the Basel committee proposal.

Consistent with the other RiskMetrics data sets, the Regulatory Data Set contains volatility estimates for a 1-day holding period. Given that the BIS rules require market risk estimates to be calculated over a 10-day holding period and a 99% confidence interval (i.e., 2.33 standard deviations), users will need to rescale the 1-day volatility (see Eq. [3.1]). The Basel proposals allow for this adjustment of data (they actually refer to scaling up VaR estimates but exclude this practice in the case of options since it only works for instruments' whose pricing formulae are linear). Scaling up volatility estimates is perfectly legitimate, assuming no autocorrelation in the data. Scaling up Value-at-Risk does not work for options, though using scaled up volatilities to estimate the market risks of options with adequate pricing algorithms poses no problem.

As in the other data sets, volatilities and correlations are measured as daily log changes in rates and prices. However, contrary to the exponential weighting schemes used for the other data sets, estimates in the Regulatory Data Set are based on simple moving averages of 1 year of historical data, sampled daily.

To make it comparable to the standard data sets, the RiskMetrics Regulatory Data Set is based on 95% confidence. Including the adjustment for the holding period, users downloading the data sets will need to rescale the volatility estimates according to the following equation, in order to meet the requirements set forth in the Basel proposals (this adjustment assumes a normal distribution. More refined methods incorporating the characteristics of fat tailed distributions are outlined in the statistics section of this document):

$$[3.1] \qquad \begin{aligned} V_{\text{Basel}} &= \frac{2.33}{1.65} \cdot V_{\text{RiskMetrics RD}} \cdot \sqrt{10} \\ &= 4.45 \cdot V_{\text{RiskMetrics RD}} \end{aligned}$$

where

$V_{\text{RiskMetrics RD}}$ = volatilities provided in RiskMetrics Regulatory Dataset

V_{Basel} = volatilities suggested by Basel Committee for use in internal models

Correlations across asset classes (i.e., foreign exchange to government bonds for example) are supplied in the RiskMetrics Regulatory Data Set, despite the fact that actual use of empirical correlations in the VaR estimates is subject to regulatory approval. The BIS has stated that the use of correlations across asset classes would be based on whether the supervisory authority was satisfied with the integrity of the estimation methodology.

Table 3.1
Comparing the Basel Committee proposal with RiskMetrics

Issue	Basel Committee proposal	RiskMetrics
Mapping: how positions are described in summary form	• Fixed Income: at least 6 time buckets, differentiate government yield curves and spread curves.	• Fixed Income: data for 7–10 buckets of government yield curves in 16 markets, 4 buckets money market rates in 27 markets, 4–6 buckets in swap rates in 18 markets.
	• Equities: country indices, individual stocks on basis of beta equivalent.	• Equities: country indices in 27 markets, individual stocks on beta (correction for non-systematic risk).
	• Commodities: to be included, not specified how.	• Commodities: 80 volatility series in 11 commodities (spot and term).
Volatility: how statistics of future price movement are estimated	• Volatility expressed in standard deviation of normal distribution proxy for daily historical observations year or more back. Equal weights or alternative weighting scheme provided effective observation period is at least one year.	• Volatility expressed in standard deviation of normal distribution proxy for exponentially weighted daily historical observations with decay factors of .94 (for trading, 74 day cutoff 1%) and .97 (for investing, 151 day cutoff at 1%). • Special Regulatory Data Set, incorporating Basel Committee 1-year moving average assumption.
	• Estimate updated at least quarterly.	• Estimates updated daily.
Adversity: size of adverse move in terms of normal distribution	• Minimum adverse move expected to happen with probability of 1% (2.32 standard deviations) over 10 business days. Permission to use daily statistics scaled up with square root of 10 (3.1). Equivalent to 7.3 daily standard deviations.	• For trading: minimum adverse move expected to happen with probability of 5% (1.65 standard deviation) over 1 business day. • For investment: minimum adverse move expected to happen with probability of 5% (1.65 standard deviation) over 25 business days.
Options: treatment of time value and non-linearity	• Risk estimate must consider effect of non-linear price movement (gamma-effect).	• Non-linear price movement can be estimated analytically (delta-gamma) or under simulation approach. Simulation scenarios to be generated from estimated volatilities and correlations.
	• Risk estimate must include effect of changes in implied volatilities (vega-effect).	• Estimates of volatilities of implied volatilities currently not provided, thus limited coverage of options risk.
Correlation: how risks are aggregated	• Portfolio effect can be considered within asset classes (Fixed Income, Equity, Commodity, FX). Use of correlations across asset classes subject to regulatory approval.	• Full portfolio effect considered across all possible parameter combinations.
	• Correlations estimated with equally weighted daily data for more than one year.	• Correlations estimated using exponentially weighted daily historical observations with decay factors of 0.94 (for trading, 74 day cutoff 1%) and 0.97 (for investing, 151 day cutoff at 1%).
Residuals: treatment of instrument specific risks	• Instrument specific risks not covered by standard maps should be estimated. • Capital requirements at least equal to 50% of charge calculated under standard methodology.	• Does not deal with specific risks not covered in standard maps.

Part II

Statistics of Financial Market Returns

Chapter 4. Statistical and probability foundations

Chapter 4. Statistical and probability foundations

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This chapter presents the statistical and probability underpinnings of the RiskMetrics model. It explains the assumptions commonly applied to forecast the distribution of portfolio returns and investigates the empirical validity of these assumptions. While we have tried to make this chapter self-contained, its subject matter does require a thorough grasp of elementary statistics. We have included many up-to-date references on specific topics so that the interested reader may pursue further study in these areas.

This chapter is organized as follows:

- Section 4.1 presents definitions of financial price returns and explains the type of returns applied in RiskMetrics.
- Section 4.2 describes the basic random walk model for financial prices to serve as background to introducing the RiskMetrics model of returns.
- Section 4.3 looks at some observed time series properties of financial returns in the context of the random walk model.
- Section 4.4 summarizes the results presented in Sections 4.1 through 4.3.
- Section 4.5 reviews some popular models of financial returns and presents a review of the normal and lognormal distributions.
- Section 4.6 presents the RiskMetrics model as a modified random walk. This section lists the assumptions of the RiskMetrics model—that is, what RiskMetrics assumes about the evolution of financial returns over time and the distribution of returns at any point in time.
- Section 4.7 is a chapter summary.

4.1 Definition of financial price changes and returns[1]

Risk is often measured in terms of price changes. These changes can take a variety of forms such as absolute price change, relative price change, and log price change. When a price change is defined relative to some initial price, it is known as a return. **RiskMetrics measures change in value of a portfolio (often referred to as the adverse price move) in terms of log price changes also known as continuously-compounded returns.** Next, we explain different definitions of price returns.

4.1.1 One-day (single period) horizon

Denote by P_t the price of a security at date *t*. In this document, *t* is taken to represent one business day.

The absolute price change on a security between dates *t* and $t-1$ (i.e., one day) is defined as

[4.1] $$D_t = P_t - P_{t-1}$$

[1] References for this section are, Campbell, Lo and MacKinley (1995) and Taylor, S. J. (1987).

The relative price change, or percent return[2], R_t, for the same period is

[4.2] $$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

If the gross return on a security is just $1 + R_t$, then the log price change (or continuously-compounded return), r_t, of a security is defined to be the natural logarithm of its gross return. That is,

[4.3] $$\begin{aligned} r_t &= \ln(1 + R_t) \\ &= \ln\left(\frac{P_t}{P_{t-1}}\right) \\ &= (p_t - p_{t-1}) \end{aligned}$$

where $p_t = \ln(P_t)$ is the natural logarithm of P_t.

In practice, the main reason for working with returns rather than prices is that returns have more attractive statistical properties than prices, as will be shown below. Further, returns (relative and log price changes) are often preferred to absolute price changes because the latter do not measure change in terms of the **given** price level.

To illustrate the different results that different price changes can yield, Table 4.1 presents daily USD/DEM exchange rates for the period 28-Mar-96 through 12-Apr-96 and the corresponding daily absolute, relative, and log price changes.

Table 4.1
Absolute, relative and log price changes*

Date	Price (USD/DEM), P_t	Absolute price change (%), D_t	Relative price change (%), R_t	Log price change (%,) r_t
28-Mar-96	0.67654	0.427	0.635	0.633
29-Mar-96	0.67732	0.078	0.115	0.115
1-Apr-96	0.67422	–0.310	–0.458	–0.459
2-Apr-96	0.67485	0.063	0.093	0.093
3-Apr-96	0.67604	0.119	0.176	0.176
4-Apr-96	0.67545	–0.059	–0.087	–0.087
5-Apr-96	0.67449	–0.096	–0.142	-0.142
8-Apr-96	0.67668	0.219	0.325	0.324
9-Apr-96	0.67033	–0.635	–0.938	–0.943
10-Apr-96	0.66680	–0.353	–0.527	–0.528
11-Apr-96	0.66609	–0.071	–0.106	–0.107
12-Apr-96	0.66503	–0.106	–0.159	–0.159

* RiskMetrics foreign exchange series are quoted as USD per unit foreign currency given that the datasets are standardized for users whose base currency is the USD. This is the inverse of market quotation standards for most currency pairs.

As expected, all three series of price changes have the same sign for any given day. Also, notice the similarity between the log and relative price changes. In fact, we should expect these two return series to be similar to one another for small changes in the underlying prices. In contrast, the absolute change series is quite different from the other two series.

[2] Although it is called “percent return,” the relative price change is expressed as a decimal number.

To further illustrate the potential differences between absolute and log price changes, Chart 4.1 shows daily absolute and log price changes for the U.S. 30-year government bond over the first quarter of 1996.

Chart 4.1
Absolute price change and log price change in U.S. 30-year government bond

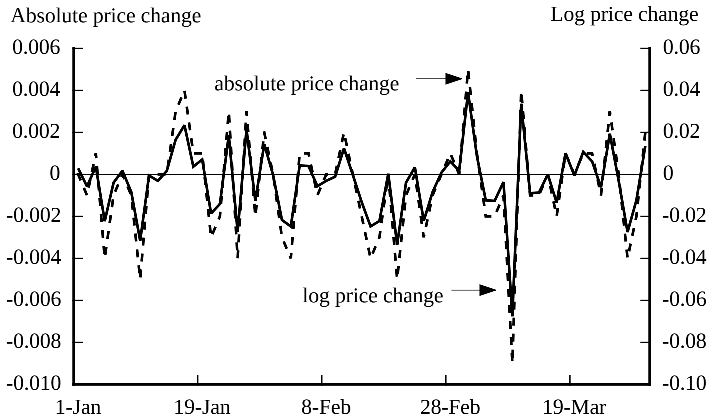


Chart 4.1 shows that movements of the two changes over time are quite similar although the magnitude of their variation is different. This latter point and the results presented in Table 4.1 should make it clear that it is important to understand the convention chosen for measuring price changes.

4.1.2 Multiple-day (multi-period) horizon

The returns R_t and r_t described above are 1-day returns. We now show how to use them to compute returns for horizons greater than one day.

Multiple-day percent returns over the most recent *k* days, $R_t(k)$, are defined simply as

[4.4] $$R_t(k) = \frac{P_t - P_{t-k}}{P_{t-k}}$$

In terms of 1-day returns, the multiple-day **gross** return $1 + R_t(k)$ is given by the product of 1-day gross returns.

[4.5] $$\begin{aligned} 1 + R_t(k) &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k-1}) \\ &= \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \cdot \ldots \cdot \frac{P_{t-k-1}}{P_{t-k}} \\ &= \frac{P_t}{P_{t-k}} \end{aligned}$$

Note that in Eq. [4.5] the *k*-day return is a discretely compounded return. For continuously compounded returns, the multiple-day return $r_t(k)$ is defined as

[4.6] $$r_t(k) = \ln\left(\frac{P_t}{P_{t-k}}\right)$$

The continuously-compounded return $r_t(k)$ is the sum of k continuously-compounded 1-day returns. To see this we use the relation $r_t(k) = \ln[1 + R_t(k)]$. The return $r_t(k)$ can then be written as

$$
\begin{aligned}
r_t(k) &= \ln[1 + R_t(k)] \\
&= \ln[(1 + R_t) \cdot (1 + R_{t-1}) \cdot (1 + R_{t-k-1})] \quad [4.7] \\
&= r_t + r_{t-1} + \ldots + r_{t-k+1}
\end{aligned}
$$

Notice from Eq. [4.7] that compounding, a multiplicative operation, is converted to an additive operation by taking logarithms. Therefore, multiple day returns based on continuous compounding are simple sums of one-day returns.

As an example of how 1-day returns are used to generate a multiple-day return, we use a 1-month period, defined by RiskMetrics as having 25 business days. Working with log price changes, the continuously compounded return over one month is given by

[4.8] $$r_t(25) = r_t + r_{t-1} + \ldots + r_{t-24}$$

That is, the 1-month return is the sum of the last 25 1-day returns.

4.1.3 Percent and continuous compounding in aggregating returns

When deciding whether to work with percent or continuously compounded returns it is important to understand how such returns aggregate both across time and across individual returns at any point in time.

In the preceding section we showed how multiple-day returns can be constructed from 1-day returns by aggregating the latter across time. This is known as temporal aggregation. However, there is another type of aggregation known as cross-section aggregation. In the latter approach, aggregation is across individual returns (each corresponding to a specific instrument) at a particular point in time. For example, consider a portfolio that consists of three instruments. Let r_i and R_i $(i = 1, 2, 3)$ be the continuously compounded and percent returns, respectively and let w_i represent the portfolio weights. (The parameter w_i represents the fraction of the total portfolio value allocated to the *i*th instrument with the condition that—assuming no short positions—$w_1 + w_2 + w_3 = 1$). If the initial value of this portfolio is P_0 the price of the portfolio one period later with continuously compounded returns is

[4.9] $$P_1 = w_1 \cdot P_0 \cdot e^{r_1} + w_2 \cdot P_0 \cdot e^{r_2} + w_3 \cdot P_0 \cdot e^{r_3}$$

Solving Eq. [4.9] for the portfolio return, $r_p = \ln\left(\frac{P_1}{P_0}\right)$, we get

[4.10] $$r_p = \ln\left(w_1 \cdot e^{r_1} + w_2 \cdot e^{r_2} + w_3 \cdot e^{r_3}\right)$$

The price of the portfolio one period later with discrete compounding, i.e., using percent returns, is

[4.11] $$P_1 = w_1 \cdot P_0 \cdot (1 + r_1) + w_2 \cdot P_0 \cdot (1 + r_2) + w_3 \cdot P_0 \cdot (1 + r_3)$$

The percent portfolio return, $R_p = \frac{(P_1 - P_0)}{P_0}$, is given by

[4.12] $$R_p = w_1 \cdot r_1 + w_2 \cdot r_2 + w_3 \cdot r_3$$

Equation [4.12] is the expression often used to describe a portfolio return—as a weighted sum of individual returns.

Table 4.2 presents expressions for returns that are constructed from temporal and cross-section aggregation for percent and continuously compounded returns.

Table 4.2
Return aggregation

Aggregation	Temporal	Cross-section
Percent returns	$R_{it}(k) = \prod_{t=1}^{T}(1 + R_{it}) - 1$	$R_{pt} = \sum_{i=1}^{N} w_i R_{it}$
Continuously compounded returns	$r_{it}(k) = \sum_{t=1}^{T} r_{it}$	$r_{pt} = \ln\left(\sum_{i=1}^{N} w_i e^{r_{it}}\right)$

The table shows that when aggregation is done across time, it is more convenient to work with continuously compounded returns whereas when aggregation is across assets, percent returns offer a simpler expression.

As previously stated, log price changes (continuously compounded returns) are used in RiskMetrics as the basis for all computations. In practice, RiskMetrics assumes that a portfolio return is a weighted average of continuously compounded returns. That is, a portfolio return is defined as follows

$$r_{pt} \cong \sum_{i=1}^{N} w_i r_{it} \qquad [4.13]$$

As will be discussed in detail in the next section, when 1-day returns are computed using r_t, then a model describing the distribution of 1-day returns extends straightforwardly to returns greater than one day.[3]

In the next two sections (4.2 and 4.3) we describe a class of time series models and investigate the empirical properties of financial returns. These sections serve as important background to understanding the assumptions RiskMetrics applies to financial returns.

4.2 Modeling financial prices and returns

A risk measurement model attempts to characterize the future change in a portfolio's value. Often, it does so by making forecasts of each of a portfolio's underlying instrument's future price changes, using only past changes to construct these forecasts. This task of describing future price changes requires that we model the following; (1) the temporal dynamics of returns, i.e., model the evolution of returns over time, and (2) the distribution of returns at any point in time.

A widely used class of models that describes the evolution of price returns is based on the notion that financial prices follow a random walk.

[3] There are two other reasons for using log price changes. The first relates to "Siegel's paradox," Meese, R.A. and Rogoff, K. (1983). The second relates to preserving normality for FX cross rates. Simply put, when using log price changes, FX cross rates can be written as differences of base currency rates. (See Section 8.4 for details.)

4.2.1 Random walk model for single-price assets

In this section we present a model for a security with a single price. Such a model applies naturally to assets such as foreign exchange rates, commodities, and equities where only one price exists per asset. The fundamental model of asset price dynamics is the random walk model,

$$P_t = \mu + P_{t-1} + \sigma\varepsilon_t$$
$$P_t - P_{t-1} = \mu + \sigma\varepsilon_t, \qquad \varepsilon_t \sim \text{IID}\ \text{N}(0,1) \qquad [4.14]$$

where IID stands for "identically and independently distributed"[4], and $\text{N}(0,1)$ stands for the normal distribution with mean 0 and variance 1. Eq. [4.14] posits the evolution of prices and their distribution by noting that at any point in time, the current price P_t depends on a fixed parameter μ, last period's price P_{t-1}, and a normally distributed random variable, ε_t. Simply put, μ and σ affect the mean and variance of P_t's distribution, respectively.

The conditional distribution of P_t, given P_{t-1}, is normally distributed.[5] An obvious drawback of this model is that there will always be a non-zero probability that prices are negative.[6] One way to guarantee that prices will be non-negative is to model the log price p_t as a random walk with normally distributed changes.

$$p_t = \mu + p_{t-1} + \sigma\varepsilon_t \qquad \varepsilon_t \sim \text{IID}\ \text{N}(0,1) \qquad [4.15]$$

Notice that since we are modeling log prices, Eq. [4.15] is a model for continuously compounded returns, i.e., $r_t = \mu + \sigma\varepsilon_t$. Now, we can derive an expression for prices, P_t given last period's price P_{t-1} from Eq. [4.15]:

$$P_t = P_{t-1}\exp(\mu + \sigma\varepsilon_t) \qquad [4.16]$$

where $\exp(x) \equiv e^x$ and $e \cong 2.718$.

Since both P_{t-1} and $\exp(\mu + \sigma\varepsilon_t)$ are non-negative, we are guaranteed that P_t will never be negative. Also, when ε_t is normally distributed, P_t follows a lognormal distribution.[7]

Notice that both versions of the random walk model above assume that the change in (log) prices has a constant variance (i.e., σ does not change with time). We can relax this (unrealistic) assumption, thus allowing the variance of price changes to vary with time. Further, the variance could be modeled as a function of past information such as past variances. By allowing the variance to vary over time we have the model

$$p_t = \mu + p_{t-1} + \sigma_t\varepsilon_t \qquad \varepsilon_t \sim N(0,1) \qquad [4.17]$$

---

[4] See Section 4.3 for the meaning of these assumptions.

[5] The unconditional distribution of P_t is undefined in that its mean and variance are infinite. This can easily be seen by solving Eq. [4.14] for P_t as a function of past ε_t's.

[6] This is because the normal distribution places a positive probability on all points from negative to positive infinity. See Section 4.5.2 for a discussion of the normal distribution.

[7] See Section 4.5.3 for a complete description of the lognormal distribution.

This version of the random walk model is important since it will be shown below that **RiskMetrics assumes that log prices evolve according to Eq. [4.17] with the parameter μ set to zero**.

4.2.2 *Random walk model for fixed income instruments*

With fixed income instruments we observe both prices and yields. When prices and yields exist, we must decide whether to model the log changes in the yields or in the prices. For example, for bonds, a well documented shortcoming of modeling price returns according to Eq. [4.15] is that the method ignores a bond's price **pull to par phenomenon**. That is, a bond has the distinct feature that as it approaches maturity, its price converges to its face value. Consequently, the bond price volatility will converge to zero.

Therefore, when modeling the dynamic behavior of bonds (and other fixed income instruments), the bond yields rather than the bond prices are often modeled according to the lognormal distribution. That is, if Y_t denotes the yield on a bond at period *t*, then $y_t = \ln(Y_t)$ is modeled as

[4.18] $$y_t = \mu + y_{t-1} + \sigma\varepsilon_t \qquad \varepsilon_t \sim IID \ \mathrm{N}(0, 1)$$

(Note that similar to Eq. [4.17] we can incorporate a time-varying variance into Eq. [4.18]). In addition to accounting for the pull to par phenomenon, another important reason for modeling the yield rather than the price according to Eq. [4.18] is that positive yields are guaranteed. In the context of bond option pricing, a strong case can often be made for modeling yields as lognormal.[8]

4.2.3 *Time-dependent properties of the random walk model*

Each of the random walk models presented in Sections 4.2.1 and 4.2.2 imply a certain movement in financial prices over time. In this section we use Eq. [4.15]—the random walk model in log prices, p_t—to explain some important properties of price dynamics implied by the random walk model. Specifically, we discuss the properties of stationary (mean-reverting) and nonstationary time series.

A stationary process is one where the mean and variance are constant and finite over time.[9] In order to introduce the properties of a stationary time series we must first generalize Eq. [4.15] to the following model.

[4.19] $$p_t = \mu + c \cdot p_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim IID \ \mathrm{N}(0, 1), \ p_0 = 0$$

where c is a parameter. Here, a stationary time series is generated when $-1 < c < 1$. For example, if we set $c = 0.5$, we can simulate a stationary time series using

[4.20] $$p_t = 0.01 + 0.5 p_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim IID \ \mathrm{N}(0, 1), \ p_0 = 0$$

[8] For a discussion on the potential advantages of modeling yield levels as lognormal, see Fabozzi (1989, Chapter 3).

[9] Stationarity also requires that the (auto-)covariance of returns at different times is only a function of the time between the returns, and not the times at which they occur. This definition of stationarity is known as weak or covariance stationarity.

Chart 4.2 shows the simulated stationary time series based on 500 simulations.

Chart 4.2
Simulated stationary/mean-reverting time series

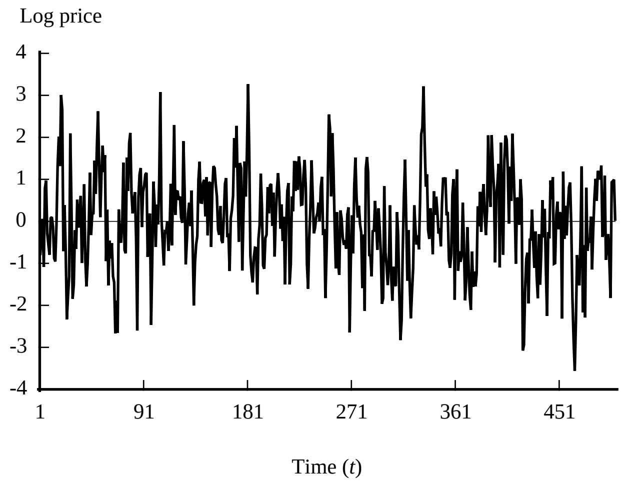


Chart 4.2 shows how a stationary series fluctuates around its mean, which in this model is 0.02. Hence, stationary series are **mean-reverting** since, regardless of the fluctuations' amplitudes, the series reverts to its mean.

Unlike a mean-reverting time series, a nonstationary time series does not fluctuate around a fixed mean. For example, in Eq. [4.15] the mean and variance of the log price p_t conditional on some original observed price, say p_0, are given by the following expressions

[4.21]
$$E_0[p_t|p_0] = p_0 + \mu t \qquad \text{(mean)}$$
$$V_0[p_t|p_0] = \sigma^2 t \qquad \text{(variance)}$$

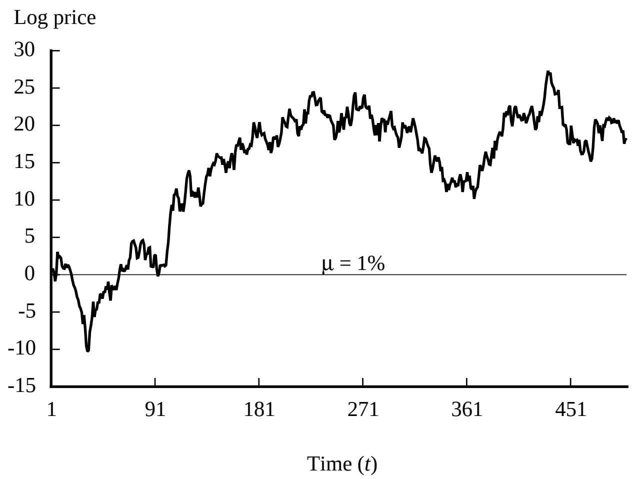
where $E_0[\ ]$ and $V_0[\ ]$ are the expectation and variance operators taken at time 0. Eq. [4.21] shows that both the mean and variance of the log price are a function of time such that, as time t increases, so does p_t's conditional mean and variance. The fact that its mean and variance change with time and "blow-up" as time increases is a characteristic of a nonstationary time series.

To illustrate the properties of a nonstationary time series, we use the random walk model, Eq. [4.15], to simulate 500 data points. Specifically, we simulate a series based on the following model,

[4.22] $$p_t = 0.01 + p_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim IID\ \mathrm{N}(0, 1),\ p_0 = 0$$

The simulated series is shown in Chart 4.3.

Chart 4.3
Simulated nonstationary time series

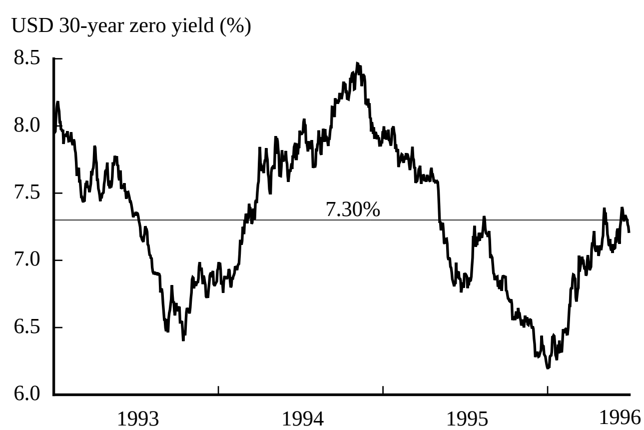


Notice how this series has a positive drift that grows with time, representing the term μt in Eq. [4.21]. This is a typical feature of a nonstationary time series.

In the preceding examples, notice that the difference between these stationary and nonstationary series is driven by the coefficient on last period's log price p_{t-1}. When this coefficient is 1, as in Eq. [4.22], the process generating log prices is known to have a "unit root". As should be expected, given the differences between stationary and non-stationary times series and their implications for statistical analysis, there is a large body of literature devoted to testing for the presence of a unit root.[10]

Real world examples of stationary and nonstationary series are shown in Charts 4.4 and 4.5. For the same period, Chart 4.4 plots the USD 30-year rate, a stationary time series.

Chart 4.4
Observed stationary time series
USD 30-year yield

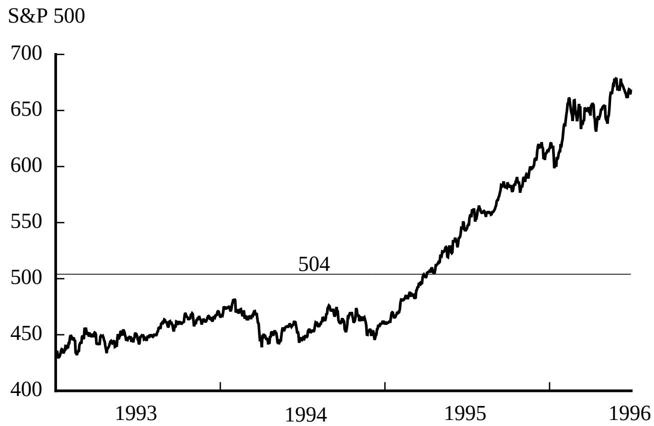


[10] A common statistical test for a unit root is known as the augmented Dickey-Fuller test. See Greene, (1993).

Notice how the 30-year rates fluctuate around the sample average of 7.30%, signifying that the time series for this period is mean-reverting.

Chart 4.5 plots the S&P 500 index for the period January 4, 1993 through June 28, 1996.

Chart 4.5
Observed nonstationary time series
S&P 500 index



Notice that the S&P 500 index does not fluctuate around the sample mean of 504, but rather has a distinct trend upwards. Comparing the S&P 500 series to the simulated nonstationary data in Chart 4.3, we see that it has all the markings of a nonstationary process.

4.3 Investigating the random-walk model

Thus far we have focused on a simple version of the random walk model (Eq. [4.15]) to demonstrate some important time series properties of financial (log) prices. Recall that this model describes how the prices of financial assets evolve over time, assuming that logarithmic price changes are identically and independently distributed (IID). These assumptions imply:

1. At each point in time, t, log price changes are distributed with a mean 0 and variance σ^2 (identically distributed). This implies that the mean and variance of the log price changes are **homoskedastic**, or unchanging over time.
2. Log price changes are statistically independent of each other over time (independently distributed). That is to say, the values of returns sampled at different points are completely unrelated

In this section we investigate the validity of these assumptions by analyzing real-world data. We find evidence that the IID assumptions do not hold.[11]

[11] Recent (nonparametric) tests to determine whether a time series is IID are presented in Campbell and Dufour (1995).

4.3.1 Is the distribution of returns constant over time?

Visual inspection of real-world data can be a useful way to help understand whether the assumptions of IID returns hold. Using a time series of returns, we investigate whether the first assumption of IID, identically distributed returns, is indeed valid. We find that it is violated and present the following data as evidence.

Charts 4.6 and 4.7 show time series plots of continuously compounded returns for the USD/DEM and USD/FRF exchange rates, respectively.[12]

Chart 4.6
USD/DEM returns

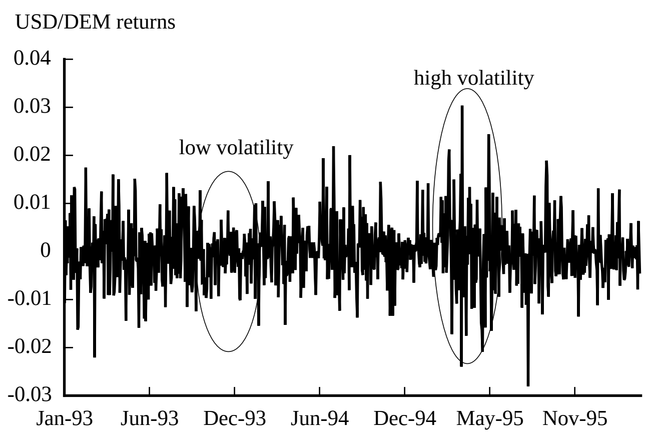
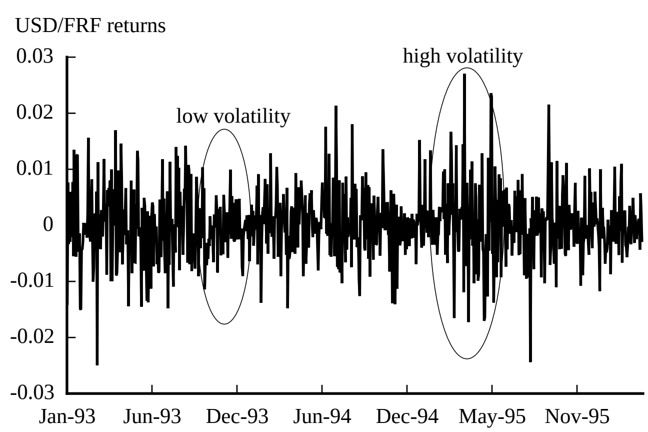


Chart 4.7
USD/FRF returns



These time series show clear evidence of volatility clustering. That is, periods of large returns are clustered and distinct from periods of small returns, which are also clustered. If we measure such volatility in terms of variance (or its square root, i.e., the standard deviation), then it is fair to think that variance changes with time, reflecting the clusters of large and small returns. In terms of the model in Eq. [4.15], this means that σ_t^2 is changing with time (t). In statistics, changing variances are often denoted by the term **heteroscedasticity**.

[12] This notation (i.e., USD per DEM) is not necessarily market convention.

In Charts 4.6 and 4.7 we also notice not only the individual volatility clustering, but the correlation of the clusters **between** return series. For example, note that periods of high volatility in USD/DEM returns coincide with high volatility in USD/FRF returns. Such correlation between returns series motivates the development of multivariate models, that is, models of returns that measure not only individual series variance (volatility), but also the correlation **between** return series.

4.3.2 Are returns statistically independent over time?

Having established, albeit informally, the possibility of time-varying variances, and consequently, a violation of the identically distributed assumption, we now investigate the validity of the independence assumption, i.e., the second assumption of IID. From our methods and the data that we present in the following sections (4.3.2.1 through 4.3.2.3), we conclude that returns in a given series are **not** independent of each other.

In Charts 4.6 and 4.7, the persistence displayed by the volatility clusters shows some evidence of autocorrelation in variances. That is, the variances of the series are correlated across time. If returns are statistically independent over time, then they are not autocorrelated. Therefore, a natural method for determining if returns are statistically independent is to test whether or not they are autocorrelated. In order to do so, we begin by defining correlation and a method of testing for autocorrelation.

4.3.2.1 Autocorrelation of daily log price changes

For a given time series of returns, the autocorrelation coefficient measures the correlation of returns across time. In general, the standard correlation coefficient between two random variables *X* and *Y* is given by the covariance between *X* and *Y* divided by their standard deviations:

[4.23] $$\rho_{xy} = \frac{\sigma^2_{xy}}{\sigma_x \sigma_y}$$

where σ^2_{xy} represents the covariance between *X* and *Y*. A simple way to understand what covariance measures is to begin with the definition of variance. The variance of a random variable *X* is a measure of the variation of *X* around its mean, μ_X. The mathematical expression for variance is

[4.24] $$E\left[(X - \mu_X)^2 \right]$$

where the term *E*[ ] is the mathematical expectation—or more simply, the average. Whereas the variance measures the magnitude of variation of one random variable (in this case *X*), covariance measures the covariation of two random variables (say, *X* and *Y*). It follows that if the variance of *X* is the expected value of $(X - \mu_X)$ times $(X - \mu_X)$, then the covariance of *X* and *Y* is the expected value of $(X - \mu_X)$ times $(Y - \mu_Y)$, or

[4.25] $$E\left[(X - \mu_X)(Y - \mu_Y) \right]$$

Now, for a time series of observations $r_t, t = 1 \ldots T$, the *k*th order autocorrelation coefficient $\rho(k)$ is defined as:

[4.26] $$\rho_k = \frac{\sigma^2_{t,\,t-k}}{\sigma_t \sigma_{t-k}} = \frac{\sigma^2_{t,\,t-k}}{\sigma^2_t}$$

Notice that since $\rho(k)$ operates on just one series the subscripts on the covariance and standard deviation refer to the time index on the return series. For a given sample of returns, $r_t, t = 1 \ldots T$, we can estimate Eq. [4.26] using the sample autocorrelation coefficient which is given by:

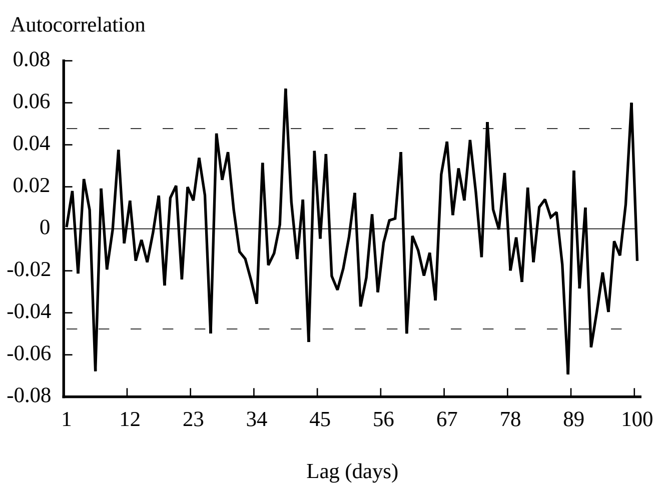
$$[4.27] \qquad \hat{\rho}_k = \frac{\sum_{t=k+1}^{T} \{ (r_t - \bar{r})(r_{t-k} - \bar{r}) \} / [T - (k-1)]}{\sum_{t=1}^{T} \{ (r_t - \bar{r})^2 \} / [T-1]}$$

where k = number of lags (days), and $\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$, is the sample mean.

If a time series is not autocorrelated then estimates of $\hat{\rho}_k$ will not be significantly different from 0. In fact, when there is a large amount of historical returns available, we can calculate a 95% confidence band around 0 for each autocorrelation coefficient[13] as $\pm\frac{1.96}{\sqrt{T}}$.

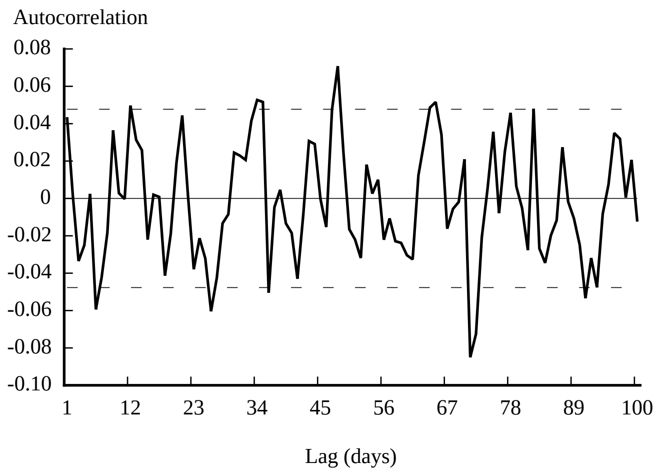
Charts 4.8 and 4.9 show the sample autocorrelation coefficient $\hat{\rho}_k$ plotted against different lags k (measured in days), along with the 95% confidence band around zero for USD/DEM foreign exchange and S&P 500 log price changes, respectively, for the period January 4, 1990 to June 24, 1996. These charts are known as correlograms. The dashed lines represent the upper and lower 95% confidence bands $\pm 4.7\%$. If there is no autocorrelation, that is, if the series are purely random, then we expect only one in twenty of the sample autocorrelation coefficients to lie outside the confidence bands.

Chart 4.8
Sample autocorrelation coefficients for USD/DEM foreign exchange returns



[13] This an asymptotic test statistic since it relies on a large value of T, say, $T > 1000$. See Harvey (p. 43, 1993).

Chart 4.9
Sample autocorrelation coefficients for USD S&P 500 returns



Overall, both charts show very little evidence of autocorrelation in daily log price changes. Even in the cases where the autocorrelations are outside the confidence bands, the autocorrelation coefficients are quite small (less than 10%).

4.3.2.2 Box-Ljung statistic for daily log price changes

While the above charts are useful for getting a general idea about the level of autocorrelation of log price changes, there are more formal methods of testing for autocorrelation. An often cited method is the Box-Ljung (BL) test statistic,[14] defined as

$$[4.28] \qquad BL(p) = T \cdot (T+2) \sum_{k=1}^{p} \frac{\rho_k^2}{T-k}$$

Under the null hypothesis that a time series is not autocorrelated, $BL(p)$, is distributed chi-squared with p degrees of freedom. In Eq. [4.28], p denotes the number of autocorrelations used to estimate the statistic. We applied this test to the USD/DEM and S&P 500 returns for $p = 15$. In this case, the 5% chi-squared critical value is 25. Therefore, values of the $BL(10)$ statistic greater than 25 implies that there is statistical evidence of autocorrelation. The results are shown in Table 4.3.

Table 4.3
Box-Ljung test statistic

Series	$\hat{BL}(15)$
USD/DEM	15
S&P 500	25

[14] See West and Cho (1995) for modifications to this statistic.

We also applied this test to the daily log price changes of a selected series of commodity futures contracts because, when plotted against time, these series appear autocorrelated. In these tests we chose $p = 10$ which implies a critical value of 18.31 at the 95% confidence level. Table 4.4 presents the results along with the first order autocorrelation coefficient, ρ_1 .

Table 4.4
Box-Ljung statistics

Contract*	**Maturity (mths.)**	$\hat{\rho}_1$	$\hat{BL}(10)$
WTI	1	–0.0338	5.24
	3	–0.0586	7.60
	6	–0.0927	13.62
	12	–0.1323	25.70
LME			
Copper	3	–0.0275	8.48
	15	–0.0900	19.04
	27	–0.1512	16.11

* Note that the higher autocorrelation associated with contracts with longer maturities may be due to the fact that such contracts are less liquid than contracts with short maturities.

The preceding tests show little evidence of autocorrelation for some daily log price change series. The fact that the autocorrelation is not strong agrees with previous research. It is often found that financial returns over the short-run (daily) are autocorrelated but the magnitudes of the autocorrelation are too small (close to zero) to be economically significant.[15] For longer return horizons (i.e., beyond a year), however, there is evidence of significant negative autocorrelation (Fama and French, 1988).

4.3.2.3 Autocorrelation of squared daily log price changes (returns)

As previously stated, although returns (log price changes) are uncorrelated, they may not be independent. In the academic literature, such dependence is demonstrated by the autocorrelation of the variances of returns. **Alternatively expressed, while the returns are not autocorrelated, their squares are autocorrelated**. And since the expected value of the squared returns are variances[16], autocorrelation in the squared returns implies autocorrelation in variances. The relationship between squared returns and variances is evident from the definition of variance, σ_t^2 .

[4.29]
$$\begin{aligned}\sigma_t^2 &= E[r_t - E(r_t)]^2 \\ &= E\left(r_t^2\right) - [E(r_t)]^2\end{aligned}$$

Assuming that the mean of the returns is zero, i.e., $E(r_t) = 0$, we get $\sigma_t^2 = E\left(r_t^2\right)$.

[15] In other words, it would be very difficult to form profitable trading rules based on autocorrelation in daily log price changes (Tucker, 1992). Also, more recent work has shown that over short horizons, autocorrelation in daily returns may be the result of institutional factors rather than purely inefficient markets (Boudoukh, Richardson and Whitelaw, 1994).

[16] This is true if the expected values of returns are zero.The plausibility of assuming a mean of zero for daily returns will be discussed in Section 5.3.1.1.

Charts 4.10 and 4.11 show time series of squared returns for the USD/DEM exchange rate and for the S&P 500 index.

Chart 4.10
USD/DEM returns squared

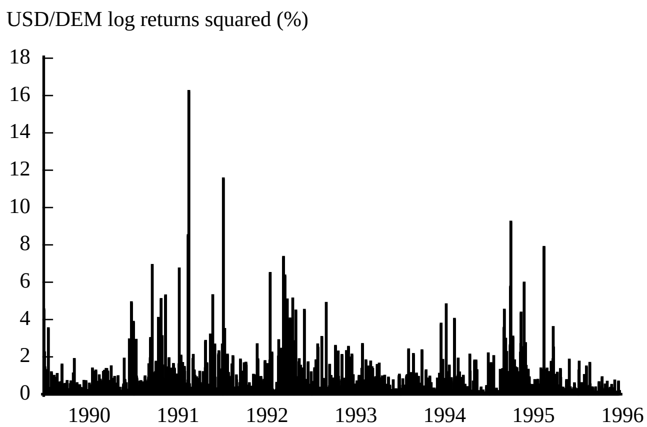
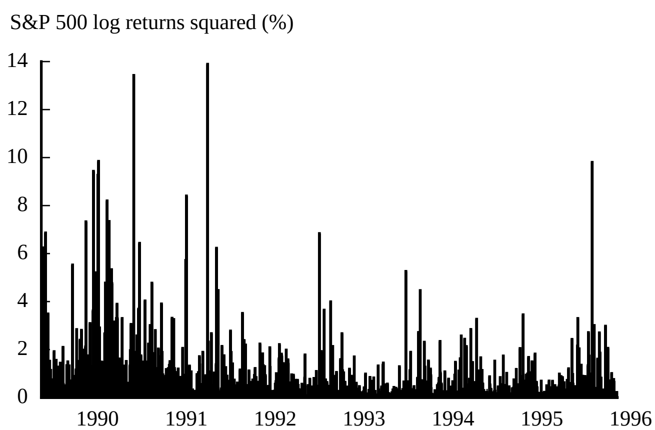


Chart 4.11
S&P 500 returns squared



Notice the clusters of large and small spikes in both series. These clusters represent periods of high and low volatility recognized in Section 4.2.1. To analyze the autocorrelation structure of the squared returns, as in the case of log price changes, we compute sample autocorrelation coefficients and the Box-Ljung statistic. Charts 4.12 and 4.13 present correlograms for the squared return series of USD/DEM foreign exchange and S&P 500, respectively.

Chart 4.12
Sample autocorrelation coefficients of USD/DEM squared returns

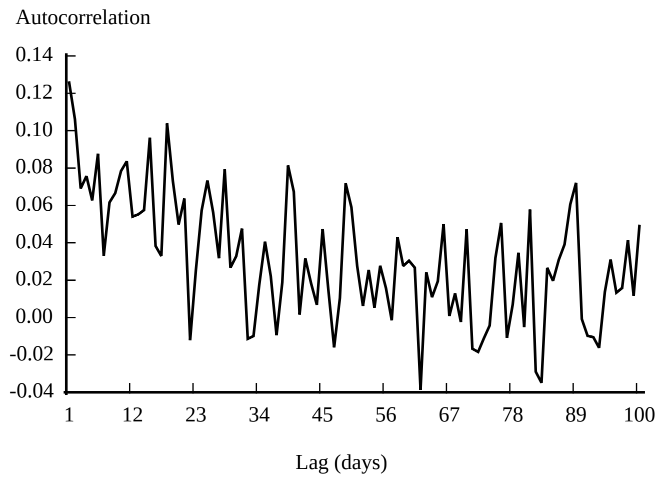
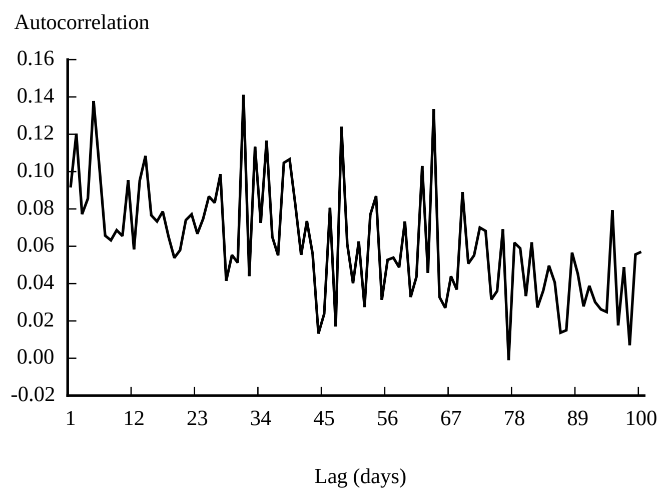


Chart 4.13
Sample autocorrelation coefficients of S&P 500 squared returns



Comparing the correlograms (Charts 4.8 and 4.9) based on daily log price changes to those based on the **squared** daily log price changes (Charts 4.12 and 4.13), we find the autocorrelation coefficients of the squared log price changes are larger and more persistent than those for log price changes. In fact, much of the significant autocorrelation in the squared log price changes is positive and well above the asymptotic 95% confidence band of 4.7%.[17] The Box-Ljung statistics for the squared log price change series are presented in Table 4.5.

Table 4.5
Box-Ljung statistics on squared log price changes (cv = 25)

Series	$\hat{BL}(15)$
USD/DEM	153
S&P 500	207

This table shows the dramatic effect that the squared log price changes has on the BL test. For all three series we reject the null hypothesis that the variances of daily returns are not autocorrelated.[18]

4.3.3 Multivariate extensions

Thus far, we have focused our attention on the empirical properties of individual returns time series. It appears that the variances of returns that were analyzed vary with time and are autocorrelated. As stated in Section 4.3.1, returns appear correlated (through their variances, at least) not only across time but also across securities. The latter finding motivates a study of the empirical properties of correlation, or more precisely, covariance between two return series.

We investigate whether covariances are autocorrelated by using the same logic applied to variances. Recall that we determined whether variances are autocorrelated by checking whether observed squared returns are autocorrelated. We used Eq. [4.29] to show the relation between variances and squared returns. Now, suppose we are interested in the covariance between two return series $r_{1,t}$ and $r_{2,t}$. We can derive a relationship between the covariance, $\sigma^2_{12,t}$, and observed returns as follows. We begin with a definition of covariance between $r_{1,t}$ and $r_{2,t}$.

[4.30]
$$\begin{aligned}\sigma^2_{12,t} &= E\{[r_{1,t}-E(r_{1,t})][r_{2,t}-E(r_{2,t})]\}\\ &= E(r_{1,t}r_{2,t})-E(r_{1,t})E(r_{2,t})\end{aligned}$$

Assuming that the mean of the returns is zero for both return series, we get

[4.31]
$$\sigma^2_{12,t} = E(r_{1,t}r_{2,t})$$

In words, Eq. [4.31] states that the covariance between $r_{1,t}$ and $r_{2,t}$ is the expectation of the cross-product of returns minus the product of the expectations. In models explaining variances, the focus is often on squared returns because of the presumption that for daily returns, squared expected returns are small. Focusing on cross-products of returns can be justified in the same way.

[17] Note that this confidence band may not be appropriate due to the fact that the underlying data are not returns, but squared returns.

[18] For a discussion on tests of autocorrelation on squared returns (residuals) see McLeod and Li (1983) and Li and Mak (1994).

Chart 4.14 presents a time series of the cross product ($r_{1,t}$ times $r_{2,t}$) of the returns on USD/DEM and USD/FRF exchange rates. This series is a proxy for the covariance between the returns on the two exchange rates.

Chart 4.14
Cross product of USD/DEM and USD/FRF returns

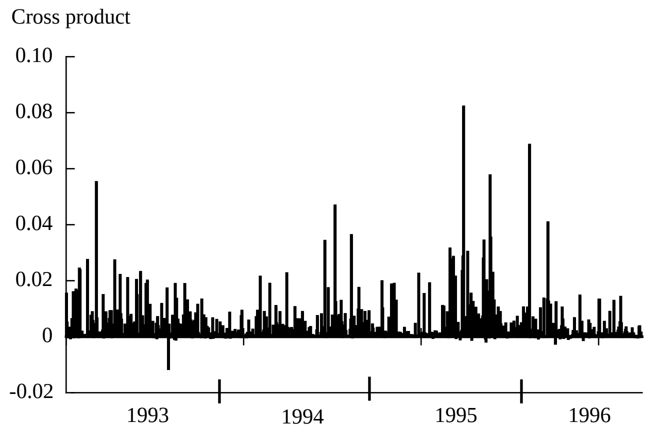
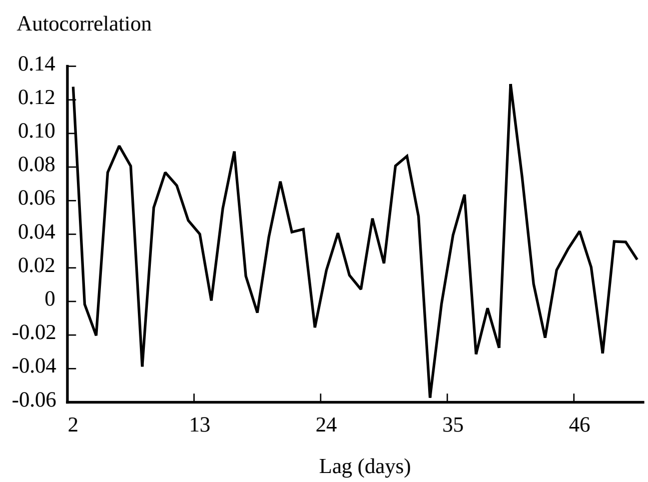


Chart 4.14 shows that the covariance (correlation) between the returns on the two exchange rates is positive over a large segment of the sample period. Time series generated from the cross product of two return series not only offers insight into the temporal dynamics of correlation but also can be used in a regression context to determine the stability of correlations over time.

Similar to the correlogram of squared returns, the correlogram of the cross product of returns on the two exchange rates can be used to determine whether the covariance of these two series are autocorrelated. Chart 4.15 shows the autocorrelations of the cross-products of returns on USD/DEM and USD/FRF exchange rates plotted against 50 daily lags.

Chart 4.15
Correlogram of the cross product of USD/DEM and USD/FRF returns



The *BL*(10) test associated with the cross product of returns on the two exchange rate series is 37, which is statistically significant (i.e., there is evidence of autocorrelation) at the 95% confidence level.

4.4 Summary of our findings

Up to this point, Chapter 4 focused on the dynamic features of daily continuously compounded returns, otherwise known as log price changes, and developed the topic as follows:

- We introduced three versions of the random walk model to describe how financial prices evolve over time. We used a particular version of this model (Eq. [4.15]) to highlight the differences between stationary (mean-reverting) and nonstationary time series.

- We investigated the assumptions that log price changes are identically and independently distributed.

  – To determine whether the distribution that generates returns is identical over time, we plotted log price changes against time. From time series plots of returns and their squares we observed the well documented phenomenon of "volatility clustering" which implies that the variance of daily log price changes vary over time (i.e., they are heteroscedastic), thus violating the identical assumption.[19]

  – To test independence, we analyzed the autocorrelation coefficients of both log price changes and squared log price changes. We found that while daily log price changes have small autocorrelations, their squares often have significant autocorrelations.

Much of this analysis has focused on short-horizon (daily) returns. In general, however, observed distributions of returns with longer horizons, such as a month or a quarter, are often different from distributions of daily returns.[20]

From this point, Chapter 4 reviews how returns are assumed to be distributed at each point in time. Specifically, we describe the normal distribution in detail. In RiskMetrics, it is assumed that returns are distributed according to the conditional normal distribution.

4.5 A review of historical observations of return distributions

As shown in Eq. [4.15] and Eq. [4.17], returns were assumed to follow, respectively, an unconditional and conditional normal distribution. The implications of the assumption that financial returns are normally distributed, at least unconditionally, has a long history in finance. Since the early work of Mandelbrot (1963) and Fama (1965), researchers have documented certain stylized facts about the statistical properties of financial returns. A large percentage of these studies focus on high frequency or daily log price changes. Their conclusions can be summarized in four basic observations:

- Financial return distributions have "fat tails." This means that extreme price movements occur more frequently than implied by a normal distribution.

- The peak of the return distribution is higher and narrower than that predicted by the normal distribution. Note that this characteristic (often referred to as the "thin waist") along with fat tails is a characteristic of a **leptokurtotic** distribution.

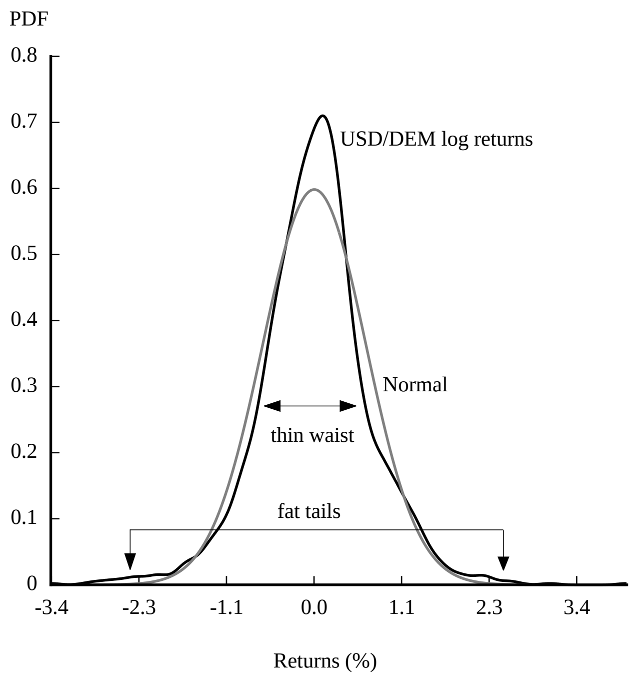
[19] See for example, Engle and Bollerslev (1986).

[20] See, for example, Richardson and Smith (1993)

- Returns have small autocorrelations.
- Squared returns often have significant autocorrelations.

Chart 4.16 illustrates a leptokurtotic distribution of log price changes in USD/DEM exchange rates for the period 28-Mar-96 through 12-Apr-96 and compares it to a normal distribution. In this chart, the leptokurtotic distribution can be thought of as a smoothed histogram, since it is obtained through a smoothing process known as “kernel density estimation.”[21] A kernel density estimate of the histogram, rather than the histogram itself, is often used since it produces a smooth line that is easier to compare to the true density function (normal, in this example).

Chart 4.16
Leptokurtotic vs. normal distribution



4.5.1 Modeling methods

Having documented the failure of the normal distribution to accurately model returns, researchers started looking for alternative modeling methods, which have since evolved into two classes: **unconditional** (time-independent) and **conditional distributions** (time-dependent) of returns.

Models in the class of unconditional distribution of returns assume that returns are independent of each other and that the return-generating process is linear with parameters that are independent of past realizations. An example of a model that falls into this class is the standard normal distribution with mean μ and variance σ^2 (note there is **no** time subscript). Other examples of unconditional distribution models include infinite-variance symmetric and asymmetric stable Paretian distributions, and finite variance distributions including the t-distribution, mixed-diffusion-jump model, and the compound normal model.

[21] See Silverman (1986).

The second class of models, the conditional distribution of returns, arises from evidence that refutes the identically and independently distributed assumptions (as presented in Sections 4.3.1 and 4.3.2). Models in this category, such as the GARCH and Stochastic Volatility, treat volatility as a time-dependent, persistent process. These models are important because they account for volatility clustering, a frequently observed phenomenon among return series.

The models for characterizing returns are presented in Table 4.6 along with supporting references.

Table 4.6
Model classes

Distribution	**Model**		**Reference**
Unconditional (time independent)	Infinite variance:	symmetric stable Paretian	Mandelbrot (1963)
		asymmetric stable Paretian	Tucker (1992)
	Finite variance:	Normal	Bachelier (1900)
		Student t	Blattberg & Gonedes (1974)
		Mixed diffusion jump	Jorion (1988)
		Compound normal	Kon (1988)
Conditional (time dependent)	GARCH:	Normal	Bollerslev (1986)
		Student t	Bollerslev (1987)
	Stochastic Volatility:	Normal	Ruiz (1994)
		Student t	Harvey et. al (1994)
		Generalized error distribution	Ruiz (1994)

It is important to remember that while conditional and unconditional processes are based on different assumptions, except for the unconditional normal model, models from both classes generate data that possess fat tails.[22]

4.5.2 Properties of the normal distribution

All of the models presented in Table 4.6 are parametric in that the underlying distributions depend on various parameters. One of the most widely applied parametric probability distribution is the normal distribution, represented by its “bell shaped” curve.

This section reviews the properties of the normal distribution as they apply to the RiskMetrics method of calculating VaR. Recall that the VaR of a single asset (at time *t*) can be written as follows:

[4.32] $$VaR_t = [1 - \exp(-1.65\sigma_{t|t-1})] V_{t-1}$$

or, using the common approximation

[4.33] $$VaR_t \cong 1.65\sigma_{t|t-1} V_{t-1}$$

where V_{t-1} is the marked-to-market value of the instrument and $\sigma_{t|t-1}$ is the standard deviation of continuously compounded returns for time *t* made at time *t*–1.

[22] For a specific comparison between time-dependent and time-independent processes, see Ghose and Kroner (1993).

4.5.2.1 Mean and variance

If it is assumed that returns are generated according to the normal distribution, then it is believed that the entire distribution of returns can be characterized by two parameters: its mean and variance. Mathematically, the normal probability density function for a random variable r_t is[23]

[4.34] $$f(r_t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp-\left(\frac{1}{2\sigma^2}\right)(r_t - \mu)^2$$

where

μ = mean of the random variable, which affects the location of the distribution′s peak

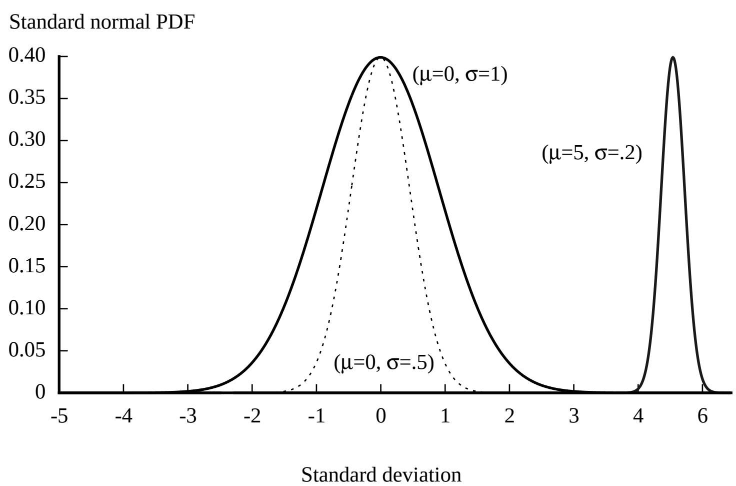
σ^2 = variance of the random variable, which affects the distribution′s width

$\pi \cong 3.1416$

Note that the normal distribution as shown in Eq. [4.34] is an unconditional distribution since the mean and variance parameters are not time-dependent and, therefore, do not have time subscripts.

Chart 4.17 shows how the mean and variance affect the shape of the normal distribution.

Chart 4.17
Normal distribution with different means and variances



Now that we have an understanding of the role of the mean and variance in the normal distribution we can present their formulae. The mathematical expression for the mean and variance of some random variable r_t, are as follows:

[4.35] $$\mu = E[r_t] \quad \text{(mean)}$$
$$\sigma^2 = E\left[(r_t - \mu)^2\right] \quad \text{(variance)}$$

[23] Note that we are abusing notation since r_t represents both a random variable and observed return. We hope that by the context in which r_t is used it will be clear what we are referring to.

where E[ ] denotes the mathematical expectation. Two additional measures that we will make reference to within this document are known as skewness and kurtosis. Skewness characterizes the asymmetry of a distribution around its mean. The expression for skewness is given by

[4.36] $$s^3 = E\left[(r_t - \mu)^3\right] \text{ (skewness)}$$

For the normal distribution skewness is zero. In practice, it is more convenient to work with the skewness coefficient which is defined as

[4.37] $$\gamma = \frac{E\left[(r_t - \mu)^3\right]}{\sigma^3} \text{ (skewness coefficient)}$$

Kurtosis measures the relative peakedness or flatness of a given distribution. The expression for kurtosis is given by

[4.38] $$s^4 = E\left[(r_t - \mu)^4\right] \text{ (kurtosis)}$$

As in the case of skewness, in practice, researchers frequently work with the kurtosis coefficient defined as

[4.39] $$\kappa = \frac{E\left[(r_t - \mu)^4\right]}{\sigma^4} \text{ (kurtosis coefficent)}$$

For the normal distribution, kurtosis is 3. This fact leads to the definition of excess kurtosis which is defined as kurtosis minus 3.

4.5.2.2 Using percentiles to measure market risk

Market risk is often measured in terms of a **percentile** (also referred to as **quantile**) of a portfolio's return distribution. The attractiveness of working with a percentile rather than say, the variance of a distribution, is that a percentile corresponds to both a magnitude (e.g., the dollar amount at risk) and an exact probability (e.g., the probability that the magnitude will not be exceeded).

The *p*th percentile of a distribution of returns is defined as the value that exceeds *p* percent of the returns. Mathematically, the pth percentile (denoted by α) of a continuous probability distribution, is given by the following formula

[4.40] $$p = \int_{-\infty}^{\alpha} f(r)\, dr$$

where $f(r)$ represents the PDF (e.g., Eq. [4.34])

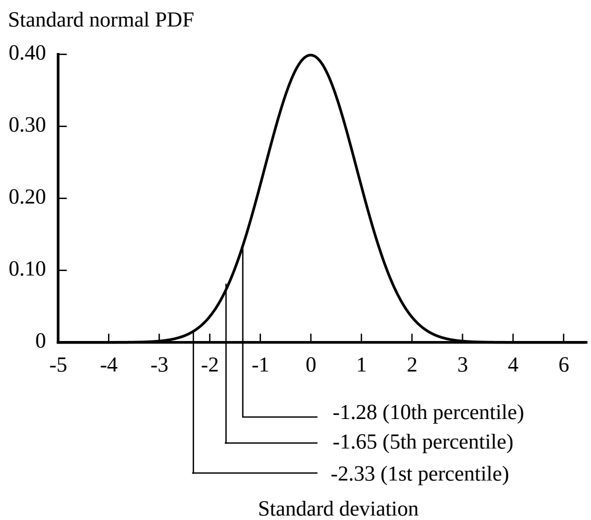
So for example, the 5th percentile is the value (point on the distribution curve) such that 95 percent of the observations lie above it (see Chart 4.18).

When we speak of percentiles they are often of the percentiles of a **standardized distribution,** which is simply a distribution of mean-centered variables scaled by their standard deviation. For example, suppose the log price change r_t is normally distributed with mean μ_t and variance σ_t^2. The standardized return $\tilde{r}_t$ is defined as

[4.41] $$\tilde{r}_t = \frac{r_t - \mu_t}{\sigma_t}$$

Therefore, the distribution of $\tilde{r}_t$ is normal with mean 0 and variance 1. An example of a standardized distribution is presented above ($\mu = 0$, $\sigma = 1$). Chart 4.18 illustrates the positions of some selected percentiles of the standard normal distribution.[24]

Chart 4.18
Selected percentile of standard normal distribution



We can use the percentiles of the standard distribution along with Eq. [4.41] to derive the percentiles of observed returns. For example, suppose that we want to find the 5th percentile of r_t, under the assumption that returns are normally distributed. We know, by definition, that

[4.42a] $$\text{Probability}\,(\tilde{r}_t < -1.65) = 5\%$$

[4.42b] $$\text{Probability}\,[\,(r_t - \mu_t)\,/\,\sigma_t < -1.65] = 5\%$$

From Eq. [4.42b], re-arranging terms yields

[4.43] $$\text{Probability}\,(r_t < -1.65\sigma_t + \mu_t) = 5\%$$

According to Eq. [4.43], there is a 5% probability that an observed return at time *t* is less than –1.65 times its standard deviation plus its mean. Notice that when $\mu_t = 0$, we are left with the standard result that is the basis for short-term horizon VaR calculation, i.e.,

[4.44] $$\text{Probability}\,(r_t < -1.65\sigma_t) = 5\%$$

[24] Note that the selected percentiles above (1%, 5%, and 10%) reside in the *tails* of the distribution. Roughly, the tails of a distribution are the areas where less then, say, 10% of the observations fall.

4.5.2.3 One-tailed and two-tailed confidence intervals

Equation [4.44] is very important as the basis of VaR calculations in RiskMetrics. It should be recognized, however, that there are different ways of stating the confidence interval associated with the same risk tolerance. For example, since the normal distribution is **symmetric**, then

[4.45]
$$\begin{aligned}\text{Probability}\,(r_t < -1.65\sigma_t + \mu_t) &= \text{Probability}\,(r_t > 1.65\sigma_t + \mu_t)\\ &= 5\%\end{aligned}$$

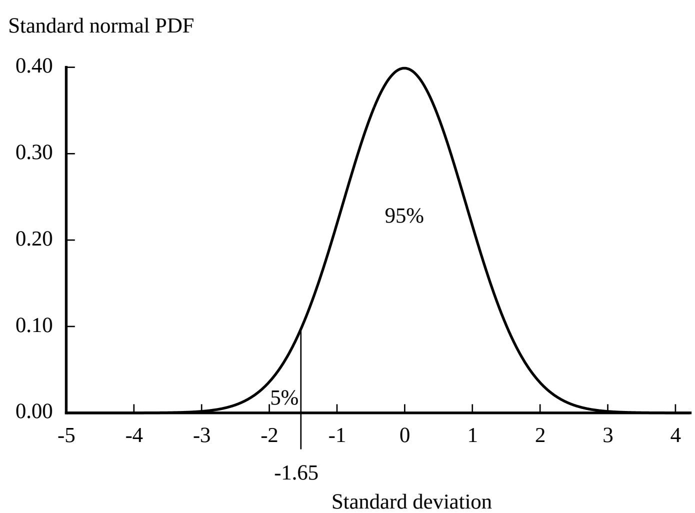
Therefore, since the entire area under the probability curve in Chart 4.18 is 100%, it follows that

[4.46a] $$\text{Probability}\,(-1.65\sigma_t + \mu_t < r_t < 1.65\sigma_t + \mu_t) = 90\%$$

[4.46b] $$\text{Probability}\,(-1.65\sigma_t + \mu_t < r_t) = 95\%$$

Charts 4.19 and 4.20 show the relationship between a one-tailed 95% confidence interval and a two-tailed 90% confidence interval. Notice that the statements in Eqs. [4.46a] and [4.46b] are consistent with Eq. [4.45], a 5% probability that the return being less than –1.65 standard deviations.[25]

Chart 4.19
One-tailed confidence interval



[25] The two statements are not equivalent in the context of formal hypothesis testing. See DeGroot (1989, chapter 8).

Chart 4.20
Two-tailed confidence interval

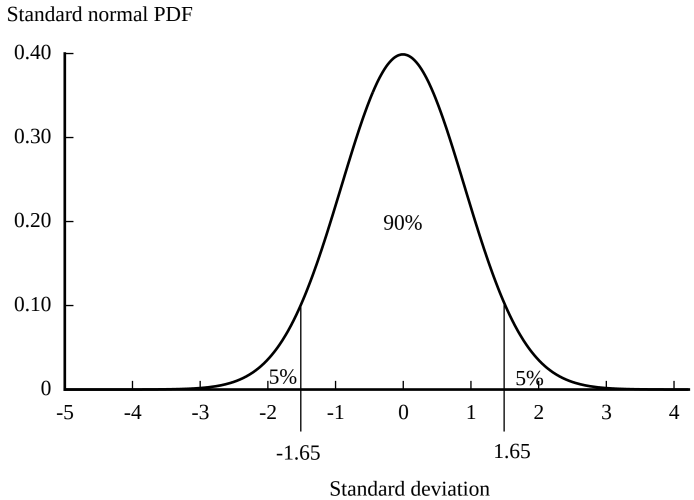


Table 4.7 shows the confidence intervals that are prescribed by standard and BIS-compliant versions of RiskMetrics, and at which the one-tailed and two-tailed tests yield the same VaR figures.[26]

Table 4.7
VaR statistics based on RiskMetrics and BIS/Basel requirements

	Confidence interval	
RiskMetrics method	**One-tailed**	**Two-tailed**
Standard	95% (−1.65σ)	90% (−/+1.65σ)
BIS/Basel Regulatory	99% (−2.33σ)	98% (−/+2.33σ)

4.5.2.4 Aggregation in the normal model
An important property of the normal distribution is that the sum of normal random variables is itself normally distributed.[27] This property is useful since portfolio returns are the weighted sum of individual security returns.

As previously stated (p. 49) RiskMetrics assumes that the return on a portfolio, $r_{p,t}$, is the weighted sum of N underlying returns (see Eq. [4.12]). For practical purposes we require a model of returns that not only relates the underlying returns to one another but also relates the distribution of the weighted sum of the underlying returns to the portfolio return distribution. To take an example, consider the case when $N = 3$, that is, the portfolio return depends on three underlying returns. The portfolio return is given by

[4.47] $$r_{pt} = w_1 r_{1,t} + w_2 r_{2,t} + w_3 r_{3,t}$$

[26] For ease of exposition we ignore time subscripts.

[27] These random variables must be drawn from a multivariate distribution.

We can model each underlying return as a random walk that is similar to Eq. [4.17]. This yields

[4.48a] $$r_{1,t} = \mu_1 + \sigma_{1,t}\varepsilon_{1,t}$$

[4.48b] $$r_{2,t} = \mu_2 + \sigma_{2,t}\varepsilon_{2,t}$$

[4.48c] $$r_{3,t} = \mu_2 + \sigma_{3,t}\varepsilon_{3,t}$$

Now, since we have three variables we must account for their movements relative to one another. These movements are captured by pairwise correlations. That is, we define measures that quantify the linear association between each pair of returns. Assuming that the ε_t's are multivariate normally (*MVN*) distributed we have the model

[4.49] $$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} \\ \rho_{21,t} & 1 & \rho_{23,t} \\ \rho_{31,t} & \rho_{32,t} & 1 \end{bmatrix} \right)$$, or more succinctly, $\varepsilon \sim MVN(\mu_t, R_t)$

where parameter matrix R_t represents the correlation matrix of $(\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t})$. Therefore, if we apply the assumptions behind Eq. [4.49] (that the sum of *MVN* random variables is normal) to the portfolio return Eq. [4.47], we know that r_{pt} is normally distributed with mean $\mu_{p,t}$ and variance $\sigma^2_{p,t}$. The formulae for the mean and variance are

[4.50a] $$\mu_{p,t} = w_1\mu_1 + w_2\mu_2 + w_3\mu_3$$

[4.50b] $$\sigma^2_{p,t} = w_1^2\sigma^2_{p,t} + w_2^2\sigma^2_{p,t} + w_3^2\sigma^2_{p,t} + 2w_1w_2\sigma^2_{12,t} + 2w_1w_3\sigma^2_{13,t} + 2w_2w_3\sigma^2_{23,t}$$

where the terms $\sigma^2_{ij,t}$ represent the covariance between returns i and j. In general, these results hold for $(N \geq 1)$ underlying returns. Since the underlying returns are distributed conditionally multivariate normal, the portfolio return is univariate normal with a mean and variance that are simple functions of the underlying portfolio weights, variances and covariances.

4.5.3 The lognormal distribution

In Section 4.2.1 we claimed that if log price changes are normally distributed, then price, P_t, conditional on P_{t-1} is lognormally distributed. This statement implies that P_t, given P_{t-1}, is drawn from the probability density function

[4.51] $$f(P_t) = \frac{1}{P_{t-1}\sigma_t\sqrt{2\pi}} \exp\left[\frac{-(\ln P_{t-1} - \mu)^2}{2\sigma_t^2}\right] \qquad P_{t-1} > 0$$

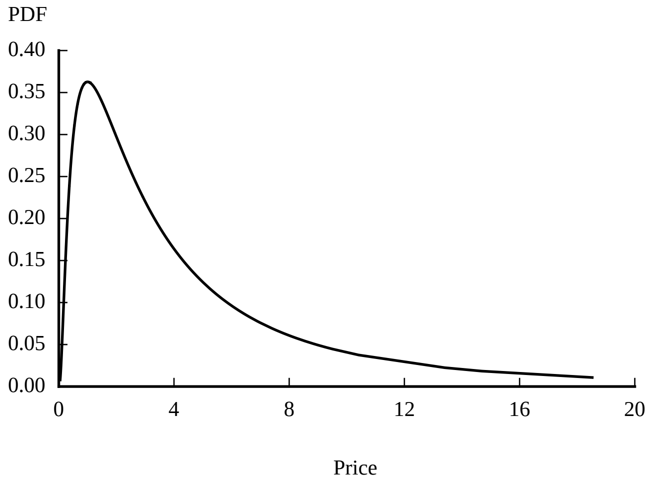
where P_t follows a lognormal distribution with a mean and variance given by

[4.52] $$E[P_t] = \exp\left(\mu + 5\sigma_t^2\right)$$

[4.53] $$V(P_t) = \left[\exp 2\mu_t \cdot \exp\left(2\sigma_t^2\right) - \exp\left(\sigma_t^2\right)\right]$$

Chart 4.21 shows the probability density function for the lognormal random variable P_t when $\mu_t = 0, \sigma_t = 1$ and $P_{t-1} = 1$.

Chart 4.21
Lognormal probability density function



Unlike the normal probability density function, the lognormal PDF has a lower bound greater than zero and is skewed to the right.

4.6 RiskMetrics model of financial returns: A modified random walk

We can now use the results of the last four sections to write down a model of how returns are generated over time. Our analysis has shown that:

- Return variances are heteroscedastic (change over time) and autocorrelated.
- Return covariances are autocorrelated and possess dynamic features.
- The assumption that returns are normally distributed is useful because of the following:

  (i) only the mean and variance are required to describe the entire shape of the distribution[28]

  (ii) the sum of multivariate normal returns is normally distributed. This fact facilitates the description of portfolio returns, which are the weighted sum of underlying returns.

Given these points, we can now state the assumptions underlying the RiskMetrics variance/covariance methodology. Consider a set of N securities, $i = 1\ldots, N$. The RiskMetrics model assumes that returns are generated according to the following model

$$\begin{aligned} & r_{i,t} = \sigma_{i,t}\varepsilon_{i,t} \qquad \varepsilon_{it} \sim N(0, 1) \\ & \varepsilon_t \sim MVN(0, R_t) \qquad \varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt}] \end{aligned} \quad [4.54]$$

[28] The covariances are also required when there is more than one return series.

where R_t is an NxN time-dependent correlation matrix. The variance of each return, $\sigma^2_{i,t}$ and the correlation between returns, $\rho_{ij,t}$, are a function of time. The property that the distribution of returns is normal given a time dependent mean and correlation matrix assumes that returns follow a conditional normal distribution—conditional on time. Notice that in Eq. [4.54] we excluded term μ_i. As will be discussed in more detail in Section 5.3.1.1, the mean return represented by μ_i is set to zero.

In Appendix A we propose a set of statistical tests to assess whether observed financial returns follow a conditional normal distribution. In Appendix B we discuss alternative distributions that relax the normality assumption.

4.7 Summary

In this chapter, we presented the statistical and probability assumptions on the evolution and distribution of financial returns in some simple models. This discussion served as background to the specification of the assumptions behind the RiskMetrics VaR methodology.

In review, this chapter covered the following subjects. The chapter began by outlining a simple version of the VaR calculation. We then:

- Defined absolute price change, relative price change, log price change, and returns.
- Showed the importance of understanding the use of different price change definitions.
- Established that RiskMetrics measures changes in portfolio value in terms of continuously-compounded returns.
- Introduced temporal aggregation and cross-section aggregation to show the implications of working with relative and log returns.
- Introduced the random walk model for:[29]
  - Single-price assets
  - Fixed income instruments
- Found evidence that contradicts the assumption that returns are IID (identically and independently) normal. In reality, continuously compounded returns are:
  - Not identical over time. (The variance of the return distribution changes over time)
  - Not statistically independent of each other over time. (Evidence of autocorrelation between return series and within a return series.)
- Explained the properties of the normal distribution, and, lastly,
- Presented the RiskMetrics model as a modified random walk that assumes that returns are conditionally normally distributed.

[29] While the random walk model serves as the basis for many popular models of returns in finance, another class of models that has received considerable attention lately is based on the phenomenon of **long-range dependence**. Briefly, such models are built on the notion that observations recorded in the distant past are correlated to observations in the distant future. (See Campbell, et. al (1995) for a review of long-range dependence models.)

Chapter 5. Estimation and forecast

Chapter 5. Estimation and forecast

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In this chapter we present a methodology for forecasting the parameters of the multivariate conditional normal distribution, i.e., variances and covariances of returns whose empirical properties were examined in Chapter 4, "Statistical and probability foundations." The reason for forecasting variances and covariances of returns is to use them to forecast a portfolio's change in value over a given horizon, which can run over one day to several months.

This chapter is organized as follows:

- Section 5.1 briefly explains why RiskMetrics forecasts of variances and covariances are generated from historical data rather than derived from option prices.
- Section 5.2 describes the RiskMetrics forecasting methodology, i.e.,
  - Use of the exponentially weighted moving average (EWMA) model to produce forecasts of variances and covariances. This includes an explanation as to why the EWMA is preferred to the simple moving average model.
  - How to compute forecasts over longer time horizons, such as one month.

  Section 5.2 also discusses alternative, more advanced methods for forecasting variances and covariances.
- Section 5.3 explains two important implementation issues involving the RiskMetrics forecasts: (1) the reliability of the forecasts in relation to the number of historical data points used to produce them, and (2) the choice of the "decay factor" used in the EWMA model.
- Section 5.4 concludes the chapter with a review of the RiskMetrics forecasting model.

Finally, practitioners often refer to the term "volatility" when speaking of movements in financial prices and rates. In what follows we use the term volatility to mean the standard deviation of continuously compounded financial returns.

5.1 Forecasts from implied versus historical information

RiskMetrics forecasts are based on historical price data, although in theory, they may be derived from option prices.

From a practical point of view, implied forecasts introduce a number of problems. For example, an implied volatility (IV) is based entirely on expectations given a particular option pricing model. Therefore, as noted in Kroner, Kneafsey and Claessens (1995), since most option pricing models assume that the standard deviation is constant, the IV becomes difficult to interpret and will not lead to good forecasts if the option formula used to derive it is not correctly specified. Moreover, IV forecasts are associated with a fixed forecast horizon. For example, the implied volatility derived from a 3 month USD/DEM option is exclusively for a 3 month forecast horizon. However, a risk manager may be interested in the VaR of this option over the next day.

If RiskMetrics were to use implied statistics, it would require observable options prices on all instruments that compose a portfolio. Currently, the universe of consistently observable options prices is not large enough to provide a complete set of implied statistics; generally only exchange-traded options are reliable sources of prices. In particular, the number of implied correlations that can be derived from traded option prices is insignificant compared to the number of correlations required to estimate risks in portfolios consisting of many types of assets.

Academic research has compared the forecasting ability of implied and historical volatility models. The evidence of the superior forecasting ability of historical volatility over implied volatility is mixed, depending on the time series considered. For example, Xu and Taylor (1995, p. 804) note that, “prior research concludes that volatility predictors calculated from options prices are better predictors of future volatility than standard deviations calculated from historical asset price data.” Kroner, Kneafsey and Claessens (1995, p. 9), on the other hand, note that researchers are beginning to conclude that GARCH (historical based) forecasts outperform implied volatility forecasts. Since implied standard deviation captures market expectations and pure time series models rely solely on past information, these models can be combined to forecast the standard deviation of returns.

5.2 RiskMetrics forecasting methodology

RiskMetrics uses the exponentially weighted moving average model (EWMA) to forecast variances and covariances (volatilities and correlations) of the multivariate normal distribution. This approach is just as simple, yet an improvement over the traditional volatility forecasting method that relies on moving averages with fixed, equal weights. This latter method is referred to as the simple moving average (SMA) model.

5.2.1 Volatility estimation and forecasting[1]

One way to capture the dynamic features of volatility is to use an exponential moving average of historical observations where the latest observations carry the highest weight in the volatility estimate. This approach has two important advantages over the equally weighted model. First, volatility reacts faster to shocks in the market as recent data carry more weight than data in the distant past. Second, following a shock (a large return), the volatility declines exponentially as the weight of the shock observation falls. In contrast, the use of a simple moving average leads to relatively abrupt changes in the standard deviation once the shock falls out of the measurement sample, which, in most cases, can be several months after it occurs.

For a given set of T returns, Table 5.1 presents the formulae used to compute the equally and exponentially weighted (standard deviation) volatility.

Table 5.1
Volatility estimators*

Equally weighted	Exponentially weighted
$\sigma = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^2}$	$\sigma = \sqrt{(1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_t - \bar{r})^2}$

* In writing the volatility estimators we intentionally do not use time subscripts.

In comparing the two estimators (equal and exponential), notice that the exponentially weighted moving average model depends on the parameter λ ($0 < \lambda < 1$) which is often referred to as the **decay factor**. This parameter determines the relative weights that are applied to the observations (returns) and the effective amount of data used in estimating volatility. Ways of estimating λ are discussed in detail in Section 5.3.2.

[1] In this section we refer loosely to the terms estimation and forecast. The reader should note, however, that these terms do have distinct meanings.

We point out that in writing the EWMA estimator in Table 5.1 we applied the approximation

[5.1] $$\sum_{j=1}^{T} \lambda^{j-1} \cong \frac{1}{(1-\lambda)}$$

These two expressions are equivalent in the limit, i.e., as $T \to \infty$. Moreover, for purpose of comparison to the equally weighted factor 1/*T*, the more appropriate version of the EWMA is

[5.2] $$\lambda^{t-1} \Big/ \sum_{j=1}^{T} \lambda^{j-1}$$

rather than $(1-\lambda)\lambda^{t-1}$. Also, notice that when $\lambda = 1$, Eq. [5.2] collapses to 1/*T*.

Charts 5.1 and 5.2 highlight an important difference between equally and exponentially weighted volatility forecasts using as an example the GBP/DEM exchange rate in the fall of 1992. In late August of that year, the foreign exchange markets went into a turmoil that led a number of Europe's currencies to leave the ERM and be devalued. The standard deviation estimate using an exponential moving average rapidly reflected this state of events, but also incorporated the decline in volatility over subsequent months. The simple 6-month moving average estimate of volatility took longer to register the shock to the market and remained higher in spite of the fact that the foreign exchange markets calmed down over the rest of the year.

Chart 5.1
DEM/GBP exchange rate

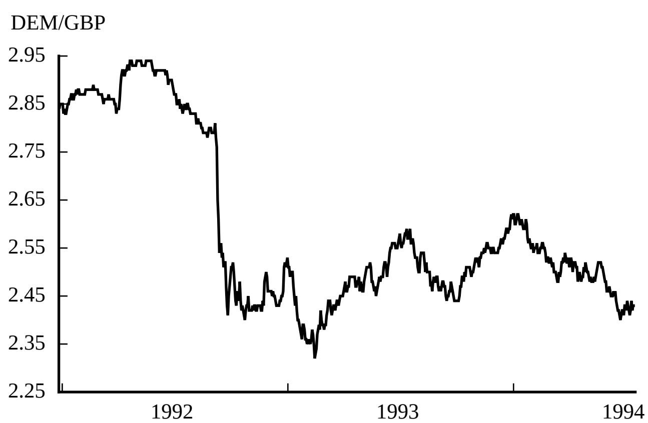
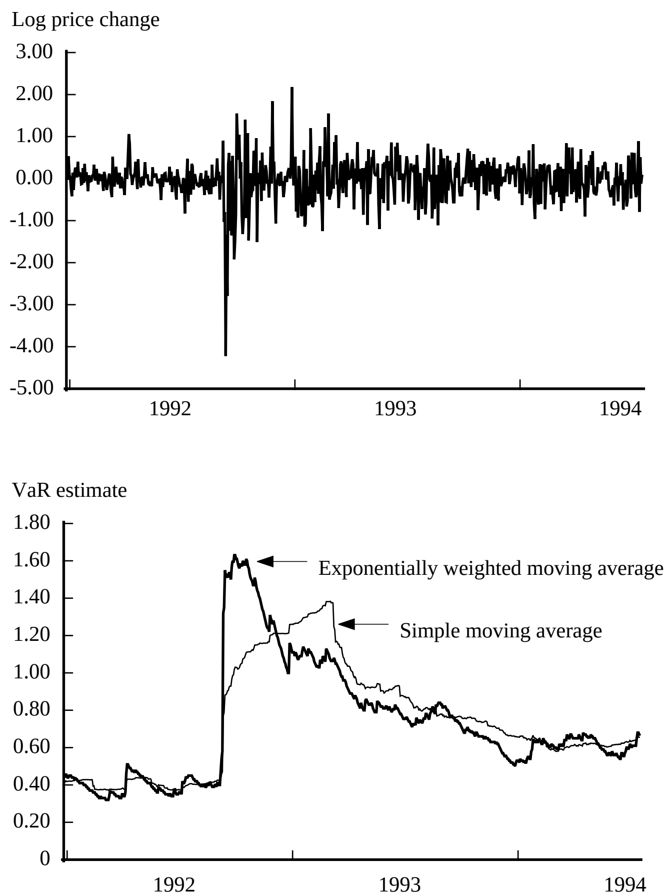


Chart 5.2
Log price changes in GBP/DEM and VaR estimates (1.65σ)



This example would suggest that EWMA is more satisfactory, given that when combined with frequent updates, it incorporates external shocks better than equally weighted moving averages, thus providing a more realistic measure of current volatility.

Although the exponentially weighted moving average estimation ranks a level above simple moving averages in terms of sophistication, it is not complex to implement. To support this point, Table 5.2 presents an example of the computation required to estimate equally and exponentially weighted moving average volatilities. Volatility estimates are based on 20 daily returns on the USD/DEM exchange rate. We arbitrarily choose $\lambda = 0.94$ and keep matters simple by setting the sample mean, $\bar{r}$, to zero.

Table 5.2
Calculating equally and exponentially weighted volatility

	A	B	C	D	Volatility	
Date	**Return USD/DEM (%)**	**Return squared (%)**	**Equal weight (T = 20)**	**Exponential weight (λ = 0.94)**	**Equally weighted, B × C**	**Exponentially weighted, B × D**
28-Mar-96	0.634	0.402	0.05	0.019	0.020	0.007
29-Mar-96	0.115	0.013	0.05	0.020	0.001	0.000
1-Apr-96	-0.460	0.211	0.05	0.021	0.011	0.004
2-Apr-96	0.094	0.009	0.05	0.022	0.000	0.000
3-Apr-96	0.176	0.031	0.05	0.024	0.002	0.001
4-Apr-96	-0.088	0.008	0.05	0.025	0.000	0.000
5-Apr-96	-0.142	0.020	0.05	0.027	0.001	0.001
8-Apr-96	0.324	0.105	0.05	0.029	0.005	0.003
9-Apr-96	-0.943	0.889	0.05	0.030	0.044	0.027
10-Apr-96	-0.528	0.279	0.05	0.032	0.014	0.009
11-Apr-96	-0.107	0.011	0.05	0.034	0.001	0.000
12-Apr-96	-0.160	0.026	0.05	0.037	0.001	0.001
15-Apr-96	-0.445	0.198	0.05	0.039	0.010	0.008
16-Apr-96	0.053	0.003	0.05	0.041	0.000	0.000
17-Apr-96	0.152	0.023	0.05	0.044	0.001	0.001
18-Apr-96	-0.318	0.101	0.05	0.047	0.005	0.005
19-Apr-96	0.424	0.180	0.05	0.050	0.009	0.009
22-Apr-96	-0.708	0.501	0.05	0.053	0.025	0.027
23-Apr-96	-0.105	0.011	0.05	0.056	0.001	0.001
24-Apr-96	-0.257	0.066	0.05	0.060	0.003	0.004
			Standard deviation:	Equally weighted	0.393	
				Exponentially weighted		0.333

Notice that the difference between the two estimated standard deviations results from the different weighting schemes. Whereas the equally weighted approach weights each squared return by 5%, the exponentially weighted scheme applies a 6% weight to the most recent squared return and 1.9% weight to the most distant observation.

An attractive feature of the exponentially weighted estimator is that it can be written in **recursive** form which, in turn, will be used as a basis for making volatility forecasts. In order to derive the recursive form, it is assumed that an infinite amount of data are available. For example, assuming again that the sample mean is zero, we can derive the period $t + 1$ variance forecast, given data available at time t (one day earlier) as

[5.3] $$\sigma^2_{1,\, t+1|t} = \lambda\sigma^2_{1,\, t|t-1} + (1-\lambda)\, r^2_{1,\, t}$$

The 1-day RiskMetrics volatility forecast is given by the expression

[5.4] $$\sigma_{1,\, t+1|t} = \sqrt{\lambda\sigma^2_{1,\, t|t-1} + (1-\lambda)\, r^2_{1,\, t}}$$

The subscript "$t + 1|t$" is read "the time $t + 1$ forecast given information up to and including time t." The subscript "$t|t - 1$" is read in a similar fashion. This notation underscores the fact that we are treating the variance (volatility) as time-dependent. The fact that this period's variance forecast depends on last period's variance is consistent with the observed autocorrelation in squared returns discussed in Section 4.3. We derive Eq. [5.3] as follows.

[5.5]
$$\begin{aligned}\sigma^2_{1,t+1|t} &= (1-\lambda)\sum_{i=0}^{\infty}\lambda^i r^2_{1,t-i} \\ &= (1\text{-}\lambda)\left(r^2_{1,t} + \lambda r^2_{1,t-1} + \lambda^2 r^2_{1,t-2} + \ldots\right) \\ &= (1\text{-}\lambda)\, r^2_{1,t} + \lambda(1-\lambda)\left(r^2_{1,t-1} + \lambda r^2_{1,t-2} + r^2_{1,t-3}\right) \\ &= \lambda\sigma^2_{1,t|t-1} + (1-\lambda)\, r^2_{1,t}\end{aligned}$$

Using daily returns, Table 5.3 presents an example of how Eq. [5.3] can be used in practice to produce a 1-day volatility forecast on USD/DEM returns for the period March 28 through April 24, 1996.

Table 5.3
Applying the recursive exponential weighting scheme to compute volatility
Daily returns on USD/DEM

	A	B		A	B
Date	**Return USD/DEM**	**Recursive variance**	**Date**	**Return USD/DEM**	**Recursive variance**
28-Mar-96	0.633	0.401	11-Apr-96	–0.107	0.296
29-Mar-96	0.115	0.378	12-Apr-96	–0.159	0.280
1-Apr-96	–0.459	0.368	15-Apr-96	–0.445	0.275
2-Apr-96	0.093	0.346	16-Apr-96	0.053	0.258
3-Apr-96	0.176	0.327	17-Apr-96	0.152	0.244
4-Apr-96	–0.087	0.308	18-Apr-96	–0.318	0.236
5-Apr-96	–0.142	0.291	19-Apr-96	0.424	0.232
8-Apr-96	0.324	0.280	22-Apr-96	–0.708	0.248
9-Apr-96	–0.943	0.316	23-Apr-96	–0.105	0.234
10-Apr-96	–0.528	0.314	24-Apr-96	–0.257	0.224

*Initial variance forecast = initial return squared. Figures following this number are obtained by applying the recursive formula.

The volatility forecast made on April 24 for the following day is the square root of 0.224% (the variance) which is 0.473%.

5.2.1.1 Covariance and correlation estimation and forecasts
We use the EWMA model to construct covariance and correlation forecasts in the same manner as we did volatility forecasts except that instead of working with the square of one series, we work with the product of two different series. Table 5.4 presents covariance estimators based on equally and exponentially weighted methods.

Table 5.4
Covariance estimators

Equally weighted	Exponentially weighted
$\sigma_{12}^2 = \frac{1}{T}\sum_{t=1}^{T}(r_{1t}-\bar{r}_1)(r_{1t}-\bar{r}_2)$	$\sigma_{12}^2 = (1-\lambda)\sum_{j=1}^{T}\lambda^{j-1}(r_{1t}-\bar{r}_1)(r_{1t}-\bar{r}_2)$

Analogous to the expression for a variance forecast (Eq. [5.3]), the covariance forecast can also be written in recursive form. For example, the 1-day covariance forecast between any two return series, $r_{1,t}$ and $r_{2,t}$ made at time t is

[5.6] $$\sigma^2_{12,\,t+1|t} = \lambda\sigma^2_{12,\,t|t-1} + (1-\lambda)\,r_{1t}\cdot r_{2t}$$

We can derive Eq. [5.6] as follows.

[5.7] $$\begin{aligned}\sigma^2_{1,\,t+1|t} &= (1-\lambda)\sum_{i=0}^{\infty}\lambda^i r_{1,\,t-i}\cdot r_{2,\,t-i}\\ &= (1-\lambda)\left(r_{1,\,t}\cdot r_{2,\,t} + \lambda r_{1,\,t-1}\cdot r_{2,\,t-1} + \lambda^2 r_{2,\,t-2}\cdot r_{2,\,t-2} + \ldots\right)\\ &= (1-\lambda)\,r_{1,\,t}\cdot r_{2,\,t}\\ &\quad + \lambda(1-\lambda)\left(r_{1,\,t-1}\cdot r_{2,\,t-1} + \lambda r_{1,\,t-2}\cdot r_{2,\,t-2} + \lambda^2 r_{1,\,t-3}\cdot r_{2,\,t-3}\right)\\ &= \lambda\sigma^2_{12,\,t|t-1} + (1-\lambda)\,r_{1,\,t-1}\cdot r_{2,\,t-1}\end{aligned}$$

In order to derive correlation forecasts we apply the corresponding covariance and volatility forecast. Recall that correlation is the covariance between the two return series, say, $r_{1,t}$ and $r_{2,t}$, divided by the product of their standard deviations. Mathematically, **the one-day RiskMetrics prediction of correlation is given by the expression**

[5.8] $$\rho_{12,\,t+1|t} = \frac{\sigma^2_{12,\,t+1|t}}{\sigma_{1,\,t+1|t}\,\sigma_{2,\,t+1|t}}$$

Table 5.5 presents an example of how to compute recursive covariance and correlation forecasts applied to the USD/DEM exchange rate and S&P 500 return series.

Table 5.5
Recursive covariance and correlation predictor

Date	Returns USD/DEM (%)	Returns S&P 500 (%)	Recursive variance USD/DEM	Recursive variance S&P 500	Recursive covariance (λ = 0.94)	Recursive correlation (λ = 0.94)
28-Mar-96	0.634	0.005	0.402	0.000	0.003	1.000
29-Mar-96	0.115	–0.532	0.379	0.017	–0.001	–0.011
1-Apr-96	-0.460	1.267	0.369	0.112	–0.036	–0.176
2-Apr-96	0.094	0.234	0.347	0.109	–0.032	–0.166
3-Apr-96	0.176	0.095	0.328	0.103	–0.029	–0.160
4-Apr-96	-0.088	–0.003	0.309	0.097	–0.028	–0.160
5-Apr-96	-0.142	–0.144	0.291	0.092	–0.025	–0.151
8-Apr-96	0.324	–1.643	0.280	0.249	–0.055	–0.209
9-Apr-96	-0.943	–0.319	0.317	0.240	–0.034	–0.123
10-Apr-96	-0.528	–1.362	0.315	0.337	0.011	0.035
11-Apr-96	-0.107	–0.367	0.296	0.325	0.013	0.042
12-Apr-96	-0.160	0.872	0.280	0.351	0.004	0.012
15-Apr-96	-0.445	0.904	0.275	0.379	–0.020	–0.063
16-Apr-96	0.053	0.390	0.259	0.365	–0.018	–0.059
17-Apr-96	0.152	–0.527	0.245	0.360	–0.022	–0.073
18-Apr-96	-0.318	0.311	0.236	0.344	–0.026	–0.093
19-Apr-96	0.424	0.227	0.233	0.327	–0.019	–0.069
22-Apr-96	-0.708	0.436	0.249	0.318	–0.036	–0.129
23-Apr-96	-0.105	0.568	0.235	0.319	–0.038	–0.138
24-Apr-96	-0.257	–0.217	0.224	0.302	–0.032	–0.124

Note that the starting points for recursion for the covariance is 0.634×0.005. From Table 5.5 we can see that the correlation prediction for the period 24-Apr-96 through 25-Apr-96 is –12.4%.

5.2.2 Multiple day forecasts

Thus far, we have presented 1-day forecasts which were defined over the period t through $t + 1$, where each t represents one business day. Risk managers, however, are often interested in forecast horizons greater than one-day. We now demonstrate how to construct variance (standard deviation) and covariance (correlation) forecasts using the EWMA model over longer time horizons. Generally speaking, the T-period (i.e., over T days) forecasts of the variance and covariance are, respectively,

$$\sigma^2_{1,\,t+T|t} = T\sigma^2_{1,\,t+1|t} \quad \text{or} \quad \sigma_{1,\,t+T|t} = \sqrt{T}\sigma_{1,\,t+1|t} \qquad [5.9]$$

and

$$\sigma^2_{12,\,t+T|t} = T\sigma^2_{12,\,t+1|t} \qquad [5.10]$$

Equations [5.9] and [5.10] imply that the correlation forecasts remain unchanged regardless of the forecast horizon. That is, [5.11] $$\rho_{t+T|t} = \frac{T\sigma^2_{12,t+1|t}}{\sqrt{T}\sigma_{1,t+1|t}\sqrt{T}\sigma_{2,t+1|t}} = \rho_{t+1|t}$$

Notice that multiple day forecasts are simple multiples of one-day forecasts. For example, if we define one month to be equivalent to 25 days, then the 1-month variance and covariance forecasts are 25 times the respective 1-day forecasts and the 1-month correlation is the same as the one-day correlation.[2] We now show how we arrive at Eq. [5.9] and Eq. [5.10].

Recall that RiskMetrics assumes that log prices p_t are generated according to the model

[5.12] $$p_{1,t} = p_{1,t-1} + \sigma_{1,t}\varepsilon_{1,t} \qquad \varepsilon_{1,t} \sim IID\ N(0,1)$$

Recursively solving Eq. [5.12] and writing the model in terms of returns, we get

[5.13] $$r_{1,t+T} = \sum_{s=1}^{T} \sigma_{1,t+s}\varepsilon_{1,t+s}$$

Taking the variance of Eq. [5.13] as of time t implies the following expression for the forecast variance

[5.14] $$\sigma^2_{1,t+T} = E_t\left[r^2_{1,t+T}\right] = \sum_{s=1}^{T} E_t\left[\sigma^2_{1,t+s}\right]$$

Similar steps can be used to find the T days-ahead covariance forecast, i.e.,

[5.15] $$\sigma^2_{12,t+T} = E_t\left[r_{1,t+T} \cdot r_{2,t+T}\right] = \sum_{s=1}^{T} E_t\left[\sigma^2_{12,t+s}\right]$$

Now, we need to evaluate the right-hand side of Eq. [5.14] and Eq. [5.15]. To do so, we work with the recursive form of the EWMA model for the variance and covariance. To make matters concrete, consider the case where we have two (correlated) return series, $r_{1,t}$ and $r_{2,t}$. In vector form[3], let's write the 1-day forecast of the two variances and covariance as follows:

[2] In RiskMetrics, 1-day and 1-month forecasts differ because we use different decay factors when making the forecasts.

[3] We use the "vec representation" as presented in Engle and Kroner (1995).

[5.16]
$$\sigma^2_{t+1|t} = \begin{bmatrix} \sigma^2_{1,t+1|t} \\ \sigma^2_{12,t+1|t} \\ \sigma^2_{2,t+1|t} \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} \sigma^2_{1,t|t-1} \\ \sigma^2_{12,t|t-1} \\ \sigma^2_{2,t|t-1} \end{bmatrix} + \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} r^2_{1,t} \\ r_{1,t} \cdot r_{2,t} \\ r^2_{2,t} \end{bmatrix}$$

Using the expectation operator at time t, write the forecast over S days as

[5.17]
$$E_t\left[\sigma^2_{t+s}\right] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} E_t\left[\sigma^2_{1,t+s-1}\right] \\ E_t\left[\sigma^2_{12,t+s-1}\right] \\ E_t\left[\sigma^2_{2,t+s-1}\right] \end{bmatrix} + \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} E_t\left[r^2_{1,t+s-1}\right] \\ E_t\left[r_{1,t+s-1} \cdot r_{2,t+s-1}\right] \\ E_t\left[r^2_{2,t+s-1}\right] \end{bmatrix}$$

Evaluating the expectations of the squared returns and their cross product yields

[5.18]
$$E_t\left[\sigma^2_{t+s}\right] = \left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} + \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \right) \begin{bmatrix} E_t\left[\sigma^2_{1,t+s-1}\right] \\ E_t\left[\sigma^2_{12,t+s-1}\right] \\ E_t\left[\sigma^2_{2,t+s-1}\right] \end{bmatrix} = E_t\left[\sigma^2_{t+s-1}\right]$$

That is, the variance forecasts for two consecutive periods are the same. Consequently, the T-period forecast is defined as

[5.19]
$$\sigma^2_{t+T|t} = \sum_{s=1}^{T} E_t\left[\sigma^2_{t+s}\right] = T \cdot E_t\left[\sigma^2_{t+1}\right]$$

so that the T-period forecast of the variance/covariance vector is

[5.20]
$$\sigma^2_{t+T|t} = T \cdot \sigma^2_{t+1|t}$$

This leads to the "square root of time" relationship for the standard deviation forecast

$$[5.21] \qquad \sigma_{1,\, t+T|t} = \sqrt{T} \cdot \sigma_{1,\, t+1|t}$$

Having found that volatility and covariance forecasts scale with time, a few points are worth noting about Eq. [5.21]. Typically, the "square root of time rule" results from the assumption that variances are constant. Obviously, in the above derivation, volatilities and covariances vary with time. Implicitly, what we are assuming in modeling the variances and covariances as exponentially weighted moving averages is that the variance process is nonstationary. Such a model has been studied extensively in the academic literature (Nelson 1990, Lumsdaine, 1995) and is referred to as the IGARCH model.[4]

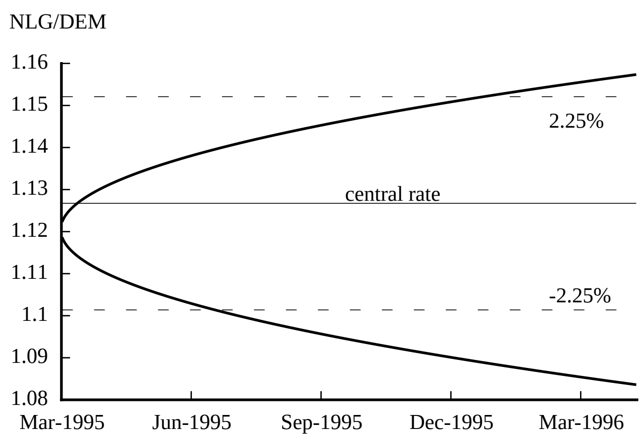
In practice, scaling up volatility forecasts may sometimes lead to results that do not make much sense. Three instances when scaling up volatility estimates prove problematic are:

- When rates/prices are mean-reverting (see Section 4.2.3)
- When boundaries limit the potential movements in rates and prices
- When estimates of volatilities optimized to forecast changes over a particular horizon are used for another horizon (jumping from daily to annual forecasts, for example).

Take the simple example of the Dutch guilder to Deutsche mark exchange rate. On March 22, 1995, the cross rate as quoted at London close of business was 1.12048 NLG/DEM. The RiskMetrics daily volatility estimate was 0.1648%, which meant that over the next 24 hours, the rate was likely to move within a 1.1186 to 1.1223 range with 90% probability (the next day's rate was 1.1213 NLG/DEM).

The Netherlands and Germany have maintained bilateral 2.25% bands within the ERM so scaling up a daily volatility estimate can quickly lead to exchange rate estimates which are extremely unlikely to occur in reality. An example of this is shown by Chart 5.3:

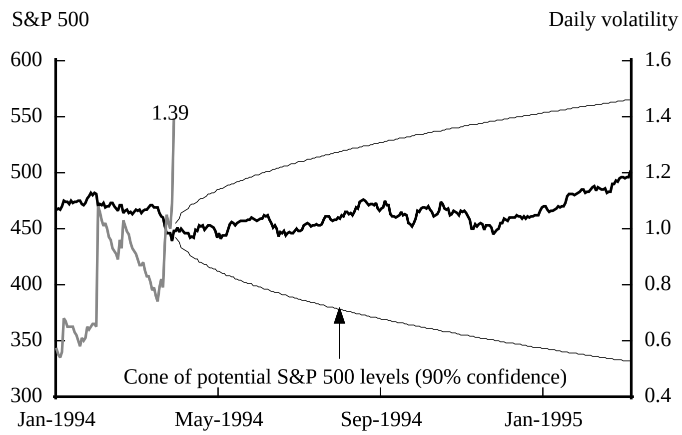
Chart 5.3
NLG/DEM exchange rate and volatility



[4] Note that whereas we essentially arrive at a model that reflects an IGARCH (without an intercept), our motivation behind its derivation was more "bottom up" in the sense that we wanted to derive a model that is generally consistent with observed returns while being simple to implement in practice. The formal approach to IGARCH is more "top down" in that a formal statistical model is written down which then maximum likelihood estimation is used to estimate its parameters.

Applying the square root of time rule with caution does not apply exclusively to exchange rates that are constrained by political arrangements. Suppose you had been trying to forecast the S&P 500's potential annual volatility on April 5, 1994. The index stood at 448.3 and its previous declines had increased the daily volatility estimate to 1.39%. Chart 5.4 extends this daily volatility estimate out to the end of the first quarter of 1995 using the square root of time rule. The chart shows how a short term increase in daily volatility would bias an estimate of volatility over any other time horizon, for example, a year.

Chart 5.4
S&P 500 returns and VaR estimates (1.65σ)



The preceding two examples underscore the importance of understanding how volatility estimates for horizons longer than a day are calculated. When daily volatility forecasts are scaled, nonsensical results may occur because the scale factor does not account for real-world restrictions.

5.2.3 More recent techniques

Research in finance and econometrics has devoted significant efforts in recent years to come up with more formal methods to estimate standard deviations and correlations. These are often referred to as volatility models. The methods range from extreme value techniques (Parkinson, 1980) and two step regression analysis (Davidian and Carroll, 1987), to more complicated nonlinear modelling such as GARCH (Bollerslev, 1986), stochastic volatility (Harvey et. al, 1994) and applications of chaotic dynamics (LeBaron, 1994). Among academics, and increasingly among practitioners, GARCH-type models have gained the most attention. This is due to the evidence that time series realizations of returns often exhibit time-dependent volatility. This idea was first formalized in Engle's (1982) ARCH (Auto Regressive Conditional Heteroscedasticity) model which is based on the specification of conditional densities at successive periods of time with a time-dependent volatility process.

Of the methods just mentioned, the least computationally demanding procedures for estimating volatility are the extreme value and regression methods. Extreme value estimators use various types of data such as high, low, opening and closing prices and transaction volume. While this approach is known for its relative efficiency (i.e., small variance), it is subject to bias. On the other hand, the two step regression method treats the underlying volatility model as a regression involving the absolute value of returns on lagged values. Applications of this method to monthly volatility can be found in Schwert (1989) and Pagan and Schwert (1990).

Since the introduction of the basic ARCH model, extensions include generalized ARCH (GARCH), Integrated GARCH (IGARCH), Exponential GARCH (EGARCH) and Switching

Regime ARCH (SWARCH), just to name a few. Numerous tests of GARCH-type models to foreign exchange and stock markets have demonstrated that these relatively sophisticated approaches can provide somewhat better estimates of volatility than simple moving averages, particularly over short time horizons such as a day or a week.

More recent research on modeling volatility involves Stochastic Volatility (SV) models. In this approach, volatility may be treated as an unobserved variable, the logarithm of which is modeled as a linear stochastic process, such as an autoregression. Since these models are quite new, their empirical properties have yet to be established. However, from a practical point of view, an appealing feature of the SV models is that their estimation is less daunting than their counterpart EGARCH models.[5]

In a recent study, West and Cho (1995) found that GARCH models did not significantly outperform the equally weighted standard deviation estimates in out-of-sample forecasts, except for very short time horizons. In another study on foreign exchange rates and equity returns, Heynen and Kat (1993) showed that while GARCH models have better predictive ability for foreign exchange, the advantage over a simple random walk estimator disappears when the outlook period chosen is more than 20 days.

We have elected to calculate the volatilities and correlations in the RiskMetrics data set using exponential moving averages. This choice is viewed as an optimal balance given the constraints under which most risk management practitioners work.

Since the GARCH models are becoming more popular among practitioners, we demonstrate the behavior of the daily volatility estimator by comparing its forecasts to those produced by a GARCH(1,1) volatility model with normal disturbances. If r_t represents the time t daily return, then the return generating process for the GARCH(1,1) volatility model is given by

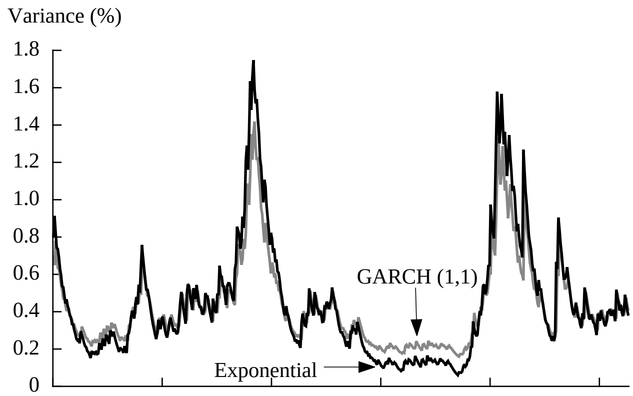
[5.22]
$$\begin{aligned} r_t &= \sigma_t \varepsilon_t \qquad \varepsilon_t \sim IID\ N(0,1) \\ \sigma_t^2 &= 0.0147 + 0.881\sigma_{t-1}^2 + 0.0828 r_{t-1}^2 \end{aligned}$$

This model is parameterized according to the results produced in Ruiz (1993). They were estimated from daily return data for the British pound. The following graph shows variance forecasts produced by this model and the exponential estimator with the decay factor set to 0.94. The forecasts from the EWMA are based on the following equation:

[5.23]
$$\sigma_{t+1|t}^2 = 0.94\sigma_{t|t-1}^2 + 0.06 r_t^2$$

[5] Bayesian SV models, on the other hand, are computationally intensive.

Chart 5.5
GARCH(1,1)-normal and EWMA estimators
GBP parameters



Notice from Chart 5.5, the dynamics of the exponential model's forecasts closely mimic those produced by the GARCH(1,1) model. This should not be surprising given our findings that the exponential model is similar in form to the IGARCH model.

A natural extension of univariate GARCH and Stochastic Volatility models has been to model conditional covariances and correlations. With the ability to estimate more parameters of the return generating process comes growing computational complexity.[6] Often, to make models tractable, restrictions are placed on either the process describing the conditional covariance matrix or the factors that explain covariance dynamics. Recent discussion and applications of multivariate GARCH models include Engle and Kroner (1995), Karolyi (1995), King, Sentena and Wadhwani (1994). Harvey (1993) presents work on multivariate extensions to the stochastic volatility models.

5.3 Estimating the parameters of the RiskMetrics model

In this section we address two important issues that arise when we estimate RiskMetrics volatilities and correlations. The first issue concerns the estimation of the sample mean. In practice, when we make volatility and correlation forecasts we set the sample mean to zero. The second issue involves the estimation of the exponential decay factor which is used in volatility and correlation forecasts.

5.3.1 Sample size and estimation issues

Whenever we must estimate and/or forecast means, standard deviations and correlations, we would like to be reasonably confident in the results. Here, confidence is measured by the standard error of the estimate or forecast; in general, the smaller the standard error, the more confident we are about its value. It is important, therefore, to use the largest samples available when computing these statistics. We illustrate the relationship between sample size and confidence intervals next. For ease of exposition we use equally weighted statistics. The results presented below carry over to the case of exponentially weighted statistics as well.

[6] With respect to the required computation of the bivariate EGARCH model, Braun, Nelson and Sunier (1991) note that, “ease of computation is, alas, not a feature even of the bivariate model. For, example, the FORTRAN code for computing the analytic derivatives … ran to forty pages.”

5.3.1.1 The sample mean

Table 5.6 shows that the mean estimates for USD/DEM foreign exchange returns and S&P 500 returns are –0.114 and –0.010 percent, respectively. To show the variability of the sample mean, Chart 5.6 presents historical estimates of the sample mean for USD/DEM exchange rate returns. Each estimate of the mean is based on a 74-day rolling window, that is, every day in the sample period we estimate a mean based on returns over the last 74 days.

Table 5.6
Mean, standard deviation and correlation calculations
USD/DEM and S&P500 returns

	Returns	
Date	**USD/DEM**	**S&P 500**
28-Mar-96	0.634	0.005
29-Mar-96	0.115	–0.532
1-Apr-96	–0.460	1.267
2-Apr-96	0.094	0.234
3-Apr-96	0.176	0.095
4-Apr-96	–0.088	–0.003
5-Apr-96	–0.142	–0.144
8-Apr-96	0.324	–1.643
9-Apr-96	–0.943	–0.319
10-Apr-96	–0.528	–1.362
11-Apr-96	–0.107	–0.367
12-Apr-96	–0.160	0.872
15-Apr-96	–0.445	0.904
16-Apr-96	0.053	0.390
17-Apr-96	0.152	–0.527
18-Apr-96	–0.318	0.311
19-Apr-96	0.424	0.227
22-Apr-96	–0.708	0.436
23-Apr-96	–0.105	0.568
24-Apr-96	–0.257	–0.217
Mean	–0.114	0.010
Standard deviation	0.393	0.688
Correlation	–0.180	

Chart 5.6
USD/DEM foreign exchange

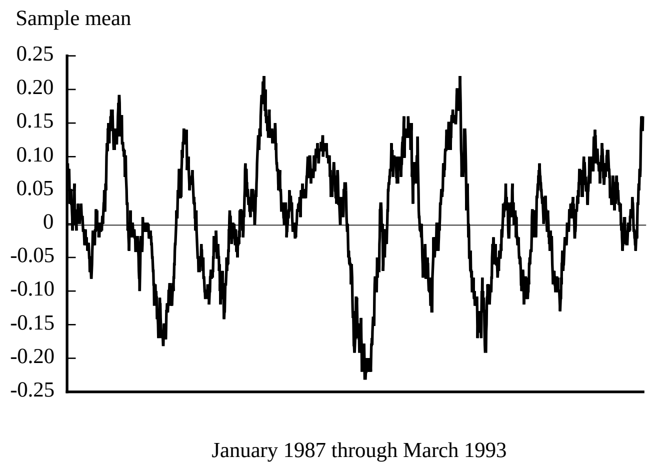


Chart 5.6 shows how the estimates of the mean of returns on USD/DEM fluctuate around zero. An interesting feature of the equally weighted sample mean estimator is that the mean estimate does not depend directly on the number of observations used to construct it. For example, recall that the 1-day log return is defined as $r_t = \ln(P_t / P_{t-1}) = p_t - p_{t-1}$. Now, the sample mean of returns for the period $t = 1, \ldots, T$ is

$$
\begin{aligned}
\bar{r} &= \frac{1}{T}\sum_{t=1}^{T} p_t - p_{t-1} \\
&= \frac{1}{T}(p_T - p_0)
\end{aligned}
\qquad [5.24]
$$

Hence, we see that the sample mean estimator depends only on the first and last observed prices; all other prices drop out of the calculation. Since this estimator does not depend on the number of observed prices between $t = 0$ and $t = T$ but rather on the length of the sample period, neither does its standard error. The implication of this effect can best be demonstrated with a simple example.[7]

Suppose a price return has a standard deviation of 10 percent and we have 4 years' of historical price data. The standard deviation of the sample mean is $10/\sqrt{4} = 5$ percent. So, if the average annual return were 20 percent over the 4-year sample (which consists of over 1000 data points), a 95 percent confidence region for the true mean would range from 10 percent to 30 percent.

In addition, recall that the variance of a returns series, r_t, can be written as $\sigma^2 = E\left(r_t^2\right) - [E(r_t)]^2$. Jorion (1995) notes that with daily data the "average term $E\left(r_t^2\right)$ dominates the term $[E(r_t)]^2$ by a typical factor of 700 to one. Therefore, ignoring expected returns is unlikely to cause a perceptible bias in the volatility estimate."

To reduce the uncertainty and imprecision of the estimated mean, it may be more accurate to set the mean to some value which is consistent with financial theory. **In RiskMetrics, we assume that the mean value of daily returns is zero. That is, standard deviation estimates are cen-**

[7] This example is adapted from Figlewski, (1994).

tered around zero, rather than the sample mean. Similarly, when computing the covariance, deviations of returns are taken around zero rather than the sample mean.

5.3.1.2 Volatility and correlation

Volatility and correlation forecasts based on the EWMA model requires that we choose an appropriate value of the decay factor λ. As a practical matter, it is important to determine the effective number of historical observations that are used in the volatility and correlation forecasts.

We can compute the number of effective days used by the variance (volatility) and covariance (correlation) forecasts. To do so, we use the metric

[5.25] $$\Omega_K^{\infty} = (1-\lambda)\sum_{t=K}^{\infty}\lambda^t$$

Setting Ω_K^{∞} equal to a value —the tolerance level (Υ_L)— we can solve for *K*, the effective number of days of data used by the EWMA. The formula for determining *K* is

[5.26] $$K = \frac{\ln\Upsilon_L}{\ln\lambda}$$

Equation [5.26] is derived as follows

[5.27] $$\Omega_K^{\infty} = (1-\lambda)\sum_{t=K}^{\infty}\lambda^t = \Upsilon_L$$

which implies

[5.28] $$\lambda^K(1-\lambda)\left(1+\lambda+\lambda^2+\ldots\right) = \Upsilon_L$$

Solving Eq. [5.28] for *K* we get Eq. [5.26].

Table 5.7 shows the relationship between the tolerance level, the decay factor, and the effective amount of data required by the EWMA.

Table 5.7
The number of historical observations used by the EWMA model
daily returns

	Days of historical data at tolerance level:			
Decay factor	**0.001%**	**0.01%**	**0.1%**	**1 %**
0.85	71	57	43	28
0.86	76	61	46	31
0.87	83	66	50	33
0.88	90	72	54	36
0.89	99	79	59	40
0.9	109	87	66	44
0.91	122	98	73	49
0.92	138	110	83	55
0.93	159	127	95	63
0.94	186	149	112	74
0.95	224	180	135	90
0.96	282	226	169	113
0.97	378	302	227	151
0.98	570	456	342	228
0.99	1146	916	687	458

For example, setting a tolerance level to 1% and the decay factor to 0.97, we see the EWMA uses approximately 151 days of historical data to forecast future volatility/correlation. Chart 5.7 depicts the relationship between the tolerance level and the amount of historical data implied by the decay factor

Chart 5.7
Tolerance level and decay factor

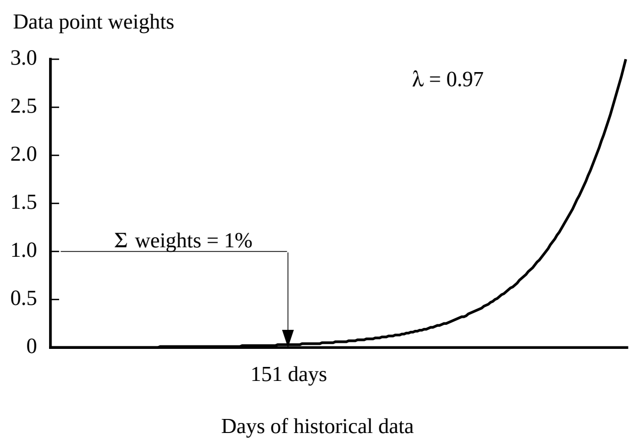
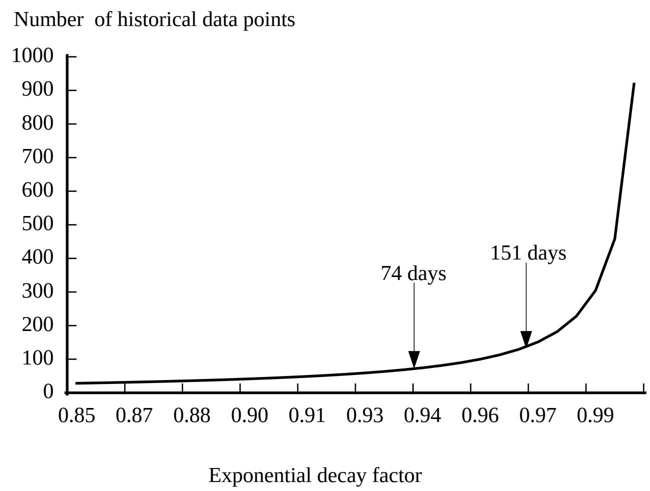


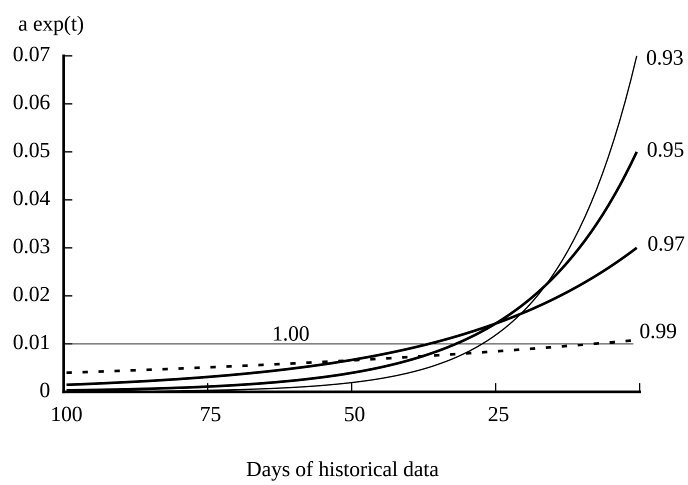
Chart 5.8 shows the relationship between the number of days of data required by EWMA and various values of the decay factor.

Chart 5.8
Relationship between historical observations and decay factor



For a different perspective on the relationship between the number of data points used and different values of the decay factor, consider Chart 5.9. It shows the weights for different decay factors over a fixed window size of T = 100 (approximately 6 months' of data).

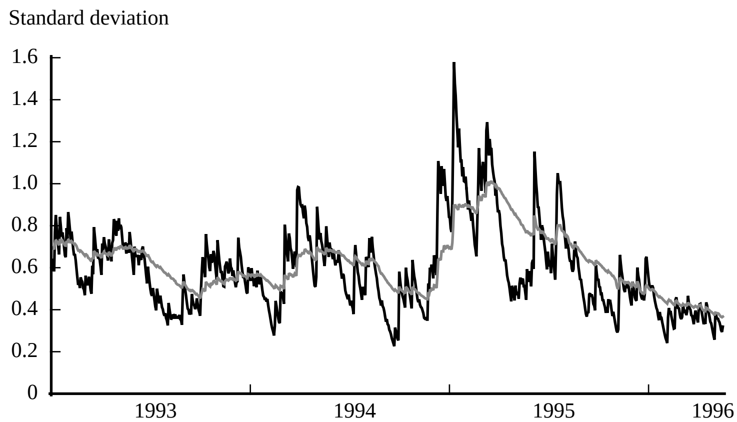
Chart 5.9
Exponential weights for *T* = 100
decay factors = 1, .99, .97, .95, .93



Note that while the decay factor of 0.93 weighs the most recent data more than the factor 0.99, after 40 days, the weight associated with the decay factor of 0.93 is below the weight of 0.99. Hence, the closer the decay factor is to 1, the less responsive it is to the most recent data.

Now we consider the effect of sample size on volatility and correlation forecasts. Chart 5.10 presents two historical time series of 1-day volatility forecasts on the returns series in USD/DEM exchange rate. One volatility series was constructed with a decay factor of 0.85, the other used 0.98. (Refer to Table 5.7 for the relationship between the decay factor and the amount of data used).

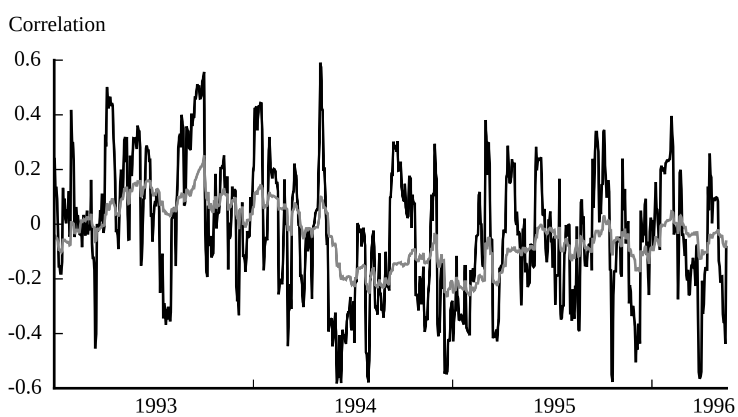
Chart 5.10
One-day volatility forecasts on USD/DEM returns
λ = 0.85 (black line), λ = 0.98 (gray line)



As expected, the volatility forecasts based on more historical observations are smoother than those that rely on much less data.

One-day forecasts of correlation between the returns on the USD/DEM foreign exchange rate and S&P 500 for two different decay factors are presented in Chart 5.11.

Chart 5.11
One-day correlation forecasts for returns on USD/DEM FX rate and on S&P500
λ = 0.85 (black line), λ = 0.98 (gray line)



Again, the time series with the higher decay factor produces more stable (though not necessarily more accurate) forecasts.

5.3.2 Choosing the decay factor

In this section we explain how we determine the decay factors (λ's) that are used to produce the RiskMetrics volatility and correlation forecasts. We begin by describing the general problem of choosing 'optimal' λ's for volatilities and correlations that are consistent with their respective covariance matrix. We then discuss **how** RiskMetrics chooses its two optimal decay factors; one for the daily data set (λ = 0.94), and the other for the monthly data set (λ = 0.97).

RiskMetrics produces volatility and correlation forecasts on over 480 time series. This requires 480 variance forecasts and 114,960 covariance forecasts. Since these parameters comprise a covariance matrix, the optimal decay factors for each variance and covariance forecast are not independent of one another. We explain this concept with a simple example that consists of two return series, $r_{1,t}$ and $r_{2,t}$. The covariance matrix associated with these returns is given by

[5.29]
$$\Sigma = \begin{bmatrix} \sigma_1^2(\lambda_1) & \sigma_{12}^2(\lambda_3) \\ \sigma_{21}^2(\lambda_3) & \sigma_2^2(\lambda_2) \end{bmatrix}$$

We write each parameter explicitly as a function of its decay factor. As we can see from Eq. [5.29], the covariance matrix, Σ, is a function of 3 decay factors, λ_1, λ_2 and λ_3. Now, Σ, to be properly defined must contain certain properties. For example, Σ must be such that the following three conditions are met:

- The variances, σ_1^2 and σ_2^2, cannot be negative
- The covariances σ_{12}^2 and σ_{21}^2 must be equal (i.e., Σ is symmetric)
- The correlation between $r_{1,t}$ and $r_{2,t}$ has the range $-1 \le \rho \le 1$. (Recall the definition of correlation, ρ, $\rho = \sigma_{12}^2 / (\sigma_1 \sigma_2)$.

It follows then that decay factors must be chosen such that they not only produce good forecasts of future variances and covariances, but that the values of these decay factors are consistent with the properties of the covariance matrix to which they belong.

In theory, while it is possible to choose optimal decays factors that are consistent with their respective covariance matrix, in practice this task is exceedingly complex for large covariance matrices (such as the kind that RiskMetrics produces that has 140,000 elements). Therefore, it becomes necessary to put some structure (restrictions) on the optimal λ's.

RiskMetrics applies one optimal decay factor to the entire covariance matrix. That is, we use one decay factor for the daily volatility and correlation matrix and one for the monthly volatility and correlation matrix. This decay factor is determined from individual variance forecasts across 450 time series (this process will be discussed in Section 5.3.2.2).

Recently, Crnkovic and Drachman (1995)[8] have shown that while it is possible to construct a covariance matrix with different decay factors that is positive semi-definite, this matrix is subject to substantial bias.[9]

We now describe a measure applied by RiskMetrics to determine the optimal decay factor, i.e., that decay factor that provides superior forecast accuracy.

[8] From personal communication.

[9] See Section 8.3 for an explanation of positive semi-definite and its relationship to covariance matrices.

5.3.2.1 Root mean squared error (RMSE) criterion[10]

The definition of the time $t+1$ forecast of the variance of the return, r_{t+1}, made one period earlier is simply $E_t\left[r_{t+1}^2\right] = \sigma_{t+1|t}^2$, i.e., the expected value of the squared return one-period earlier. Similarly, the definition of the time $t+1$ forecast of the covariance between two return series, $r_{1,t+1}$ and $r_{2,t+1}$ made one period earlier is $E_t\left[r_{1,t+1} r_{2,t+1}\right] = \sigma_{12,t+1|t}^2$. In general, these results hold for any forecast made at time $t+j$, $j \geq 1$.

Now, if we define the variance forecast error as $\varepsilon_{t+1|t} = r_{t+1}^2 - \sigma_{t+1|t}^2$ it then follows that the expected value of the forecast error is zero, i.e., $E_t\left[\varepsilon_{t+1|t}\right] = E_t\left[r_{t+1}^2\right] - \sigma_{t+1|t}^2 = 0$. Based on this relation a natural requirement for choosing λ is to minimize average squared errors. When applied to daily forecasts of variance, this leads to the (daily) root mean squared prediction error which is given by

$$[5.30] \qquad RMSE_v = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(r_{t+1}^2 - \hat{\sigma}_{t+1|t}^2(\lambda)\right)^2} \quad \text{(variance)}$$

where the forecast value of the variance is written explicitly as a function of λ.

In practice we find the optimal decay factor λ* by searching for the smallest RMSE over different values of λ. That is, we search for the decay factor that produces the best forecasts (i.e., minimizes the forecast measures).

Although RiskMetrics does not assess the accuracy of covariance forecasts, similar results to those for the variance can be derived for covariance forecasts, i.e., the covariance forecast error is $\varepsilon_{12,t+1|t} = r_{1,t+1} r_{2,t+1} - \sigma_{12,t+1|t}^2$ such that $E_t\left[\varepsilon_{12,t+1|t}\right] = E_t\left[r_{1,t+1} r_{2,t+1}\right] - \sigma_{12,t+1|t}^2 = 0$ and

$$[5.31] \qquad RMSE_c = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(r_{1,t+1} r_{2,t+1} - \hat{\sigma}_{12,t+1|t}^2(\lambda)\right)^2} \quad \text{(covariance)}$$

The measures presented above are purely statistical in nature. For risk management purposes, this may not be optimal since other factors come into play that determine the best forecast. For example, the decay factor should allow enough stability in the variance and covariance forecasts so that these forecasts are useful for risk managers who do not update their systems on a daily basis.[11]

Next, we explain how we determine the two RiskMetrics optimal decay factors, one for daily and one for monthly forecasts.

[10] See Appendix C for alternative measures to assess forecast accuracy.

[11] West, Edison and Cho (1993) suggested that an interesting alternative basis for comparing forecasts is to calculate the utility of an investor with a particular utility function investing on the basis of different variance forecasts. We plan to pursue this idea from a risk management perspective in future research.

5.3.2.2 How RiskMetrics chooses its optimal decay factor

RiskMetrics currently processes 480 time series, and associated with each series is an optimal decay factor that minimizes the root mean squared error of the variance forecast (i.e., Eq. [5.30]). We choose RMSE as the forecast error measure criterion.[12] Table 5.8 presents optimal decay factors for return series in five series.

Table 5.8
Optimal decay factors based on volatility forecasts
based on RMSE criterion

Country	Foreign exchange	5-year swaps	10-year zero prices	Equity indices	1-year money market rates
Austria	0.945	—	—	—	—
Australia	0.980	0.955	0.975	0.975	0.970
Belgium	0.945	0.935	0.935	0.965	0.850
Canada	0.960	0.965	0.960	—	0.990
Switzerland	0.955	0.835	—	0.970	0.980
Germany	0.955	0.940	0.960	0.980	0.970
Denmark	0.950	0.905	0.920	0.985	0.850
Spain	0.920	0.925	0.935	0.980	0.945
France	0.955	0.945	0.945	0.985	—
Finland	0.995	—	—	—	0.960
Great Britain	0.960	0.950	0.960	0.975	0.990
Hong Kong	0.980	—	—	—	—
Ireland	0.990	—	0.925	—	—
Italy	0.940	0.960	0.935	0.970	0.990
Japan	0.965	0.965	0.950	0.955	0.985
Netherlands	0.960	0.945	0.950	0.975	0.970
Norway	0.975	—	—	—	—
New Zealand	0.975	0.980	—	—	—
Portugal	0.940	—	—	—	0.895
Sweden	0.985	—	0.980	—	0.885
Singapore	0.950	0.935	—	—	—
United States	—	0.970	0.980	0.980	0.965
ECU	—	0.950	—	—	—

For the daily and monthly data sets we compute one optimal decay factor from the 480+ time series. Denote the *i*th optimal decay factor by $\hat{\lambda}_i$ and let N (i = 1, 2,…, N) denote the number of time series in the RiskMetrics database. Also, let τ_i denote the *i*th RMSE associated with $\hat{\lambda}_i$, i.e., τ_i is the minimum RMSE for the *i*th time series. We derive the one optimal decay factor as follows:

1. Find Π, the sum of all N minimal RMSE's, τ_i's:

$$\Pi = \sum_{i=1}^{N} \tau_i . \qquad [5.32]$$

2. Define the relative error measure:

[12] We have chosen this criterion because it penalizes large forecast errors more severely, and provides more useful results than other common accuracy statistics.

$$[5.33] \qquad \theta_i = \tau_i \Big/ \left(\sum_{i=1}^{N} \tau_i \right)$$

3. Define the weight ϕ_i:

$$[5.34] \qquad \phi_i = \theta_i^{-1} \Big/ \sum_{i=1}^{N} \theta_i^{-1}$$

where $\sum_{i=1}^{N} \phi_i = 1$

4. The optimal decay factor $\tilde{\lambda}$ is defined as

$$[5.35] \qquad \tilde{\lambda} = \sum_{i=1}^{N} \phi_i \hat{\lambda}_i$$

That is, the optimal decay factor applied by RiskMetrics is a weighted average of individual optimal decay factors where the weights are a measure of individual forecast accuracy.

Applying this methodology to both daily and monthly returns we find that the decay factor for the daily data set is 0.94, and the decay factor for the monthly data set is 0.97.

5.4 Summary and concluding remarks

In this chapter we explained the methodology and practical issues surrounding the estimation of the RiskMetrics volatilities and correlations. Table 5.9 summarizes the important results about the RiskMetrics volatility and correlation forecasts.

Table 5.9
Summary of RiskMetrics volatility and correlation forecasts

Forecast	Expression*	Decay factor	# of daily returns used in production	Effective # of daily returns used in estimation†
1-day volatility	$\sigma_{1,t+1\|t} = \sqrt{\lambda\sigma^2_{1,t\|t-1} + (1-\lambda) r^2_{1,t}}$	0.94	550	75
1-day correlation	$\rho_{12,t+1\|t} = \dfrac{\sigma^2_{12,t+1\|t}}{\sigma_{1,t+1\|t}\sigma_{2,t+1\|t}}$	0.94	550	75
1-month volatility	$\sigma_{1,t+25\|t} = 5 \cdot \sigma_{1,t+1\|t}$	0.97	550	150
1-month correlation	$\rho_{12,t+25\|t} = \rho_{12,t+1\|t}$	0.97	550	150

* Note that in all calculations the sample mean of daily returns is set to zero.

† This number is a dependent of the decay factor explained in Section 5.3.1.2.

Lastly, recall from Chapter 4 that RiskMetrics assumes that returns are generated according to the model

[5.36] $$r_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \rightarrow IID\ N(0, 1)$$

Now, given the recursive form of the EWMA model, a more complete version of the RiskMetrics model for any individual time series is

[5.37] $$\begin{aligned} r_t &= \sigma_t \varepsilon_t \qquad \varepsilon_t \rightarrow IID\ N(0, 1) \\ \sigma_t^2 &= \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \end{aligned}$$

Since Eq. [5.37] describes a process by which returns are generated, we can determine whether this model (evaluated at the optimal decay factor) can replicate the distinctive features of the observed data as presented in Chapter 4. We do so by generating a time series of daily returns from Eq. [5.37] for a given value of λ. A simulated time series from Eq. [5.37] with λ = 0.94 is shown in Chart 5.12.

Chart 5.12
Simulated returns from RiskMetrics model

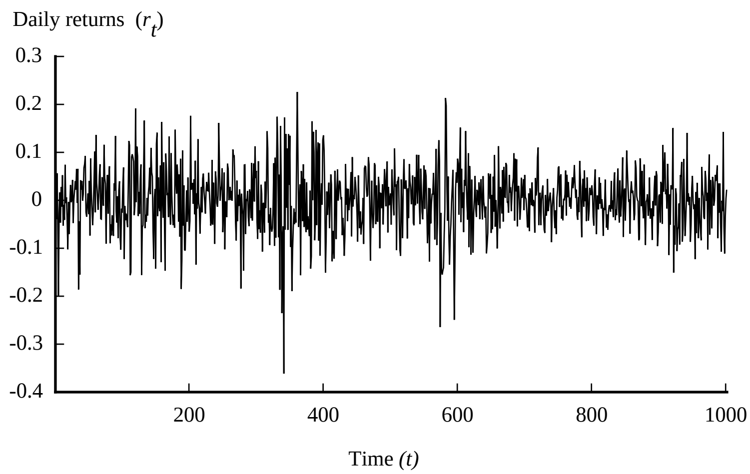


Chart 5.12 shows that the RiskMetrics model can replicate the volatility clustering feature noted in Chapter 4 (compare Chart 5.12 to Charts 4.6 and 4.7).

Part III

Risk Modeling of Financial Instruments

Chapter 6. Market risk methodology

Chapter 6. Market risk methodology

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This chapter explains the methodology RiskMetrics uses to calculate VaR for portfolios that include multiple instruments such as simple bonds, swaps, foreign exchange, equity and other positions.

The chapter is organized as follows:

- Section 6.1 describes how to decompose various positions into cash flows.
- Section 6.2 covers how to convert or map the actual cash flows onto the corresponding RiskMetrics vertices.
- Section 6.3 explains two analytical approaches to measuring VaR.
- Section 6.4 presents a number of examples to illustrate the application of the RiskMetrics methodology.

6.1 Step 1—Identifying exposures and cash flows

The RiskMetrics building block for describing any position is a cash flow. A cash flow is defined by an amount of a currency, a payment date and the credit standing of the payor.

Once determined, these cash flows are marked-to-market. Marking-to-market a position's cash flows means determining the present value of the cash flows given **current** market rates and prices. This procedure requires current market rates, including the current on-the-run yield curve for newly issued debt, and a zero-coupon yield curve on instruments that pay no cash flow until maturity.[1] The zero coupon rate is the relevant rate for discounting cash flows received in a particular future period.[2]

We now describe how to express positions in fixed income, foreign exchange, equity, and commodities in terms of cash flows. The general process of describing a position in terms of cash flows is known as mapping.

6.1.1 Fixed Income

Interest rate positions describe the distribution of cash flows over time. Practitioners have applied various methods to express, or map, the cash flows of interest rate positions, the most common three being (1) duration map, (2) principal map, and (3) cash flow map. In this book we use the cash flow map method, but for comparison, present the two other methods as viable alternatives.

- Duration map

  The first and most common method to characterize a position's cash flows is by its duration (the weighted average life of a position's interest and principal payments). Macaulay duration is a measure of the weighted average maturity of an instrument's cash flows. Modified duration is a measure of a bond's price sensitivity to changes in interest rates. In general, duration provides risk managers with a simplified view of a portfolio's market risk. Its main drawback is that it assumes a linear relationship between price changes and yield changes. Moreover,

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[1] See *The J. P. Morgan/Arthur Andersen Guide to Corporate Exposure Management* (p. 54, 1994).

[2] It is often suggested that implied forward rates are required to estimate the floating rates to be paid in future periods. In this document, however, we will show why forward rates are not necessarily required.

this approach works well when there are so-called parallel shifts in the yield curve but poorly when yield curves twist. Duration maps are used extensively in fixed income investment management. Many investment managers' activities are constrained by risk limits expressed in terms of portfolio duration.

- Principal map

  A second method, used extensively over the last two decades by commercial banks, is to describe a global position in terms of when principal payments occur. These "principal" maps form the basis for asset/liability management. ARBLs (Assets Repricing Before Liabilities) are used by banks to quantify interest rate risk in terms of cumulative assets maturing before liabilities. This method is employed most often when risks are expressed and earnings are accounted for on an accrual basis. The main problem with principal maps is that they assume that all interest payments occur at current market rates. This is often not a good assumption particularly when positions include fixed rate instruments with long maturities and when interest rates are volatile. Principal maps describe an instrument only as a function of the value and timing of redemption.

- Cash flow map

  The third method, and the one RiskMetrics applies is known as cash flow mapping. Fixed income securities can be easily represented as cash flows given their standard future stream of payments. In practice, this is equivalent to decomposing a bond into a stream of zero-coupon instruments. Complications in applying this technique can arise, however, when some of these cash flows are uncertain, as with callable or puttable bonds.

The following example shows how each of the mapping methodologies can be applied in practice. Chart 6.1 shows how a 10-year French OAT (FRF 100,000 francs nominal, 7.5% of April 2005) can be mapped under the approaches listed above:

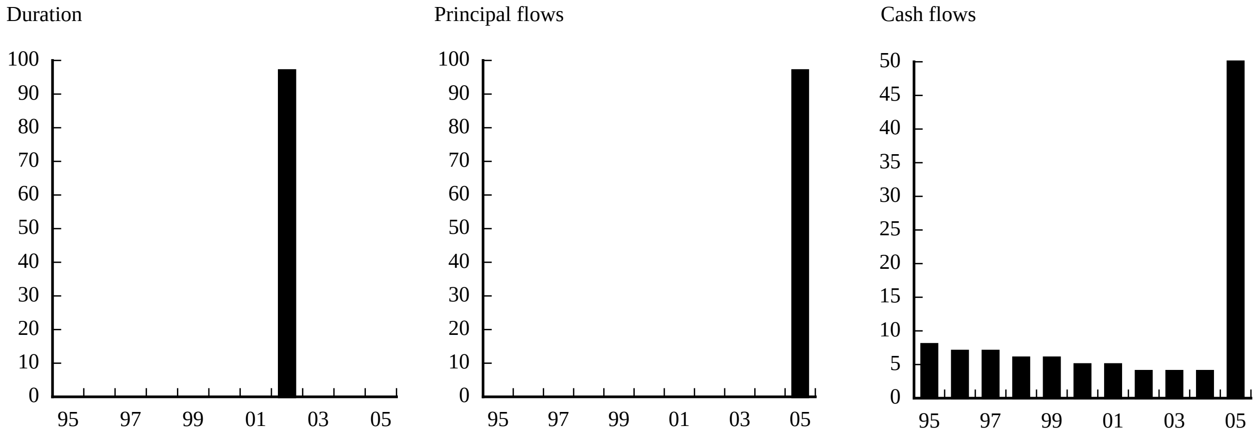
- The duration map associates the market value of the instrument against the bond's Macaulay duration of 6.88 years.

- The principal map allocates the market value of the bond to the 10-year maturity vertex.

- The cash flow map shows the distribution over time of the current market value of all future streams (coupons + principals).

  As shown in Chart 6.1, the cash flow map (present valued) treats all cash flows separately and does not group them together as do the duration and principal maps. Cash flow mapping is the preferred alternative because it treats cash flows as being distinct and separate, enabling us to model the risk of the fixed income position better than if the cash flows were simply represented by a grouped cash flow as in the duration and principal maps.

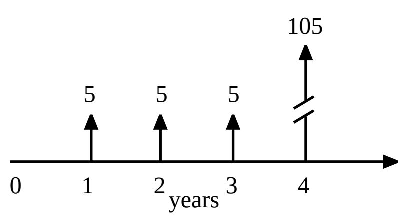
Chart 6.1
French franc 10-year benchmark maps
amounts in thousands of market value



6.1.1.1 Simple bonds
Consider a hypothetical bond with a par value of 100, a maturity of 4 years and a coupon rate of 5%. Assume that the bond is purchased at time 0 and that coupon payments are payed on an annual basis at the beginning of each year. Chart 6.2 shows the bond's cash flows.[3]

In general, arrows pointing upwards signify cash inflows and arrows pointing downwards represent outflows. Also, a cash flow's magnitude is proportional to the length of the arrow; the taller (shorter) the arrow the greater (lower) the cash flow.

Chart 6.2
Cash flow representation of a simple bond



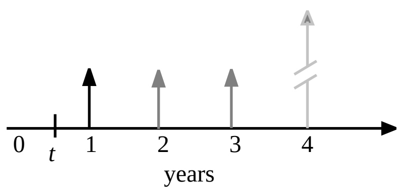
We can represent the cash flows of the simple bond in our example as cash flows from four zero-coupon bonds with maturities of 1,2,3 and 4 years. This implies that on a risk basis, there is no difference between holding the simple bond or the corresponding four zero-coupon bonds.

6.1.1.2 Floating rate notes (FRN)
A **floating rate note (FRN)** is an instrument that is based on a principal, P, that pays floating coupons. A FRN's coupon payment is defined as the product of the principal and a floating rate that is set some time in advance of the actual coupon payment. For example, if coupon payments are paid on a semiannual basis, the 6-month LIBOR rate would be used to determine the payment in 6 month's time. The coupon payments would adjust accordingly depending on the current 6-month LIBOR rate when the floating rate is reset. The principal is exchanged at both the beginning and end of the FRN's life.

[3] We ignore the payment for the bond. That is, we do not account for the initial (negative) cash flow at time 0.

Chart 6.3 shows the cash flows for a hypothetical FRN lasting 4 years. The floating payments are represented by the gray shaded arrows. The black arrows represent fixed payments. All payments are assumed to occur on a yearly basis.

Chart 6.3
Cash flow representation of a FRN



Notice that the first payment (at year 1) is known, and therefore, fixed. Also, the last payment represents the fact that the principal is known at the fourth year, but the final coupon payment is unknown. We now show how to evaluate the future floating payments.

Suppose that at time t (between 0 and 1 year), a risk manager is interested in analyzing the floating payment that will be received in year 3. The rate that determines this value is set in the second year and lasts one year. Now, implied forward rates are often used to forecast floating rates. The fundamental arbitrage relationship between current and future rates implies that the 1-year rate, as of year 2 satisfies the expression

[6.1] $$(1 + r_{2-t,t}) \cdot (1 + f_{1,2}) = (1 + r_{3-t,t})$$

where $r_{i,j}$ is the i-year rate set at time j and $f_{i,j}$ is the i period forward rate set at time j. So, for example, $f_{1,2}$ is the 1-year rate, beginning at the second year. It follows that the cash flow implied by this rate occurs in year 3. Since we know at time t both $r_{2-t,t}$ (the 2-t year rate) and $r_{3-t,t}$ (the 3-t year rate), we can solve for the implied forward rate as a function of observed rates. i.e.,

[6.2] $$f_{1,2} = \frac{(1 + r_{3-t,t})}{(1 + r_{2-t,t})} - 1$$

We can apply same technique to all other implied forward rates so that we can solve for $f_{1,1}$, $f_{1,2}$, $f_{1,3}$ and determine the expected future payments. The forecast coupon payment, for example, at time 3 is $P \cdot f_{1,2}$. The present value of this payment at time t, is simply $(P \cdot f_{1,2}) / (1 + r_{3-t,t})$. Substituting Eq. [6.2] into the expression for the discounted coupon payment yields,

[6.3] $$\frac{P \cdot f_{1,2}}{(1 + r_{3-t,t})} = \frac{P}{(1 + r_{2-t,t})} - \frac{P}{(1 + r_{3-t,t})}$$

Equation [6.3] shows that the expected coupon payment can be written in terms of known zero coupon rates. We can apply similar methods to the other coupon payments so that we can write the cash flows of the FRN as

[6.4] $$P_{FRN} = \frac{P \cdot r_{1,0}}{(1 + r_{1-t,t})} + \frac{P \cdot f_{1,1}}{(1 + r_{2-t,t})} + \frac{P \cdot f_{1,2}}{(1 + r_{3-t,t})} + \frac{P \cdot f_{1,3}}{(1 + r_{4-t,t})} + \frac{P}{(1 + r_{4-t,t})}$$

The right-hand side of Eq. [6.4] is equal to

[6.5]

$$\frac{P \cdot r_{1,0}}{(1+r_{1-t,t})} + \left(\frac{P}{(1+r_{1-t,t})} - \frac{P}{(1+r_{2-t,t})}\right) + \left(\frac{P}{(1+r_{2-t,t})} - \frac{P}{(1+r_{3-t,t})}\right) + \left(\frac{P}{(1+r_{3-t,t})} - \frac{P}{(1+r_{4-t,t})}\right) + \frac{P}{(1+r_{4-t,t})}$$

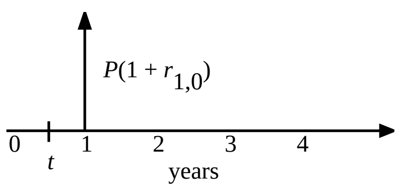
Equation [6.5] collapses to the present value

[6.6]

$$\frac{P \cdot (1+r_{1,0})}{(1+r_{1-t,t})}$$

Chart 6.4 shows that the cash flow of the FRN from the time t perspective, is $P(1+r_{1,0})$. Therefore, we would treat the FRN's cash flows as a cash flow from a zero coupon bond with maturity 1-*t* period.

Chart 6.4
Estimated cash flows of a FRN

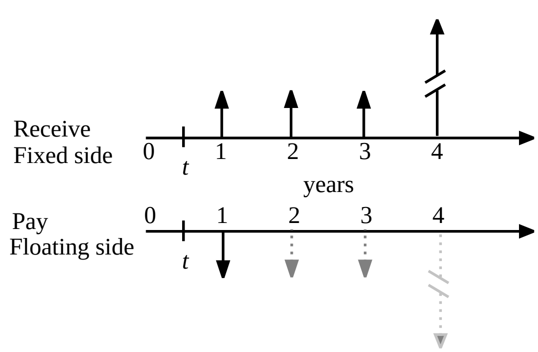


Notice that if the cash flows in Chart 6.3 were computed relative to time zero (the start of the FRN), rather than to time *t*, the cash flow would be simply *P* at *t* = 0, representing the par value of the FRN at its start.

6.1.1.3 Simple interest-rate swaps
Investors enter into interest-rate swaps to change their exposure to interest rate uncertainty by exchanging interest flows. In order to understand how to identify a simple interest-rate swap's cash flows, a swap should be thought of as a portfolio consisting of one fixed and one floating rate instrument. Specifically, the fixed leg is represented by a simple bond without an exchange of principal. The floating leg is a FRN with the caveat that the principal is used only to determine coupon payments, and is not exchanged.

Chart 6.5 shows the cash flows of an interest-rate swap that receives fixed rate and pays the floating rate.

Chart 6.5
Cash flow representation of simple interest rate swap



We compute the cash flows relative to time t, (again, between 0 and 1 year) after the start of the swap. The cash flows on the fixed side are simply the fixed coupon payments over the next 4 years which, as already explained in Section 6.1.1.1, are treated as holding four zero-coupon bonds. The cash flows on the floating side are derived in the exact manner as the payments for the FRN (except now we are short the floating payments). The present value of the cash flow map of the floating side of the swap is given by Eq. [6.7]

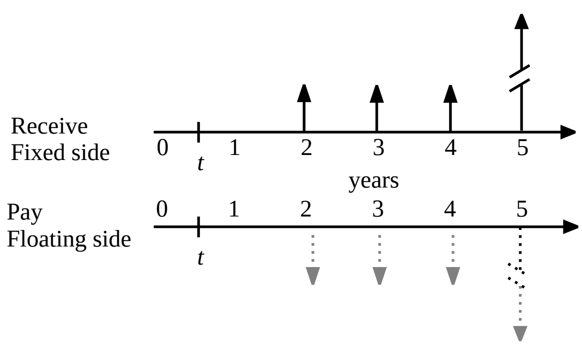
[6.7] $$-\frac{P \cdot (1 + r_{1,0})}{(1 + r_{1-t,t})},$$

where P is the principal of the swap. Notice the similarity between this cash flow and that given by Eq. [6.6] for the FRN. Hence, we can represent the cash flows on the floating side of the swap as being short a zero coupon bond with maturity 1-t.

6.1.1.4 Forward starting swap

A forward starting swap is an instrument where one enters into an agreement to swap interest payments at some future date. Unlike a simple swap none of the floating rates are fixed in advance. Chart 6.6 shows the cash flows of a forward starting swap.

Chart 6.6
Cash flow representation of forward starting swap



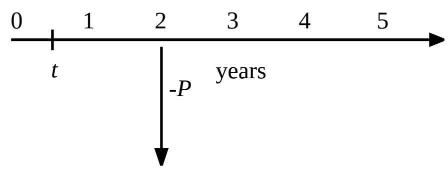
Suppose that an investor enters into a forward starting swap with 5 years to maturity at some time t (the trade date), and the start date of the swap, (i.e., the date when the floating rates are fixed) is year 2. Starting in year 3, payments are made every year until year 5. The cash flows for this instrument are essentially the same as a simple interest-rate swap, but now the first floating payment is unknown.

The cash flows on the fixed side are simply the cash flows discounted back to time t. On the floating side, the cash flows are, again, determined by the implied forward rates. The cash flow map for the (short) floating payments is represented by Eq. [6.8].

[6.8] $$-\frac{P}{(1 + r_{2-t,t})}$$

Chart 6.7 depicts this cash flow.

Chart 6.7
Cash flows of the floating payments in a forward starting swap



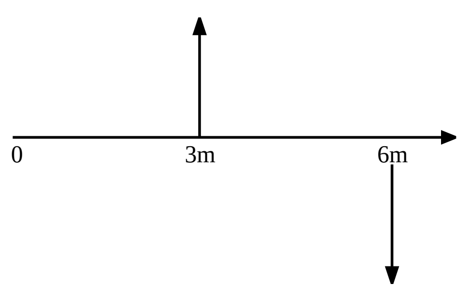
Notice that this cash flow map is equivalent to being short a 2-*t* zero coupon bond.

6.1.1.5 Forward rate agreement (FRA)
A **forward rate agreement (FRA)** is an interest rate contract. It locks in an interest rate, either a borrowing rate (buying a FRA) or a lending rate (selling a FRA) for a specific period in the future. FRAs are similar to futures but are over-the-counter instruments and can be customized for any maturity.

A FRA is a notional contract. Therefore, there is no exchange of principal at the expiry date (i.e., the fixing date). In effect, FRAs allow market participants to lock in a forward rate that equals the implied break even rate between money market and term deposits.[4] To understand how to map the cash flows of a FRA, let's consider a simple, hypothetical example of a purchase of a 3 vs. 6 FRA at r% on a notional amount P. This is equivalent to locking in a borrowing rate for 3 months starting in 3 months. The notation 3 vs. 6 thus refers to the start date of the underlying versus the end date of the underlying, with the start date being the delivery date of the contract.

Chart 6.8 depicts the cash flows of this FRA.

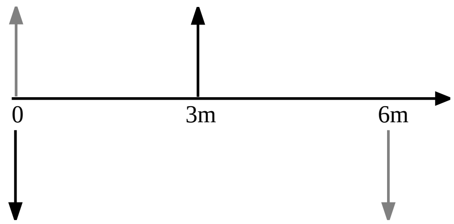
Chart 6.8
Cash flow representation of FRA



We can replicate these cash flows by going long the current 3-month rate and short the 6-month rate as shown in Chart 6.9.

[4] For more details on FRAs, refer to Valuing and Using FRAs (Hakim Mamoni, October, 1994, JP Morgan publication).

Chart 6.9
Replicating cash flows of 3-month vs. 6-month FRA

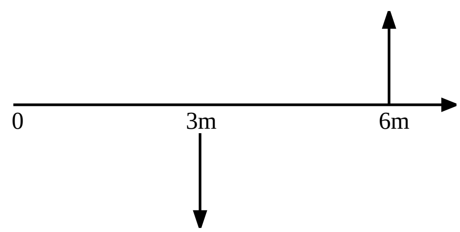


Note that the gray arrows no longer represent floating payments. The gray and black arrows represent the cash flows associated with going short a 6-month zero coupon bond and long a 3-month zero coupon bond, respectively. The benefit of working with the cash flows in Chart 6.9 rather than in Chart 6.8, is that the latter requires information on forward rates whereas the former does not.

6.1.1.6 Interest rate future

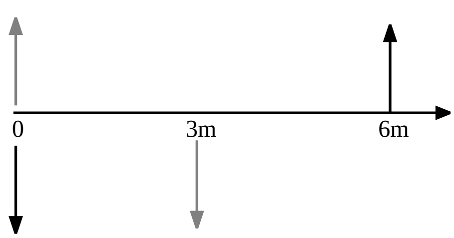
We now consider the cash flow map of a 3-month Eurodollar future contract that expires in 3 months'. Taking time 0 to represent the current date, we represent the future's cash flows by an outflow in 3 months and an inflow in 6 months, as shown in Chart 6.10.

Chart 6.10
Cash flow representation of 3-month Eurodollar future



To be more specific, if the current USD 3-month Eurodollar deposit rate is 7.20%, a purchaser of this futures contract would face a cash outflow of USD 981,800 in 3 months and a cash inflow of USD 1,000,000 in 6 months. We can then represent these cash flows as being short the current 3-month rate and investing this money in the current 6-month rate. Hence, the cash flows of this Eurodollar futures contract can be replicated by a short 3-month position and a long 6-month position as shown in Chart 6.11.

Chart 6.11
Replicating cash flows of a Eurodollar futures contract



6.1.2 *Foreign exchange*

Financial positions are described in terms of a base or "home" currency. For example, American institutions report risks in U.S. dollars, while German institutions use Deutsche marks. A risk manager's risk profile is not independent of the currency in which risk is reported. For example, consider two investors. One investor is based in US dollars, the other in Italian lira. Both investors purchase an Italian government bond. Whereas the USD based investor is exposed to both interest rate and exchange rate risk (by way of the ITL/USD exchange rate), the lira based investor is exposed only to interest rate risk. Therefore, an important step to measure foreign exchange risk is to understand how cash flows are generated by foreign exchange positions.

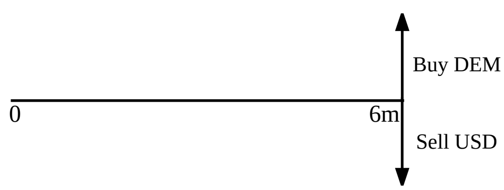
6.1.2.1 *Spot positions*

Describing cash flows of spot foreign exchange positions is trivial. Graphically, up and down arrows represent long and short positions in foreign exchange, respectively.

6.1.2.2 *Forward foreign exchange positions*

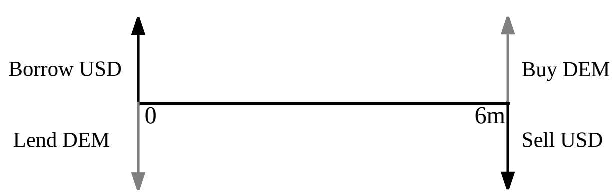
A foreign exchange (FX) forward is an agreement to exchange at a future date, an amount of one currency for another at a specified forward rate. Mapping a forward foreign exchange position is facilitated by the ability to express the forward as a function of two interest rates and a spot foreign exchange rate.[5] For example, Chart 6.12 shows the cash flows of an FX forward that allows an investor to buy Deutsche marks with US dollars in 6 months' time at a prespecified forward rate.

Chart 6.12
FX forward to buy Deutsche marks with US dollars in 6 months



We can replicate these cash flows by borrowing dollars at time 0 at the 6-month interest rate $r_{USD,\,t}$ and immediately investing these dollars in Germany at a rate $r_{DEM,\,t}$, This scenario would generate the cash flows which, at the 6-month mark, are identical to those of the forward contract. These cash flows are shown in Chart 6.13.

Chart 6.13
Replicating cash flows of an FX forward



The ability to replicate future foreign exchange cash flows with interest rate positions results from what is known as interest rate parity (IRP). We now demonstrate this condition. Let the spot rate, S_t, of the home currency expressed in units of foreign currency, (e.g., if the home currency is the US dollars and the foreign currency is Deutsche marks, S_t is expressed in US dollars per Deutsche

[5] For simplicity, we ignore other factors such as transaction costs and possible risk premia.

marks (USD/DEM)). The forward rate, F_t, is the exchange rate observed at time t, which guarantees a spot rate at some future time T. Under interest rate parity the following condition holds

[6.9] $$F_t = S_t \frac{(1 + r_{USD,t})}{(1 + r_{DEM,t})}$$

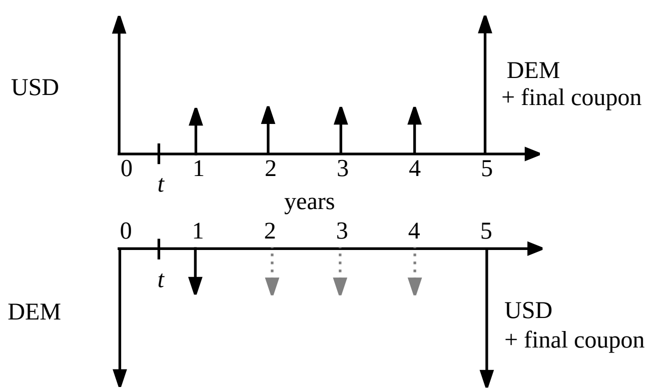
It follows from IRP that the ability to convert cash flows of an FX forward into equivalent borrowing and lending positions implies that holding an FX forward involves cash flows that are exposed to both foreign exchange and interest risk.

6.1.2.3 Currency swaps

Currency swaps are swaps for which the two legs of the swap are each denominated in a different currency. For example, one party might receive fixed rate Deutsche marks, the other floating rate US dollars. Unlike an interest-rate swap, the notional principal in a currency swap is exchanged at the beginning and end of the swap.[6]

Chart 6.14 shows the cash flows for a hypothetical currency swap with a maturity of 4 years and paying fixed rate Deutsche marks and floating rate US dollars on an annual basis. For completeness, we present the cash flows associated with the initial exchange of principal.

Chart 6.14
Actual cash flows of currency swap



From the perspective of holding the swap at time t between 0 and year 1, the fixed leg of the swap has the same cash flows as the simple bond presented in Section 6.1.1.1. The cash flows of the floating leg are the same as that as a short position in a FRN.

6.1.3 Equities

The cash flows of equity are simple spot positions expressed in home currency equivalents. Equity positions held in foreign countries are subject to foreign exchange risk in addition to the risk from holding equity.

6 There are currency swaps where one or both of the notional amounts are not exchanged.

6.1.4 Commodities

Exposures to commodities can be explained using a framework similar to that of interest rates. Risks arise in both the spot market (you purchase a product today and store it over time) and from transactions that take place in the future (e.g., physical delivery of a product in one month's time).

6.1.4.1 Commodity futures contract

Commodity futures contracts enable investors to trade products for future delivery with relative ease and also serve as a price setting and risk transferring mechanisms for commodity producers. These contracts provide market participants with valuable information about the term structure of commodities prices.

6.1.4.2 Commodity swap

Institutions do not have to limit themselves to futures contracts when they participate in the commodity markets. They can enter into swaps to change their exposure to interest rates, currency, and/or commodity risks. A typical commodity swap entails an institution to paying (receiving) fixed amounts in exchange for receiving (paying) variable amounts with respect to an index (e.g., an average of the daily price of the nearby natural gas futures contract).

In many respects, commodity swaps are similar to interest-rate swaps. Unlike an interest-rate swap the underlying instrument of a commodity swap can be of variable quality thereby making the terms of the transaction more complex.

6.2 Step 2—Mapping cash flows onto RiskMetrics vertices

In the last section we described cash flows generated by particular classes of instruments. Financial instruments, in general, can generate numerous cash flows, each one occurring at a unique time. This gives rise to an unwieldy number of combinations of cash flow dates when many instruments are considered. As a result, we are faced with the impractical task of having to compute an intractable number of volatilities and correlations for the VaR calculation. To more easily estimate the risks associated with instruments' cash flows, we need to simplify the time structure of these cash flows.

The RiskMetrics method of simplifying time structure involves cash flow mapping, i.e., redistributing (mapping) the observed cash flows onto so-called RiskMetrics vertices, to produce RiskMetrics cash flows.

6.2.1 RiskMetrics vertices

All RiskMetrics cash flows use one or more of the 14 RiskMetrics vertices shown below (and on page 107).

1m 3m 6m 12m 2yr 3yr 4yr 5yr 7yr 9yr 10yr 15yr 20yr 30yr

These vertices have two important properties:

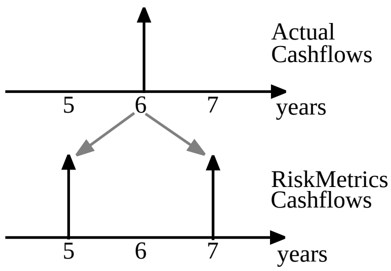
- They are fixed and hold at any time now and in the future for all instruments, linear and non-linear. (J.P. Morgan can occasionally redefine these vertices to keep up with market trends.)

- RiskMetrics data sets provide volatilities and correlations for each of these vertices (and only for these vertices).

Mapping an actual cash flow involves splitting it between the two closest RiskMetrics vertices (unless the cash flow happens to coincide with a RiskMetrics vertex). For example, a cash flow occurring in 6 years is represented as a combination of a 5-year and a 7-year cash flow. Chart 6.15

shows how the actual cash flow occurring at year 6 is split into the synthetic (RiskMetrics) cash flows occurring at the 5- and 7-year vertices.

Chart 6.15
RiskMetrics cash flow mapping



The two fractions of the cash flow are weighted such that the following three conditions hold:

1. **Market value is preserved.** The total market value of the two RiskMetrics cash flows must be equal to the market value of the original cash flow.
2. **Market risk is preserved.** The market risk of the portfolio of the RiskMetrics cash flows must also be equal to the market risk of the original cash flow.
3. **Sign is preserved.** The RiskMetrics cash flows have the same sign as the original cash flow.

In the trivial case that the actual vertex and RiskMetrics vertex coincide, 100% of the actual cash flow is allocated to the RiskMetrics vertex.

It is important to understand that RiskMetrics cash flow mapping differs from conventional mapping methods in the three conditions that it stipulates. A common practice used to date throughout the financial industry has been to follow **two** standard rules when allocating cash flows between vertices:

1. **Maintain present value.** For example, the sum of the cash flows maturing in 5 and 7 years must be equal to the original cash flow occurring in year 6.
2. **Maintain duration.** The duration of the combination of 5- and 7-year cash flows must also be equal to the duration of the 6-year cash flow.

Cash flow maps like these are similar to a barbell type trade, where an existing position is replaced by a combination of two instruments distributed along the yield curve under the condition that the trade remains duration neutral. Barbell trades are entered into by investors who are duration-constrained but have a view on a shift in the yield curve. What is a perfectly defensible investment strategy, however, cannot be simply applied to risk estimation.

6.2.2 Computing RiskMetrics cash flows

For allocating actual cash flows to RiskMetrics vertices, RiskMetrics proposes a methodology that is based on the variance (σ^2) of financial returns. The advantage of working with the variance is that it is a risk measure closely associated with one of the ways RiskMetrics computes VaR, namely the simple VaR method as opposed to the delta-gamma or Monte Carlo methods.

In order to facilitate the necessary mapping, the RiskMetrics data sets provide users with volatilities on, and correlations across many instruments in 33 markets. For example, in the US government bond market, RiskMetrics data sets provide volatilities and correlations on the 2-, 3-, 4-, 5-, 7-, 9-, 10-, 15-, 20-, and 30-year zero coupon bonds.

We now demonstrate how to convert actual cash flows to RiskMetrics cash flows, continuing with the example of allocating a cash flow in year 6 to the 5- and 7-year vertices (Chart 6.15). We denote the allocations to the 5- and 7-year vertices by α and $(1-\alpha)$, respectively. The procedure presented below is not restricted to fixed income instruments, but applies to all future cash flows.

1. Calculate the actual cash flow's interpolated yield:

   We obtain the 6-year yield, y_6, from a linear interpolation of the 5- and 7-year yields provided in the RiskMetrics data sets. Using the following equation,

   [6.10] $$y_6 = \hat{a} y_5 + (1-\hat{a}) y_7 \qquad 0 \le \hat{a} \le 1$$

   where y_6 = interpolated 6-year zero yield
   $\hat{a}$ = linear weighting coefficient, $\hat{a}$ = 0.5 in this example
   y_5 = 5-year zero yield
   y_7 = 7-year zero yield

   If an actual cash flow vertex is not equidistant between the two RiskMetrics vertices, then the greater of the two values, $\hat{a}$ and $(1-\hat{a})$, is assigned to the closer RiskMetrics vertex.

2. Determine the actual cash flow's present value:

   From the 6-year zero yield, y_6, we determine the present value, P_6, of the cash flow occurring at the 6-year vertex. (In general, P_i denotes the present value of a cash flow occurring in *i* years.)

3. Calculate the standard deviation of the price return on the actual cash flow:

   We obtain the standard deviation, σ_6, of the return on the 6-year zero coupon bond, by a linear interpolation of the standard deviations of the 5- and 7-year price returns, i.e., σ_5 and σ_7, respectively.

   Note that σ_5 and σ_7 are provided in the RiskMetrics data sets as the VaR statistics $1.65\sigma_5$ and $1.65\sigma_7$, respectively. Hence, $1.65\sigma_6$ is the interpolated VaR.

   To obtain σ_6, we use the following equation:

   [6.11] $$\sigma_6 = \hat{a}\sigma_5 + (1-\hat{a})\sigma_7 \qquad 0 \le \hat{a} \le 1$$

   where

   $\hat{a}$ = linear weighting coefficient from Eq. [6.10]
   σ_5 = standard deviation of the 5-year return
   σ_7 = standard deviation of the 7-year return

4. Compute the allocation, α and (1-α), from the following equation:

[6.12]
$$\begin{aligned}\text{Variance}\,(r_{6\text{yr}}) &= \text{Variance}\,[\alpha r_{5\text{yr}} + (1-\alpha)\,r_{7\text{yr}}],\ \text{or the equivalent}\\ \sigma_6^2 &= \alpha^2\sigma_5^2 + 2\alpha(1-\alpha)\,\rho_{5,7}\sigma_5\sigma_7 + (1-\alpha)^2\sigma_7^2\end{aligned}$$

where $\rho_{5,7}$, is the correlation between the 5- and 7- year returns. (Note that $\rho_{5,7}$ is provided in the correlation matrix in RiskMetrics data sets).

Equation [6.12] can be written in the quadratic form

[6.13] $$a\alpha^2 + b\alpha + c = 0$$

where

$$\begin{aligned}a &= \sigma_5^2 + \sigma_7^2 - 2\rho_{5,7}\sigma_5\sigma_7\\ b &= 2\rho_{5,7}\sigma_5\sigma_7 - 2\sigma_7^2\\ c &= \sigma_7^2 - \sigma_6^2\end{aligned}$$

The solution to α is given by

[6.14] $$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice that Eq. [6.14] yields two solutions (roots). We choose the solution that satisfies the three conditions listed on page 118.

5. Distribute the actual cash flow onto the RiskMetrics vertices:

   Split the actual cash flow at year 6 into two components, α and (1-α), where you allocate α to the 5-year RiskMetrics vertex and (1-α) to the 7-year RiskMetrics vertex.

Using the steps above, we compute a RiskMetrics cash flow map from the following real-world data. Suppose that on July 31, 1996, the cash flow occurring in 6 years is USD 100. The RiskMetrics daily data sets provide the statistics shown in Table 6.1, from which we calculate the data shown in Table 6.2.[7]

[7] Recall that RiskMetrics provides VaR statistics—that is, 1.65 times the standard deviation.

Table 6.1
Data provided in the daily RiskMetrics data set

y_5	5-year yield	6.605%
y_7	7-year yield	6.745%
$1.65\sigma_5$	volatility on the 5-year bond price return	0.5770%
$1.65\sigma_7$	volatility on the 7-year bond price return	0.8095%
$\rho_{5,\,7}$	correlation between the 5- and 7-year bond returns	0.9975

Table 6.2
Data calculated from the daily RiskMetrics data set

y_6	6-year yield (from Eq. [6.10], where $\hat{a} = 0.5$)	6.675%
σ_6	standard deviation on the 6-year bond price return	0.4202%
σ_6^2	variance on the 6-year bond price return (from Eq. [6.11])	1.765×10^{-3}%
σ_5^2	variance on the 5-year bond price return	1.223×10^{-3}%
σ_7^2	variance on the 7-year bond price return	2.406×10^{-3}%

To solve for α, we substitute the values in Tables 6.1 and 6.2 into Eq. [6.13] to find the following values:

$$a = 2.14 \times 10^{-6}$$
$$b = -1.39 \times 10^{-5}$$
$$c = 6.41 \times 10^{-6}$$

which in Eq. [6.14], yields the solutions $\alpha = 5.999$ and $\alpha = 0.489$. We disqualify the first solution since $(1-\alpha)$ violates the sign preservation condition (page 118).

From Step 2 on page 119, the present value of USD 100 in 6 years is USD 93.74, i.e., the 6-year cash flow. We allocate 49.66% (USD 46.55) of it to the 5-year vertex and 50.33% (USD 47.17) to the 7-year vertex. We thus obtain the RiskMetrics cash flow map shown in Chart 6.15.

The preceding example demonstrated how to map a single cash flow to RiskMetrics vertices. In practice portfolios often contain many cash flows, each of which has to be mapped to the RiskMetrics vertices. In such instances, cash flow mapping simply requires a repeated application of the methodology explained in this section.

6.3 Step 3—Computing Value-at-Risk

This section explains two analytical approaches to measuring Value-at-Risk: simple VaR for linear instruments, and delta-gamma VaR for nonlinear instruments, where the terms “linear” and “nonlinear” describe the relationship of a position’s underlying returns to the position’s relative change in value. (For more information about simple VaR methodology, see Section 6.3.2. For more information about delta-gamma methodology, see Section 6.3.3.)

In the simple VaR approach, we assume that returns on securities follow a conditionally multivariate normal distribution (see Chapter 4) and that the relative change in a position’s value is a linear function of the underlying return. Defining VaR as the 5th percentile of the distribution of a portfolio's relative changes, we compute VaR as 1.65 times the portfolio's standard deviation, where the multiple 1.65 is derived from the normal distribution. This standard deviation depends on the volatilities and correlations of the underlying returns and on the present value of cash flows.

In the delta-gamma approach, we continue to assume that returns on securities are normally distributed, but allow for a nonlinear relationship between the position's value and the underlying returns. Specifically, we allow for a second-order or gamma effect, which implies that the distribution of the portfolio's relative change is no longer normal. Therefore, we cannot define VaR as 1.65 times the portfolio's standard deviation. Instead, we compute VaR in two basic steps. First, we calculate the first four moments of the portfolio's return distribution, i.e., the mean, standard deviation, skewness and kurtosis. Second, we find a distribution that has the same four moments and then calculate the fifth percentile of this distribution, from which we finally compute the VaR.

The choice of approach depends on the type of positions that are at risk, i.e., linear or non-linear positions, as defined above.

6.3.1 Relating changes in position values to underlying returns

This section explains the linearity and nonlinearity of instruments in the context of RiskMetrics methodology.

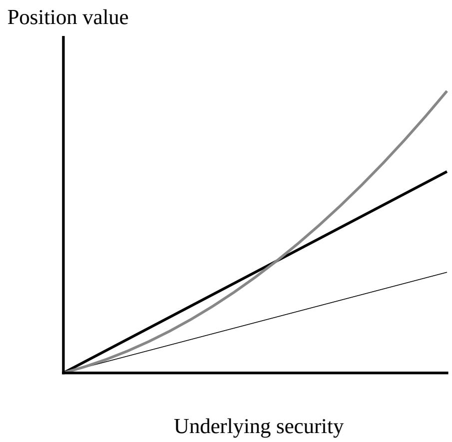
Value-at-Risk measures the market risk of a portfolio. We define a portfolio as a set of positions, each of which is composed of some underlying security. In order to calculate the risk of the portfolio, we must be able to compute the risks of the positions that compose the portfolio. This requires an understanding of how a position's value changes when the value on its underlying security changes. Thus, we classify positions into simple positions, which are linear, and into derivative positions, which can be further broken down into linear and nonlinear derivative positions.

As an example of a simple position, the relative change in value of a USD 100 million dollar position in DEM is a linear function of the relative change in value in the USD/DEM exchange rate (i.e., the return on the USD/DEM exchange rate).

The value of derivative positions depends on the value of some other security. For example, the value of a forward rate agreement, a linear derivative, depends on the value of some future interest rate. In contrast, other derivatives may have a nonlinear relationship between the relative change in value of the derivative position and the value of the underlying security. For example, the relative change in value of an option on the USD/FRF exchange rate is a nonlinear function of the return on that rate.

Chart 6.16 shows how the return on a position varies with the return on the underlying security.

Chart 6.16
Linear and nonlinear payoff functions



The straight lines in Chart 6.16 signify a constant relationship between the position and underlying security. The black line represents a one-to-one relationship between the position value and the underlying security. Note that for a payoff to be linear, the movement between the position value and the underlying security's value does not have to be one-to-one. For example, the change in value of a simple option can be expressed in terms of the "delta" (slope) of the underlying security, where the delta varies between −1 and +1. Chart 6.16 shows a payoff function where delta is 0.5 (gray line).

When payoffs are nonlinear there is no longer a "straight line" relationship between the position value and the underlying security's value. Chart 6.16 shows that the payoff line is curved such that the position value can change dramatically as the underlying security value increases. The convexity of the line is quantified by the parameter "gamma".

In summary, linear payoffs are characterized by a constant slope, delta. Their convexity, gamma, is always equal to zero. VaR for such instruments is calculated from the simple VaR methodology (Section 6.3.2). For nonlinear payoffs, delta can take on any value between –1 and +1, while gamma is always non-zero, accounting for the observed curvature of the payoff function. Nonlinear instruments are thus treated by the delta-gamma methodology (although the same methodology can also be used to handle linear instruments. See Section 6.3.3 on page 129).

Table 6.3 lists selected positions (instruments), their underlying returns, and the relationship between the two.

Table 6.3
Relationship between instrument and underlying price/rate

Type of position	Instrument[*]	Underlying price/rate[†]
Simple (linear):	Bond	Bond price[§]
	Stock	Local market index
	Foreign exchange	FX rate
	Commodity	Commodity price
	IR swap	Swap price
Linear derivative:	Floating rate note	Money market price

Table 6.3 (continued)
Relationship between instrument and underlying price/rate

Type of position	Instrument*	Underlying price/rate†
	FX forward	FX rate/money market price
	Forward rate agreement	Money market price
	Currency swap	Swap price/FX rate
Nonlinear derivative:	Stock Option	Stock price
	Bond Option	Bond price
	FX Option	FX rate

* Treated by $\hat{r}_t$. See the remainder of Section 6.3.
† Treated by r_t. See the remainder of Section 6.3.
§ Note, however, the relationship between a bond price and its yield is nonlinear.

6.3.1.1 Linear positions
Using the qualitative information in the preceding section, we now formally derive the relationship between the relative change in the value of a position and an underlying return for linear instruments.

We denote the relative change in value of the *i*th position, at time *t*, as $\hat{r}_{i,t}$. In the simple case where there is a linear one-to-one correspondence between the relative change in value of this position and some underlying return $r_{j,t}$, we have $\hat{r}_{i,t} = r_{j,t}$.[8] In general, we denote a position that is linearly related to an underlying return as $\hat{r}_{i,t} = \delta r_{j,t}$, where δ is a scalar.

Notice that in the case of fixed income instruments, the underlying value is defined in terms of prices on zero equivalent bonds (Table 6.3). Alternatively, underlying returns could have been defined in terms of yields. For example, in the case of bonds, there is no longer a one-to-one correspondence between a change in the underlying yield and the change in the price of the instrument. In fact, the relationship between the change in price of the bond and yield is nonlinear. Since we only deal with zero-coupon bonds we focus on these. Further, we work with continuous compounding.

Assuming continuous compounding, the price of an N-period zero-coupon bond at time *t*, P_t, with yield y_t is

[6.15] $$P_t = e^{-y_t N}$$

A second order approximation to the relative change in P_t yields

[6.16] $$\hat{r}_t = -y_t N(\Delta y_t / y_t) + \frac{1}{2}(y_t N)^2 (\Delta y_t / y_t)^2$$

Now, if we define the return r_t in terms of relative yield changes, i.e., $r_t = (\Delta y_t / y_t)$, then we have

[6.17] $$\hat{r}_t = -y_t N(r_t) + \frac{1}{2}(y_t N)^2 (r_t)^2$$

8 Technically, this results from the fact that the derivative of the price of the security with respect to the underlying price is 1.

Equation [6.17] reveals two properties: (1) If we ignore the second term on the right-hand side we find that the relative price change is linearly, but not in one-to-one correspondence, related to the return on yield. (2) If we include the second term, then there is a nonlinear relationship between return, r_t, and relative price change.

6.3.1.2 Nonlinear positions (options)

In options positions there is a nonlinear relationship between the change in value of the position and the underlying return. We explain this relationship with a simple stock option. For a given set of parameters denote the option's price by $V(P_t, K, \tau, \rho, \sigma)$ where P_t is the spot price on the underlying stock at time t, K is the option's exercise price, τ is the time to maturity of the option in terms of a year, ρ is the riskless rate of a security that matures when the option does, and σ is the standard deviation of the log stock price change over the horizon of the option.

In order to obtain an expression for the return on the option, $\widehat{r}_{i,t}$, we approximate the future value of the option $V(P_{t+1}, K, \tau, \rho, \sigma)$ with a second-order Taylor series expansion around the current values (spot rates), $P_t, K, \tau, \rho,$ and σ. This yields,

[6.18] $$V(P_{t+1}, K, \tau, \rho, \sigma) \cong V_0(P_t, K, \tau, \rho, \sigma) + \left(\frac{\partial V}{\partial P}\right)(P_{t+1} - P_t) + \frac{1}{2}\left(\frac{\partial^2 V}{\partial P^2}\right)(P_{t+1} - P_t)^2$$

which can be rewritten in the more concise form

[6.19] $$dV \cong \delta(dP) + \frac{1}{2}\Gamma(dP)^2$$

Notice that dV, the change in value of the option, is in units of price P thus δ is unitless and Γ is in units of $1/P$. Writing Eq. [6.19] in terms of relative changes, we get

[6.20] $$\widehat{r}_t \cong \eta \cdot \left[\delta r_t + \frac{1}{2}\Gamma P_t r_t^2\right]$$

where η measures the leverage effect of holding the option, δ measures the relative change in the value of the option given a change in the value of the price P_t, Γ measures the relative change in the value of the option given a change in the value of δ.

As Eq. [6.20] shows, the relative change, $\widehat{r}_t$, in the option is a nonlinear function of r_t, the return on the underlying stock price, since it involves the term r_t^2.

6.3.2 Simple VaR calculation

In this section we provide the general formula to compute VaR for linear instruments. (These instruments include the first nine listed in Table 6.3.) The example provided below deals exclusively with the VaR calculation at the 95% confidence interval using the data provided by RiskMetrics.

Consider a portfolio that consists of N positions and that each of the positions consists of one cash flow on which we have volatility and correlation forecasts. Denote the relative change in value of the nth position by $\widehat{r}_{n,t}$. We can write the change in value of the portfolio, $\widehat{r}_{p,t}$, as

[6.21] $$\widehat{r}_{p,t} = \sum_{n=1}^{N} \omega_n \widehat{r}_{n,t} = \sum_{n=1}^{N} \omega_n \delta_n r_{n,t}$$

where ω_n is the total (nominal) amount invested in the *n*th position. For example, suppose that the total current market value of a portfolio is $100 and that $10 is allocated to the first position. It follows that $\omega_1 = \$10$.

Now, suppose that the VaR forecast horizon is one day. In RiskMetrics, the VaR on a portfolio of simple linear instruments can be computed by 1.65 times the standard deviation of $\hat{r}_{p,t}$—the portfolio return, one day ahead.The expression of VaR is given as follows.

[6.22] $$VaR_t = \sqrt{\tilde{V}_{t|t-1} R_{t|t-1} \tilde{V}^T_{t|t-1}} \quad \text{(Value-at-Risk estimate)}$$

where

[6.23] $$\tilde{V}_{t|t-1} = [1.65\sigma_{1,t|t-1}\omega_1\delta_1 \quad 1.65\sigma_{2,t|t-1}\omega_2\delta_2 \ \ldots \quad 1.65\sigma_{N,t|t-1}\omega_N\delta_N]$$

is the individual VaR vector (1xN) and

[6.24] $$R_{t|t-1} = \begin{bmatrix} 1 & \rho_{12,t|t-1} & \ldots & \rho_{1N,t|t-1} \\ \rho_{21,t|t-1} & 1 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{N1,t|t-1} & \ldots & \ldots & 1 \end{bmatrix}$$

is the NxN correlation matrix of the returns on the underlying cash flows.

The above computations are for portfolios whose returns can be reasonably approximated by the conditional normal distribution. In other words, it is assumed that the portfolio return follows a conditional normal distribution.

6.3.2.1 Fixed income instruments

In this section we address two important issues related to calculating the VaR on a portfolio of fixed income instruments. The first issue relates to what variable should be used to measure volatility and correlation. In other words, should we compute volatilities and correlations on prices or on yields? The second issue deals with incorporating the "roll down" and "pull-to-par" effects of bonds into VaR calculations.

We discussed in Section 6.3.1.1 that one may choose to model either the yield (interest rate) or the price of a fixed income instrument. **RiskMetrics computes the price volatilities and correlations on all fixed income instruments**. This is done by first computing zero rates for all instruments with a maturity of over one year, and then constructing prices from these series using the expression (continuous compounding).

[6.25] $$P_t = e^{-y_t N}$$

where y_t is the current yield on the N-period zero-coupon bond.

For money market rates, i.e., instruments with a maturity of less than one-year, prices are constructed from the formula

[6.26] $$P_t = \frac{1}{(1 + y_t)^N}$$

Since practitioners often think of volatilities on fixed income instruments in terms of yield, we present the price volatility in terms of yield volatility. Starting with Eq. [6.25], we find the price return to be

$$r_t = \ln(P_t/P_{t-1}) = N(y_{t-1} - y_t) \qquad [6.27]$$

Therefore, the standard deviation of price returns under continuous compounding is given by the expression

$$\sigma_t = N\sigma(y_{t-1} - y_t) \qquad [6.28]$$

where $\sigma(y_{t-1} - y_t)$ is the standard deviation of $y_{t-1} - y_t$. What Eq. [6.28] states is that price return volatility is the maturity on the underlying instrument times the standard deviation of the absolute changes in yields.

Performing the same exercise on Eq. [6.26] we find the price return to be

$$\begin{aligned} r_t &= \ln(P_t/P_{t-1}) \\ &= N\left[\ln\left(\frac{1+y_{t-1}}{1+y_t}\right)\right] \end{aligned} \qquad [6.29]$$

In this case (discrete compounding) the standard deviation of price returns is

$$\sigma_t = N\sigma\left[\ln\left(\frac{1+y_t}{1+y_{t-1}}\right)\right] \qquad [6.30]$$

where $\sigma\left[\ln\left(\frac{1+y_t}{1+y_{t-1}}\right)\right]$ is the standard deviation of $\ln\left(\frac{1+y_t}{1+y_{t-1}}\right)$.

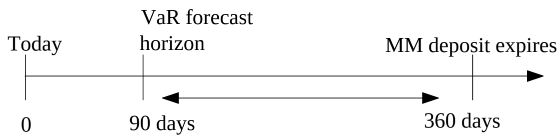
We now explain how to incorporate the unique features of fixed income instruments in VaR calculations.[9] Traditionally, RiskMetrics treats a cash flow as a zero coupon bond and subjects it to two assumptions: (1) There is no expected change in the market value of such a bond, and (2) the volatility of the bond's market value scales up with the square root of the time horizon. In reality, the bond's market value systematically increases toward its par value (the "pull to par" effect), and its daily volatility decreases as it moves closer to par (the "roll down" effect). The two assumptions imply that the cash flow is treated as a generic bond (a bond whose maturity is always the same) rather than as an instrument whose maturity decreases with time. While this leads to an accurate depiction of the risk of the future cash flow for short forecast horizons, for longer horizons, it can result in a significant overstatement of risk.

Suppose that as of today, a USD based investor currently holds a one-year USD money market deposit and is interested in computing a Value-at-Risk estimate of this instrument over a 3-month forecast horizon. That is, the investor would like to know the maximum loss on this deposit (at a 95% confidence level) if he held the deposit for 3 months. To compute the risk of this position we compute the VaR of holding 9-month deposit with a forecast horizon 3-months. In other words, we are measuring the volatility on the 9-month deposit over a 3-month forecast horizon. To do this we use the current 9-month money market rates. This addresses the "roll down effect". In addition, the expected value of holding a one-year deposit for 3 months is not zero. Instead, the mean return is

[9] This section is based on the article by Christopher C. Finger, "Accounting for the "pull to par" and "roll down" for RiskMetrics cash flows", *RiskMetrics Monitor*, September 16, 1996.

non-zero reflecting the pull-to-par phenomenon. Chart 6.17 presents a visual description of the situation.

Chart 6.17
VaR horizon and maturity of money market deposit



In general, the methodology to measure the VaR of a future cash flow(s) that occurs in T days over a forecast horizon of t days ($t < T$) is as follows.

1. Use the T-t rate, y_{T-t}, to discount the cash flow occurring in T days time. Denote the present value of this cash flow by V_{T-t}
2. Compute VaR as $V_{T-t} \cdot \left(\sigma_{T-t} \sqrt{t} \right)$.

Note that in the preceding example, $T = 360$, $t = 90$, y_{T-t} is the 270-day rate and σ_{T-t} is the standard deviation of the distribution of returns on the 270-day rate.

6.3.2.2 Equity positions

The market risk of the stock, VaR_t, is defined as the market value of the investment in that stock, V_t, multiplied by the price volatility estimate of that stock's returns, $1.65\sigma_t$.

[6.31] $$VaR_t = V_t \cdot 1.65\sigma_t$$

Since RiskMetrics does not publish volatility estimates for individual stocks, equity positions are mapped to their respective local indices. This methodology is based upon the principles of single-index models (the Capital Asset Pricing Model is one example) that relate the return of a stock to the return of a stock (market) index in order to attempt to forecast the correlation structure between securities. Let the return of a stock, r_t, be defined as

[6.32] $$r_t = \beta_t r_{m,t} + \alpha_t + \varepsilon_t$$

where

r_t = return of the stock
$r_{m,t}$ = the return of the market index
β_t = beta, a measure of the expected change in r_t given a change in $r_{m,t}$
α_t = the expected value of a stock's return that is firm-specific
ε_t = the random element of the firm-specific return with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t]^2 = \sigma^2_{\varepsilon_t}$

As such, the returns of an asset are explained by market-specific components $(\beta_t R_{m,t})$[3] and by stock-specific components $(\alpha_t + \varepsilon_t)$. Similarly, the total variance of a stock is a function of the market- and firm-specific variances.

Since the firm-specific component can be diversified away by increasing the number of different equities that comprise a given portfolio, the market risk, VaR_t, of the stock can be expressed as a function of the stock index

[6.33] $$\sigma_t^2 = \beta_t^2 \sigma_{m,t}^2 + \sigma_{\varepsilon_t}^2 .$$

Substituting Eq. [6.33] into Eq. [6.31] yields

[6.34] $$VaR_t = V_t \cdot \beta_t \cdot 1.65\sigma_{m,t} ,$$

where

$$1.65\sigma_{m,t} = \text{the RiskMetrics VaR estimate for the appropriate stock index} .$$

As with individual stocks, Eq. [6.34] should also be used to calculate the VaR for positions that consist of issue that themselves are part of the EMBI+.

6.3.3 Delta-gamma VaR methodology (for portfolios containing options)

In this section we describe a methodology known as delta-gamma that allows users to compute the Value-at-Risk of a portfolio that contains options. Specifically, we provide a methodology to incorporate the delta, gamma and theta of individual options in the VaR calculation. We explain this methodology by first showing how it applies to a single option and then to a portfolio that contains three options. To keep our examples simple, we assume that each option is a function of one cash flow. In other words, we can write the return on each option as

[6.35] $$\widehat{r}_{1,t} = \tilde{\delta}_1 r_{1,t} + 0.5\tilde{\Gamma}_1 r_{1,t}^2 + \tilde{\theta}_1 n$$

where
$$\tilde{\delta}_1 = \eta_1 \delta_1$$
$$\tilde{\Gamma}_1 = \eta_1 P_{1,t} \Gamma_1$$
$$\tilde{\theta}_1 = \theta_1 / V_1$$
$$n = \text{VaR forecast horizon}$$
$$V_1 = \text{option's premium}$$

For a complete derivation of Eq. [6.35], see Appendix D. Similarly, we can write the returns on the other two options as

[6.36] $$\widehat{r}_{2,t} = \tilde{\delta}_2 r_{2,t} + 0.5\tilde{\Gamma}_2 r_{2,t}^2 + \tilde{\theta}_2 n \quad \text{and} \quad \widehat{r}_{3,t} = \tilde{\delta}_3 r_{3,t} + 0.5\tilde{\Gamma}_3 r_{3,t}^2 + \tilde{\theta}_3 n$$

Let's begin by demonstrating the effect of incorporating gamma and theta components on the return distribution of the option. We do so by comparing the statistical features on the return on option 1, $\widehat{r}_{1,t}$, and the return of its underlying cash flow, $r_{1,t}$. Recall that RiskMetrics assumes that the returns on the underlying assets, $r_{1,t}$, are normally distributed with mean 0 and variance

$\sigma^2_{1,t}$. Table 6.4 shows the first four moments[10]—the mean, variance, skewness, and kurtosis—for $\hat{r}_{1,t}$ and $r_{1,t}$.

Table 6.4
Statistical features of an option and its underlying return

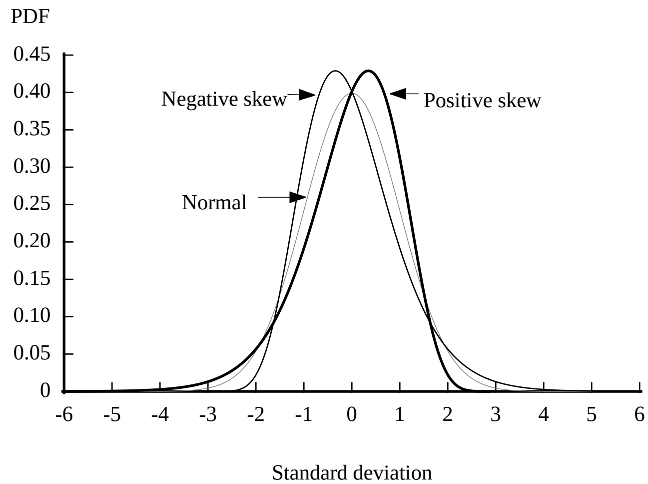
Statistical parameter	**Option**	**Underlying**
Return	$\hat{r}_{1,t}$	$r_{1,t}$
Mean	$0.5\tilde{\Gamma}\sigma^2_{1,t} + \tilde{\theta}_1 n$	0
Variance	$\tilde{\delta}_1^2\sigma^2_{1,t} + 0.5\tilde{\Gamma}^2\sigma^4_{1,t}$	$\sigma^2_{1,t}$
Skewness	$3\tilde{\delta}_1^2\tilde{\Gamma}_1\sigma^4_{1,t} + \tilde{\Gamma}_1^3\sigma^6_{1,t}$	0
Kurtosis	$12\tilde{\delta}_1^2\tilde{\Gamma}_1^2\sigma^6_{1,t} + 3\tilde{\Gamma}_1^4\sigma^8_{1,t} + 3\sigma^4_{1,t}$	$3\sigma^4_{1,t}$

The results presented in Table 6.4 point to three interesting findings.

- First, even though it is assumed that the return on the underlying has a zero mean return, this is not true for the option's return unless both gamma and theta are zero. Also, the sign of the option's mean will be determined by the relative magnitudes and signs of both gamma and theta and whether one is long or short the option.
- Second, the variance of the return on the option differs from the variance of the return on the underlying instrument by the factor $\left(\tilde{\delta}_1^2 + 0.5\tilde{\Gamma}^2\sigma^2_{1,t}\right)$.
- And third, depending on whether one is long or short the option determines whether the return on the option distribution is negatively or positively skewed. To see this, on a short option position, $V_1 < 0$ and therefore $\tilde{\Gamma}_1 < 0$. Consequently, the term $\tilde{\Gamma}^3$ in the skewness expression will be negative. As an example of this point, Chart 6.18 shows the probability density functions for long and short options positions (along with the normal probability curve).

[10] See Section 4.5.2.1 for the definition of these moments.

Chart 6.18
Long and short option positions
negative and positive skewness



Note that in variance and kurtosis, the sign of $\tilde{\Gamma}$ is immaterial since in these expressions $\tilde{\Gamma}$ is raised to an even power.

Now, to determine the numerical values of the moments presented in Table 6.4 we need estimates of $\tilde{\delta}_1$, $\tilde{\Gamma}_1$, $\tilde{\theta}_1$ and $\sigma^2_{1,t}$. Estimates of the first three parameters are easily found by applying a Black-Scholes type valuation model. The variance, $\sigma^2_{1,t}$, is given in the RiskMetrics data sets.

Having obtained the first four moments of $\widehat{r}_{1,t}$'s distribution, we find a distribution that has the same moments but whose distribution we know exactly. In other words, we match the moments of $\widehat{r}_{1,t}$'s distribution to one of a set of possible distributions known as Johnson distributions. Here, "matching moments" simply means finding a distribution that has the same mean, standard deviation, skewness and kurtosis as $\widehat{r}_{1,t}$'s. The name Johnson comes from the statistician Norman Johnson who described a process of matching a particular distribution to a given set of moments.

Matching moments to a family of distributions requires that we specify a transformation from the option's return $\widehat{r}_{1,t}$ to a return, $r_{1,t}$, that has a standard normal distribution. For example, Johnson (1949) suggested the general transformation

[6.37] $$r_{1,t} = a + bf\left(\frac{\widehat{r}_{1,t} - c}{d}\right)$$

where $f(\ )$ is a monotonic function and a, b, c and d are parameters whose values are determined by $\widehat{r}_{1,t}$'s first four moments. In addition to the normal distribution, the Johnson family of distributions consists of three types of transformations.

[6.38] $$r_{1,t} = a + b\log\left(\frac{\widehat{r}_{1,t} - c}{d}\right) \quad \text{(Lognormal)}$$

[6.39] $$r_{1,t} = a + b\sinh^{-1}\left(\frac{\widehat{r}_{1,t} - c}{d}\right) \quad \text{(Unbounded)}$$

and

[6.40] $$r_{1,t} = a + b\log\left(\frac{\tilde{r}_{1,t} - c}{c + d - \tilde{r}_{1,t}}\right) \quad \text{(Bounded)}$$

with the restriction $(c < \tilde{r}_{1,t} < c + d)$.

To find estimates of a, b, c and d, we apply a modified version of Hill, Hill and Holder's (1976) algorithm.[11] Given these estimates we can calculate any percentile of $\tilde{r}_{1,t}$'s distribution given the corresponding standard normal percentile (e.g., –1.65). This approximate percentile is then used in the VaR calculation. For example, suppose that we have estimates of $\tilde{\delta}_1$, $\tilde{\Gamma}_1$, $\tilde{\theta}_1$ and $\sigma^2_{1,t}$ and that they result in the following moments: mean = 0.2, variance = 1, skewness coefficient = 0.75 and kurtosis coefficient = 7. Note that these numbers would be derived from the formulae presented in Table 6.3. Applying the Hill et. al. algorithm we find that the selected distribution is "Unbounded" with parameter estimates: a = –0.582, b = 1.768, c = –0.353, and d = 1.406.

Therefore, the percentile of $\tilde{r}_{1,t}$'s distribution is based on the transformation

[6.41] $$\tilde{r}_{1,t} = \sinh\left(\frac{(r_{1,t} - a)}{b}\right) \cdot d + c$$

Setting $r_{1,t}$ = –1.65, the estimated 5th percentile of $\tilde{r}_{1,t}$'s distribution is –1.45. That is, the option's fifth percentile is increased by 0.20. In this hypothetical example, the incorporation of gamma and theta reduces the risk relative to holding the underlying.

We now show that it is straightforward to compute the VaR of a portfolio of options. In particular, we show this for the case of a portfolio that contains three options. We begin by writing the portfolio return as

[6.42] $$\tilde{r}_{p,t} = \omega_1 \tilde{r}_{1,t} + \omega_2 \tilde{r}_{2,t} + \omega_3 \tilde{r}_{3,t}$$

$$\text{where } \omega_i = \frac{V_i}{\sum_{i=1}^{3} V_i}$$

To compute the moments of $\tilde{r}_{p,t}$'s distribution we need the RiskMetrics covariance matrix, Σ, of the underlying returns $\{r_{1,t}, r_{2,t}, r_{3,t}\}$, and the delta, gamma and theta cash flow vectors that are defined as follows:

[6.43] $$\tilde{\delta} = \begin{bmatrix} \tilde{\delta}_1 \\ \tilde{\delta}_2 \\ \tilde{\delta}_3 \end{bmatrix}, \quad \tilde{\Gamma} = \begin{bmatrix} \tilde{\Gamma}_1 & 0 & 0 \\ 0 & \tilde{\Gamma}_2 & 0 \\ 0 & 0 & \tilde{\Gamma}_3 \end{bmatrix}, \text{ and } \tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{bmatrix}$$

We find the 5th percentile $\tilde{r}_{p,t}$'s distribution the same way we found the 5th percentile of $\tilde{r}_{1,t}$'s distribution, as shown previously. The only difference is that now the expressions for the four moments are more complicated. For example, the mean and variance of the portfolio return are

[11] These original algorithms (numbers 99 and 100) are available in their entirety on the Web at the StatLib—Griffiths and Hill Archive. The URL is http://lib.stat.cmu.edu/griffiths-hill/.

[6.44] $$\mu_{p,t} = 0.5 \cdot \text{trace}\,[\tilde{\Gamma}\Sigma] + \sum_{i=1}^{3} \tilde{\theta}_i \text{ and}$$

[6.45] $$\sigma^2_{p,t} = \tilde{\delta}'\Sigma\tilde{\delta} + 0.5 \cdot \text{trace}\left[(\tilde{\Gamma}\Sigma)^2\right]$$

where “trace” is an operator that sums the diagonal elements of a matrix.

The delta-gamma methodology described in this section extends to options that have more than one underlying cash flow (e.g., bond option). We have presented a simple example purposely to facilitate our exposition of the methodology. See, Appendix D for an assessment of the methodology.

Finally, the methodologies presented in Section 6.3 do not require simulation. All that is necessary for computing VaR is a covariance matrix, financial parameters (such as delta, gamma and theta) and position values. In the next chapter we present a methodology known as structured Monte Carlo that computes VaR by first simulating future paths of financial prices and/or rates.

6.4 Examples

In this section we present nine examples of VaR calculations for the various instruments discussed in this chapter. Note that the diskette symbol placed to the left of each example means that the example appears on the enclosed diskette at the back of the book.

Ex. 6.1 Government bond mapping of a single cash flow

Suppose that on March 27, 1995, an investor owns FRF 100,000 of the French OAT benchmark 7.5% maturing in April 2005. This bond pays coupon flows of FRF 7,500 each over the next 10 years and returns the principal investment at maturity. One of these flows occurs in 6.08 years, between the standard vertices of 5 and 7 years (for which volatilities and correlations are available).

All the data required to compute the cash flow map is readily available in the RiskMetrics data sets except for the price volatility ($1.65\sigma_{6.08}$) of the original cash flow's present value. This must be interpolated from the price volatilities already determined for the RiskMetrics vertices.

Applying the three conditions on page 118 and using Eqs. [6.10]–[6.14] with the RiskMetrics data in Table 6.5, we solve for the allocation α (and $(1-\alpha)$), and obtain the values $\alpha = 4.30$ and $\alpha = 0.4073$. Given the constraint that both of the allocated cash flows must have the same sign as the original cash flow, we reject the first solution, which would lead to a short position in the second proxy cash flow. As a result, our original cash flow of FRF 7,500, whose present value is FRF 4,774, must be mapped as a combination of a 5-year maturity cash flow of FRF 1,944 (40.73% of the original cash flow's PV) and a 7-year maturity cash flow of FRF 2,829 (59.27% of the original cash flow's PV).

The cash flow map is shown in Table 6.6.

Table 6.5
RiskMetrics data for 27, March 1995

RiskMetrics Vertex	Yield,%	P. Vol,* ($1.65\sigma_t$)	Yield Vol,† ($1.65\sigma_t$)	Correlation Matrix, ρ_{ij} 5yr	7yr
5yr	7.628	0.533	1.50	1.000	0.963
7yr	7.794	0.696	1.37	0.963	1.000

* P. Vol = price volatility, also called the VaR statistic.
† While this data is provided in the data set, it is not used in this calculation.

Table 6.6
RiskMetrics map of single cash flow

			Step 1*	Step 2*†		Step 3*		Step 4*	Step 5*
Coupon Flow	**Term**	**Yield,% (Actual)**	**Yield,% ($y_{6.08}$) (Interpolated)**	**$(PV)_{6.08}$‡**	**P. Vol, ($1.65\sigma_t$)§ (RiskMetrics)**	**P. Vol, ($1.65\sigma_{6.08}$)§ (Interpolated)**	**RiskMetrics Vertex**	**Allocation**	**RiskMetrics Cash flow**
		7.628			0.533		5yr	0.4073	1,950
7,500	6.08yr		7.717	4,774		0.624			
		7.794			0.696		7yr	0.3927	2,824

* Step from the mapping procedure on pages 119–121. Also, data in this column is calculated from the data in Table 6.5. Note that in Step 3 the price volatility, $1.65\sigma_{6.08}$, rather than the standard deviation alone, is computed.
† In this example $\hat{a} = 0.46$.
‡ PV = present value.
§ P. Vol = price volatility, also called the VaR statistic.

Ex. 6.2 Government bond mapping of multiple cash flows

A full set of positions can easily be mapped in the same fashion as the single cash flow in the last example.

The example below takes the instrument in Ex. 6.1, i.e., the 10-year French OAT benchmark on March 27, 1995, and decomposes all of the component cash flows according to the method described on pages 119–121, to create a detailed RiskMetrics cash flow map. Table 6.7 shows how the 100,000 French franc nominal position whose market value is FRF 97,400, is decomposed into nine representative present value cash flows. The table also shows the VaR for the cash flow at each RiskMetrics vertex and the diversified Value-at-Risk.

In this example, note that the first cash flow (on 25-Apr-95) occurs in less than one month's time relative to March 27, but is allocated at 100% weight to the 1-month RiskMetrics vertex. The reason for 100% allocation is that vertices shorter than one month are not defined in the RiskMetrics data sets.

Table 6.7
RiskMetrics map for multiple cash flows

Bond data	
Settlement	30-Mar
Principal	100,000
Price	97.4
Coupon	7.50
Basis	365

RiskMetrics™ vertices		**1m**	**1y**	**2y**	**3y**	**4y**	**5y**	**7y**	**10y**	**15y**
Yield volatility		7.00	3.16	2.10	1.74	1.63	1.50	1.37	1.36	1.29
Current yield		8.25	7.04	7.28	7.39	7.54	7.63	7.79	7.92	8.15
Price volatility		0.04	0.21	0.29	0.36	0.46	0.53	0.70	1.00	1.46
Correlation Matrix	1m	**1.00**	**0.75**	0.53	0.48	0.45	0.42	0.33	0.33	0.33
	1y	**0.75**	**1.00**	**0.88**	0.81	0.78	0.74	0.61	0.63	0.58
	2y	0.53	**0.88**	**1.00**	**0.99**	0.96	0.92	0.80	0.82	0.76
	3y	0.48	0.81	**0.99**	**1.00**	**0.98**	0.95	0.85	0.87	0.81
	4y	0.45	0.78	0.96	**0.98**	**1.00**	**0.99**	0.91	0.93	0.88
	5y	0.42	0.74	0.92	0.95	**0.99**	**1.00**	**0.96**	0.96	0.93
	7y	0.33	0.61	0.80	0.85	0.91	**0.96**	**1.00**	**1.00**	0.99
	10y	0.33	0.63	0.82	0.87	0.93	0.96	**1.00**	**1.00**	**0.99**
	15y	0.33	0.58	0.76	0.81	0.88	0.93	0.99	**0.99**	**1.00**

Date	Flow	Term	Yield	PV	Md. Dur	P.Vol	1m	1y	2y	3y	4y	5y	7y	10y	15y
25-Apr-95	7,500	0.071	8.204	7,456	0.066	0.032	7,456								
25-Apr-96	7,500	1.074	7.056	6,970	1.003	0.218		5,594	1,376						
25-Apr-97	7,500	2.074	7.284	6,482	1.933	0.292			5,780	703					
25-Apr-98	7,500	3.074	7.402	6,022	2.862	0.366				5,505	517				
25-Apr-99	7,500	4.074	7.543	5,577	3.788	0.463					5,105	472			
25-Apr-00	7,500	5.077	7.635	5,162	4.717	0.539						4,923	240		
25-Apr-01	7,500	6.077	7.720	4,773	5.641	0.624						1,944	2,829		
25-Apr-02	7,500	7.077	7.798	4,408	6.565	0.703							4,302	107	
25-Apr-03	7,500	8.077	7.855	4,072	7.488	0.805							2,589	1,483	
25-Apr-04	7,500	9.079	7.895	3,762	8.415	0.905							1,131	2,631	
25-Apr-05	107,500	10.079	7.919	49,863	9.340	1.004								49,019	844

RiskMetrics™ vertices	**1m**	**1y**	**2y**	**3y**	**4y**	**5y**	**7y**	**10y**	**15y**
Total Vertex Mapping	7,456	5,594	7,156	6,207	5,622	7,339	11,091	53,239	844
RiskMetrics™ Vertex VaR	3	12	20	22	26	39	77	530	12

Diversified Value at Risk 727 FRF over the next 24 hours

% of market value 0.7%

Money market rate volatilities are used for vertices below 2 years. Government bond zero volatilities are used for 2-year and other vertices.

Ex. 6.3 Forward rate agreement cash flow mapping and VaR

A forward rate agreement is an interest-rate contract. It locks in an interest rate, either a borrowing rate (buying a FRA) or a lending rate (selling a FRA) for a specific period in the future. FRAs are similar to futures, but are over-the-counter instruments and can be customized for any maturity.

Because a FRA is a notional contract, there is no exchange of principal at the expiry date (i.e., the fixing date). If the rate is higher at settlement than the FRA rate agreed by the counterparties when they traded, then the seller of a FRA agrees to pay the buyer the present value of the interest rate differential applied to the nominal amount agreed upon at the time of the trade. The interest rate differential is between the FRA rate and the observed fixing rate for the period. In most cases this is the LIBOR rate for any given currency.

The general FRA pricing equation is given by

$$[6.46] \qquad Y_{T_S \times T_L FRA} = \left[\frac{1 + \left(Y_L \cdot \frac{T_L}{B} \right)}{1 + \left(Y_S \cdot \frac{T_S}{B} \right)} - 1 \right] \cdot \frac{B}{T_L - T_S}$$

where

Y_L = yield of the longer maturity leg

T_L = maturity of the longer maturity leg in days

B = Basis (360 or 365)

Y_S = yield of the shorter maturity leg

T_S = maturity of the shorter maturity leg in days

In effect, FRAs allow market participants to lock in a forward rate that equals the implied break-even rate between money market term deposits.

Given that a FRA is a linear combination of money market rates, it is simple to express its degree of risk as a function of the combination of these rates.

Suppose that on January 6, 1995 you sold a 6x12 FRA on a notional 1 million French francs at 7.24%. This is equivalent to locking in an investment rate for 6 months starting in 6 months' time. The rate of 7.24% is calculated by combining the 6- and 12-month money market rates using the general pricing equation, Eq. [6.46], which can be rewritten as follows to reflect the no-arbitrage condition:

$$[6.47] \qquad \left(1 + Y_{12m} \cdot \frac{365}{365} \right) = \left(1 + Y_{6m} \cdot \frac{181}{365} \right) \cdot \left(1 + Y_{6 \times 12 FRA} \cdot \frac{184}{365} \right)$$

where

Y_{6m}, Y_{12m} = 6- and 12-month French franc yields, respectively

Y_{6m}, Y_{12m} = 6 x 12 FRA rate

This FRA transaction is equivalent to borrowing FRF 1 million for 6 months on a discount basis (i.e., total liability of FRF 1 million in 6 months' time) and investing the proceeds (FRF 969,121) for 12 months. This combination can be mapped easily into the RiskMetrics vertices as shown in Table 6.8. The current present value of these two positions is shown in column (6). The Value-at-

Example 6.3 (continued)

Risk of each leg of the FRA over a 1-month horizon period, shown in column (9), is obtained by multiplying the absolute present value of the position by the monthly price volatility of the equivalent maturity deposit. The portfolio VaR is obtained by applying the 6- to 12-month correlation to the VaR estimate from column (9).

Table 6.8

Mapping a 6x12 short FRF FRA at inception

Observed data		**RiskMetrics data set**					**Calculated values**			
(1)	(2)	(3)	(4)		(5)		(6)	(7)	(8)	(9)
			Volatilities		Correlation matrix					
Cash flow	Term (mths.)	Yield,%	Yield	Price	6m	12m	Present value	RiskMetrics vertex	RiskMetrics cash flow	VaR estimate
−1,000,000	6	6.39	6.94	0.21	1.00	0.70	-969,121	6m	−969,121	2,081
1,036,317	12	6.93	7.42	0.48	0.70	1.00	969,121	12m	969,121	4,662
						Total	0		0	
						Portfolio VaR				3,530

One month into the trade, the mapping becomes somewhat more complex as the cash flows have now shorter maturities (the instrument is now in fact a 5x11 FRA). The 5-month cash flow must be mapped as a combination of 3-month and 6-month RiskMetrics vertices (Table 6.9), while the 11-month cash flow must be split between the 6-month and 12-month vertices.

Table 6.9

Mapping a 6x12 short FRF FRA held for one month

Observed data		**RiskMetrics data set**					**Calculated values**				
		Volatilities		Correlation matrix							
Cash flow	Term (mths.)	Yield	Price	3m	6m	12m	Yield,%	Present value	RiskMetrics vertex	RiskMetrics cash flow	VaR estimate
		6.77	0.1	1	0.81	0.67			3m	−302,232	−296
−1,000,000	5						6.12	−975,302			
		7.91	0.19	0.81	1.00	0.68			6m	−549,300	−1,048
1,036,317	11						6.65	976,894			
		7.14	0.41	0.67	0.68	1.00			12m	853,124	3,533
						Total		1,592		1,592	
						Portfolio VaR					2,777

Example 6.3 (continued)

One month into the trade, the change in market value of the contract is a positive FRF 1,592. This is well within the range of possible gains or losses predicted (with a 95% probability) by the previous month's Value-at-Risk estimate of FRF 3,530.

Unwinding a FRA, i.e., hedging out the interest rate risk between the FRA rate and the market rate, before maturity requires entering into a contract of opposite sign at dates that no longer qualify as standard maturities. If you wanted to unwind the position in this example one month after the dealing date, you would have to ask a quote to buy a 5x11 FRA, a broken dated instrument that is less liquid and therefore is quoted at higher bid-offer spreads. The rates in column (1) above do not take this into account. They were derived by interpolating rates between standard maturities. Actual market quotes would have been slightly less favorable, reducing the profit on the transaction. This risk is liquidity related and is not identified in the VaR calculations.

Ex. 6.4 Structured note

The basic concepts used to estimate the market risk of simple derivatives can be extended to more complex instruments. Suppose that in early 1994, when the market consensus was that German rates were to continue to decline, you had purchased a “one year index note linked to two year rates”. This 1-year instrument leveraged a view that the DEM 2-year swap rate in 1-year’s time would be below the forward rate measured at the time the transaction was entered into.

The characteristics of the instrument are shown in Table 6.10.

Table 6.10
Structured note specification

Issuer	Company A
Format	Euro Medium Term Note
Issue date	9 February 94
Maturity date	9 February 95
Issue price	100%
Amount	DEM 35,000,000
Coupon	5.10%
Strike	5.10%
Redemption value	100%+20*(Strike- 2-year DEM swap rate)

* Although these details are hypothetical, similar products were marketed in 1994.

While seemingly complex, this transaction is in fact little more (disregarding minor convexity issues) than a bond to which a leveraged long-dated FRA had been attached. As a holder of the note, you were long the 3-year swap rate and short the 1-year rate, with significant leverage attached to the difference.

Table 6.11 shows how the leveraged note can be decomposed into the cash flows of the two underlying building blocks:

- The 1-year DEM 35 million bond with a 5.10% coupon.
- The forward swap (2-year swap starting in one year). The forward principal cash flows of the swap are equal to 20 times the notional amount of the note divided by the PVBP (price value of a basis point) of a 2-year instrument, or 1.86 in this case. The forward coupons are equal to the forward principal times the coupon rate of 5.10%.

Table 6.11
Actual cash flows of a structured note

Term (years)	Bond		Swap		Total cash flow
	Principal	Coupon	Principal	Coupon	
1	35,000,000	1,785,000	–376,996,928		–340,211,928
2				19,226,843	19,226,843
3			376,996,928	19,226,843	396,223,771

Combining the bond and the swap creates three annual cash flows where the investor is short DEM 340 million in the 1-year, and long DEM 19 and DEM 396 million in the 2- and 3-year maturities. At issue, the market value of these cash flows is equal to DEM 35 million, the instrument’s issue price.

Example 6.4 (continued)

Each of the three cash flows is mapped to RiskMetrics vertices to produce the cash flow map shown in Table 6.12.

Table 6.12
VaR calculation of structured note
One month forecast horizon

Observed data		**RiskMetrics data set**					**Calculated values**			
				Correlation matrix						
Cash flow	Term (years)	Yield,%	Price volatility	1y	2y	3y	Present value	RiskMetrics vertex	RiskMetrics cash flow	VaR estimate
−340,211,928	1	5.48	0.33	1.00	0.46	0.43	−322,536,906	1y	322,536,906	1,067,597
19,226,843	2	5.15	0.46	0.46	1.00	0.95	17,389,594	2y	17,389,594	79,644
396,223,772	3	5.22	0.68	0.43	0.95	1.00	340,147,312	3y	340,147,312	2,309,600
						Total	35,000,000			
						Portfolio VaR				2,155,108

Using the appropriate volatilities and correlations as of February 9, 1994, the Value-at-Risk of such a position over a 1-month horizon was around DEM 2.15 million.

One month into the life of the instrument on March 9, the mapping and risk estimation could have been repeated using updated interest rates as well as updated RiskMetrics volatilities and correlations. Table 6.13 shows the result.

Table 6.13
VaR calculation on structured note
One-month into life of instrument

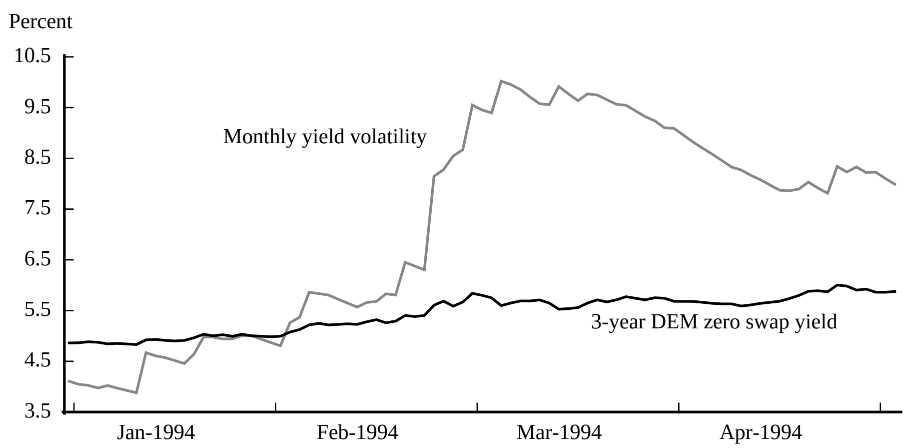
Observed data		**RiskMetrics data set**						**Calculated values**			
				Correlation matrix							
Cash flow	Term (years)	Yield,%	Price volatility	6m	1y	2y	3y	Present value	RiskMetrics vertex	RiskMetrics cash flow	VaR estimate
			0.16	1.00	0.83	0.58	0.54		6m	−43,218,017	68,023
−340,211,928	0.9	5.53						−975,302			
			0.35	0.83	1.00	0.58	0.54		1y	−277,979,823	960,250
19,226,843	1.9	5.53						17,399,085			
			0.65	0.58	0.58	0.94	1.00		2y	39,059,679	252180
19,226,843	2.9	5.68						337,230,727			
			1.03	0.54	0.54	0.94	1.00		3y	312,934,965	3,218,971
							Total	30,976,801		−4,203,199	
							Portfolio VaR				3,018,143

The movement in market rates led the market value of the note to fall by over DEM 4 million, twice the maximum amount expected to happen with a 95% probability using the previous month’s RiskMetrics volatilities and correlations. Why?

Example 6.4 (continued)

Chart 6.19 shows the 3-year DEM swap rate moved 45+bp from 5.22% to slightly above 5.68% during the month—twice the maximum amount expected with a 95% probability (4.56% × 5.22%; i.e., 23 basis points). This was clearly a large rate move. The RiskMetrics volatility estimate increased from 4.56% to 6.37% as of March 9. This reflects the rapid adjustment to recent observations resulting from the use of an exponential moving average estimation method. Correspondingly the VaR of the structured note increased 44% over the period to just over DEM 3 million.

Chart 6.19
DEM 3-year swaps in Q1-94



The message in these examples is that with proper cash flow mapping, the risks in complex derivatives can be easily estimated using the RiskMetrics methodology and data sets.

Ex. 6.5 Interest-rate swap

Investors enter into swaps to change their exposure to interest rate uncertainty by exchanging interest flows. In order to understand how to map its cash flows, a swap should be thought of as a portfolio of one fixed- and one floating-rate transaction. Specifically, the fixed leg of a swap is mapped as if it were a bond, while the floating leg is considered to be a FRN.

Market risk estimation is straightforward if the value of each leg is considered separately. The fixed leg exposes an investor to interest rate variability as would a bond. Since the floating leg is valued as if it were a FRN, if interest rates change, then forward rates used to value the leg change and it will revalue to par. Once a floating payment is set, the remaining portion of the floating leg will revalue to par, and we need only consider interest rate exposure with respect to that set cash flow. The details of this will be provided in a forthcoming edition of the *RiskMetrics Monitor.*

Consider the following example. A company that enters into a 5-year USD interest-rate swap pays 9.379% fixed and receives floating cash flows indexed off of 1-year USD LIBOR flat on a notional amount of USD 1,000,000. For simplicity, the reset/payment dates are annual. Table 6.14 presents the data used to estimate the market risk of this transaction.

Table 6.14
Cash flow mapping and VaR of interest-rate swap

Observed data				**RiskMetrics data set**						**Calculated values**
		Cash flow			Correlation matrix					
Term	Zero rate	Fixed	Floating	Price volatility, %	1yr	2yr	3yr	4yr	5yr	Net present value, USD
1yr	8.75	–86,247	1,000,000	0.027	1.000	0.949	0.933	0.923	0.911	913,753
2yr	9.08	–85,986		0.067	0.949	1.000	0.982	0.978	0.964	–85,986
3yr	9.24	–85,860		0.112	0.933	0.982	1.0000	0.995	0.984	85,860
4yr	9.34	–85,782		0.149	0.923	0.978	0.995	1.000	0.986	85,782
5yr	9.42	–999,629		0.190	0.911	0.964	0.984	0.986	1.000	–999,629
							Total			–343,505
							Portfolio VaR			1,958

Ex. 6.6 Foreign exchange forward

Below is an example of how to calculate the market risk of buying a 1-year 153,000,000 DEM/USD foreign exchange forward. Note that buying a DEM/USD foreign exchange forward is equivalent to borrowing US dollars for 1-year (short money market position) and using them to purchase Deutsche marks in one year's time (short foreign exchange position). We take the holding period to be one day. Based on a 1-day volatility forecast, the foreign exchange risk, in USD, is $904,922 (94,004,163 × 0.963%) as shown in Table 6.15. The interest rate risk is calculated by multiplying the current market value of each 12-month leg (the short in USD and the long in DEM) times its respective interest rate volatility. Therefore, the Value-at-Risk for a 1-day holding period is $912,880.

Table 6.15
VaR on foreign exchange forward

Observed data			**RiskMetrics data set**					**Calculated values**	
					Correlation matrix				
Instrument	Cash flow	Term (years)	Yield,%	Price volatility	DEM FX	DEM 1y	USD 1y	Present value, USD	VaR estimate
DEM Spot FX	—			0.963	1.0000	–0.0035	–0.0042	–94,004,163	–904,922
DEM 1y	153,000,000	1	6.12	0.074	–0.0035	1.0000	0.1240	94,004,163	45,855
USD 1y	–99,820,670	1	6.65	0.116	–0.0042	0.1240	1.0000	–94,004,163	–108,624
							Total	94,004,163	
							Portfolio VaR		3,530

Ex. 6.7 Equity

Consider a three-asset portfolio in which an investor holds stocks ABC and XYZ (both U.S. companies) as well as a basket of stocks that replicate the S & P 500 index. The market risk of this portfolio, VaR_p, is

[6.48]
$$\begin{aligned} VaR_p &= (V_{ABC} \cdot 1.65\sigma_{R_{ABC}}) + (V_{XYZ} \cdot 1.65\sigma_{R_{XYZ}}) \\ &\quad + V_{SP500} \cdot 1.65\sigma_{R_{SP500}} \end{aligned}$$

Rewriting this equation in terms of Eq. [6.48]

[6.49]
$$VaR_s = V_s \cdot \beta_s \cdot 1.65\sigma_{R_M}$$

where

$1.65\sigma_{R_M}$ = the RiskMetrics volatility estimate for the appropriate stock index,

yields

[6.50]
$$\begin{aligned} VaR_p &= (V_{ABC} \cdot \beta_{ABC} \cdot 1.65\sigma_{R_{SP500}}) + (V_{XYZ} \cdot \beta_{XYZ} \cdot 1.65\sigma_{R_{SP500}}) \\ &\quad + (V_{SP500} \cdot 1.65\sigma_{R_{SP500}}) \end{aligned}$$

Factoring the common term and solving for the portfolio VaR results in

[6.51]
$$\begin{aligned} VaR_p &= 1.65\sigma_{R_{SP500}} [(V_{ABC} \cdot \beta_{ABC}) + (V_{XYZ} \cdot \beta_{XYZ}) + V_{SP500}] \\ &= 4.832\% \, [1,000,000\,(0.5) + (1,000,000)\,(1.5) + 1,000,000] \\ &= 4.832\% \, (3,000,000) \\ &= \text{USD } 144,960 \end{aligned}$$

The methodology for estimating the market risk of a multi-index portfolio is similar to the process above and takes into account correlation among indices as well as foreign exchange rates. Since all positions must be described in a base or "home" currency, you need to account for foreign exchange risk.

Ex. 6.8 Commodity futures contract

Suppose on July 1, 1994 you bought a 6-month WTI future on a notional USD 18.3 million (1 million barrels multiplied by a price of USD 18.30 per barrel). The market and RiskMetrics data for that date are presented in Table 6.16.

Table 6.16
Market data and RiskMetrics estimates as of trade date July 1, 1994

			WTI future		Correlation matrix	
Vertex	LIBOR	Term	Price	Volatility	3m	6m
3m	5.563	0.250		10.25	1.000	
6m	5.813	0.500	18.30	9.47	0.992	1.000

The initial Value-at-Risk for a 1-month horizon is approximately USD 1.7 million. This represents the maximum amount, with 95% confidence, that one can expect to lose from this transaction over the next 25 business days. Since the flow occurs in 6 months, the entire position is mapped to the 6-month WTI vertex, therefore calculating the Value-at-Risk of this transaction on the trade date is simply

$$
\begin{aligned}
VaR_{6\text{m Future}} &= \textit{PV of cash flow} \cdot \textit{RiskMetrics volatility estimate} \\
&= \left[\frac{18,300,000}{1+\left(\frac{5.813\%}{100}\right)\cdot 0.5}\right]\cdot 9.47\% \qquad [6.52] \\
&= \text{USD 1.7 million}
\end{aligned}
$$

One month into the trade, the cash flow mapping becomes slightly more complex. Table 6.17 shows the new VaR of this transaction.

Table 6.17
Cash flow mapping and VaR of commodity futures contract

Term	Zero rate	Cash flow (PV)	Price volatility, %	Correlation matrix 3m	Correlation matrix 6m	RiskMetrics vertex	RiskMetrics cash flow
	4.810		6.212	1.00	0.992	3m	13,084,859
4m	5.068	17,924,465					
	5.190		5.739	0.992	1.00	6m	4,839,605
					Portfolio VaR		1,417,343

Ex. 6.9 Delta-gamma methodology

Consider the situation where a USD (US dollar) based investor currently holds a USD 1million equivalent position in a French government bond that matures in 6 years and a call option on Deutsche marks that expires in 3 months. Since market risk is measured in terms of a portfolio's return distribution, the first step to computing VaR is to write down an expression for this portfolio's return, R_P, which consists of one French government bond and one foreign exchange option. Here, return is defined as the one-day relative price change in the portfolio's value. The return on the portfolio is given by the expression

[6.53] $$\widehat{r}_p = \widehat{r}_o + r_B$$

where r_o is the return on the option, and r_B is the return on the French government bond.

We now provide a more detailed expression for the returns on the bond and option. Since the cash flow generated by the bond does not coincide with a specific RiskMetrics vertex, we must map it to the two nearest RiskMetrics vertices. Suppose we map 49% of the cash flow that arrives in 6 years' time to the 5-year vertex and 51 percent of the cash flow to the 7-year vertex. If we denote the returns on the 5 and 7-year bonds by r_5 and r_7, respectively, we can write the return on the French government bond as

[6.54] $$r_B = 0.49r_5 + 0.51r_7$$

Writing the return on the option is more involved. We write the return on the option as a function of its delta, gamma and theta components. The one-day return on the option is given by the expression[12]

[6.55] $$\widehat{r}_0 = \alpha\delta r_{\text{USD/DEM}} + 0.5\alpha\Gamma P_{\text{USD/DEM}} r^2_{\text{USD/DEM}} + V^{-1}\theta n$$

where

$r_{\text{USD/DEM}}$ is the one-day return on the DEM/USD exchange rate

$P_{\text{USD/DEM}}$ is the spot position in USD/DEM

V is the price of the option, or premium.

α is the ratio of $P_{\text{USD/DEM}}$ to V. The parameter α measures the leverage from holding the option.

δ is the "delta" of the option. Delta measures the change in the value of the option given a change in the underlying exchange rate.

Γ is the "gamma" of the option. Gamma measures the change in δ given a change in the underlying exchange rate.

θ is the "theta" of the option. Theta measures the change in the value of the option for a given change in the option's time to expiry.

n is the forecast horizon over which VaR is measured. In this example n is 1 for one day.

[12] We derive this expression in Appendix D.

Example 6.9 (continued)

We can now write the return on the portfolio as

[6.56] $$\hat{r}_p = 0.49r_5 + 0.51R_7 + \alpha\delta r_{USD/DEM} + 0.5\alpha\Gamma P_{USD/DEM} r^2_{USD/DEM} + V^{-1}\theta n$$

In particular, we find the first four moments of r_p's distribution that correspond to the mean, variance, skewness and kurtosis (a measure of tail thickness). These moments depend only on the price of the option, the current market prices of the underlying securities, the option's "greeks" δ, Γ, θ, and the RiskMetrics covariance matrix, Σ. In this example, Σ is the covariance matrix of returns r_5, r_7 and $r_{USD/DEM}$.

Let's take a simple hypothetical example to describe the delta-gamma methodology. Table 6.18 presents the necessary statistics on the bond and option positions to apply delta-gamma.

Table 6.18
Portfolio specification

Bond	**Option**
$\sigma_5 = 0.95\%$	$\sigma_{USD/DEM} = 1\%$
$\sigma_7 = 1\%$	$\delta = 0.9032$
$\rho_{5,7} = 0.85$	$\Gamma = 0.0566$
$P_B = \text{USD } 100$	$\theta = -0.9156$
	V = USD 3.7191
	$P_{USd/DEM} = \text{USD } 346.3$
Portfolio P*V* = USD 103.719	

To compute VaR we require the covariance matrix

[6.57] $$\Sigma = \begin{bmatrix} \sigma^2_{6y} & \sigma^2_{6y,\,USD/DEM} \\ \sigma^2_{6y,\,USD/DEM} & \sigma^2_{USD/DEM} \end{bmatrix}$$

which, when using the information in Table 6.18 yields

[6.58] $$\Sigma = \begin{bmatrix} 0.009025 & 0.008075 \\ 0.008075 & 0.01000 \end{bmatrix} \text{ (in percent)}$$

Also, we need the cash flows corresponding to the delta components of the portfolio,

[6.59] $$\tilde{\delta} = \begin{bmatrix} 100 \\ 81.352 \end{bmatrix} \begin{matrix} \text{Delta cash flow on bond} \\ \text{Delta cash flow on FX option} \end{matrix}$$

the cash flows corresponding to the gamma components of the portfolio,

Example 6.9 (continued)

[6.60] $$\tilde{\Gamma} = \begin{bmatrix} 0 \\ 1708.47 \end{bmatrix} \begin{matrix} \text{Zero gamma cash flow on bond} \\ \text{Gamma cash flow for FX option} \end{matrix}$$

and, the cash flows corresponding to the theta components of the portfolio.

[6.61] $$\tilde{\theta} = \begin{bmatrix} 0 \\ -0.2462 \end{bmatrix} \begin{matrix} \text{Zero theta cashflow on bond} \\ \text{Theta cashflow for FX option} \end{matrix}$$

The implied moments of this portfolio's distribution are presented in Table 6.19.

Table 6.19
Portfolio statistics

Moments	Including theta	Excluding theta
mean	−0.1608	0.0854
variance	2.8927	2.8927
skewness coefficient	0.2747	0.2747
kurtosis coefficient	3.0997	3.0997

Based on the information presented above, VaR estimates of this portfolio over a one-day forecast horizon are presented in Table 6.20 for three confidence levels. For comparison we also present VaR based on the normal model and VaR that excludes the theta effect.

Table 6.20
Value-at-Risk estimates (USD)
one-day forecast horizon: total portfolio value is 103.719

VaR percentile	Normal	Delta-gamma (excluding theta)	Delta-gamma (including theta)
5.0%	−2.799	−2.579	−2.826
2.5%	−3.325	−3.018	−3.265
1.0%	−3.953	−3.523	−3.953

Chapter 7. Monte Carlo

Chapter 7. Monte Carlo

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In the previous chapter, we illustrated how to combine cash flows, volatilities, and correlations analytically to compute the Value-at-Risk for a portfolio. We have seen that this methodology is applicable to linear instruments, as well as to non-linear instruments whose values can be well approximated by a Taylor series expansion (that is, by its "greeks").

In this chapter, we outline a Monte Carlo framework under which it is possible to compute VaR for portfolios whose instruments may not be amenable to the analytic treatment. We will see that this methodology produces an estimate for the entire probability distribution of portfolio values, and not just one risk measure.

The Monte Carlo methodology consists of three major steps:

1. **Scenario generation**—Using the volatility and correlation estimates for the underlying assets in our portfolio, we produce a large number of future price scenarios in accordance with the lognormal models described previously.
2. **Portfolio valuation—**For each scenario, we compute a portfolio value.
3. **Summary—**We report the results of the simulation, either as a portfolio distribution or as a particular risk measure.

We devote one section of this chapter to each of the three steps above.

To better demonstrate the methodology, we will consider throughout this section a portfolio comprised of two assets: a future cash flow stream of DEM 1M to be received in one year's time and an at the money put option with contract size of DEM 1M and expiration date one month in the future. Assume the implied volatility at which the option is priced is 14%. We see that our portfolio value is dependent on the USD/DEM exchange rate and the one year DEM bond price. (Technically, the value of the option also changes with USD interest rates and the implied volatility, but we will not consider these effects.) Our risk horizon for the example will be five days.

7.1 Scenario generation

We first recall the lognormal model which we assume for all underlying instruments. Consider a forecast horizon of t days. If an instrument's price today is P_0, and our estimate for the one day volatility of this instrument is σ, then we model the price of the instrument in t days by

[7.1] $$P_t = P_0 e^{\sigma \sqrt{t} Y}$$

where Y is a standard normal random variable. Thus, the procedure to generate scenarios is to generate standard normal variates and use Eq. [7.1] to produce future prices. The procedure for the multivariate case is similar, with the added complication that the Y's corresponding to each instrument must be correlated according to our correlation estimates.

In practice, it is straightforward to generate independent normal variates; generating arbitrarily correlated variates is more involved, however. Suppose we wish to generate n normal variates with unit variance and correlations given by the $n \times n$ matrix Λ. The general idea is to generate n independent variates, and then combine these variates is such a way to achieve the desired correlations. To be more precise, the procedure is as follows:

- Decompose Λ using the Cholesky factorization, yielding a lower triangular matrix A such that $\Lambda = AA'$. We provide details on this factorization below and in Appendix E.

- Generate an $n \times 1$ vector Z of independent standard normal variates.
- Let $Z = AY$. The elements of Z will each have unit variance and will be correlated according to Λ.

To illustrate the intuition behind using the Cholesky decomposition, consider the case where we wish to generate two variates with correlation matrix

[7.2] $$\Lambda = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The Cholesky factorization of Λ is given by

[7.3] $$A = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}$$

(It is easy to check that $AA' = \Lambda$.) Now say that Y is a 2×1 vector containing independent standard normal random variables Y_1 and Y_2. If we let $Z = AY$, then the elements of Z are given by

[7.4a] $$Z_1 = Y_1 \text{ and}$$

[7.4b] $$Z_2 = \rho Y_1 + \sqrt{1-\rho^2}\, Y_2$$

Clearly, Z_1 has unit variance, and since Y_1 and Y_2 are independent, the variance of Z_2 is given by

[7.5] $$\rho^2 Var(Y_1) + \left(\sqrt{1-\rho^2}\right)^2 Var(Y_2) = 1$$

Again using the fact that Y_1 and Y_2 are independent, we see that the expected value of $Z_1 Z_2$ is just ρ, and so the correlation is as desired.

Note that it is not necessary to use the Cholesky factorization, since any matrix A which satisfies $\Lambda = AA'$ will serve in the procedure above. A singular value or eigenvalue decomposition would yield the same results. The Cholesky approach is advantageous since the lower triangular structure of A means that fewer operations are necessary in the AZ multiplication. Further, there exist recursive algorithms to compute the Cholesky factorization; we provide details on this in Appendix E. On the other hand, the Cholesky decomposition requires a positive-definite correlation matrix; large matrices obtained from the RiskMetrics data do not always have this property.

Using the procedure above to generate random variates with arbitrary correlations, we may generate scenarios of asset prices. For example, suppose we wish to model the prices of two assets t days into the future. Let $P_0^{(1)}$ and $P_0^{(2)}$ indicate the prices of the assets today, let σ_1 and σ_2 indicate the daily volatilities of the assets, and let ρ indicate the correlation between the two assets. To generate a future price scenario, we generate correlated standard normal variates Z_1 and Z_2 as outlined above and compute the future prices by

[7.6a] $$P_t^{(1)} = P_0^{(1)} e^{\sigma_1 \sqrt{t} Z_1} \text{ and}$$

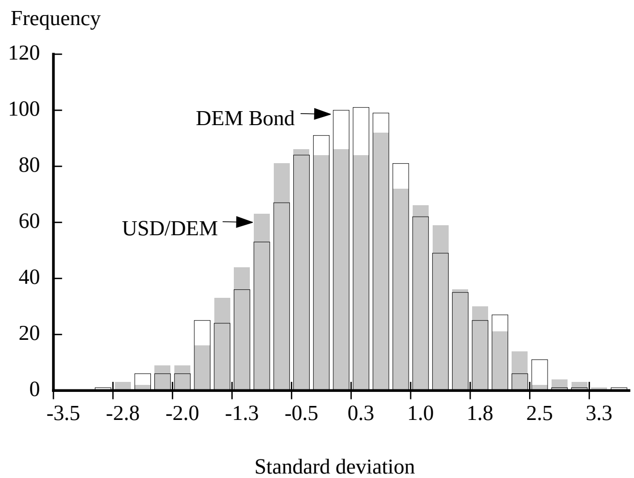
[7.6b] $$P_t^{(2)} = P_0^{(2)} e^{\sigma_2 \sqrt{t} Z_2}$$

To generate a collection of scenarios, we simply repeat this procedure the required number of times.

For our example portfolio, the two underlying assets to be simulated are the USD/DEM exchange rate and the one year DEM bond price. Suppose that the current one year German interest rate is 10% (meaning the present value of a one year 1M DEM notional bond is DEM 909,091) and that the current USD/DEM exchange rate is 0.65. We take as the daily volatilities of these two assets $\sigma_{FX} = 0.0042$ and $\sigma_B = 0.0008$ and as the correlation between the two $\rho = -0.17$.

To generate one thousand scenarios for values of the two underlying assets in five days, we first use the approach above to generate one thousand pairs of standard normal variates whose correlation is ρ . Label each pair Z_{FX} and Z_B . We present histograms for the results in Chart 7.1. Note that the distributions are essentially the same.

Chart 7.1
Frequency distributions for Z_{FX} and Z_B
1000 trials



The next step is to apply Eq. [7.6a] and Eq. [7.6b]. This will create the actual scenarios for our assets. Thus, for each pair Z_{FX} and Z_B , we create future prices P_{FX} and P_B by applying

[7.7a] $$P_{FX} = 0.65 e^{0.0042 \times \sqrt{5} \times Z_{FX}}$$

and

[7.7b] $$P_B = 909,091 e^{0.0008 \times \sqrt{5} \times Z_B}$$

Of course, to express the bond price in USD (accounting for both the exchange rate and interest rate risk for the bond), it is necessary to multiply the bond price by the exchange rate in each scenario. Charts 7.2 and 7.3 show the distributions of future prices, P_B and P_{FX} , respectively, obtained by one thousand simulations. Note that the distributions are no longer bell shaped, and for the bond price, the distribution shows a marked asymmetry. This is due to the transformation we make from normal to lognormal variates by applying Eq. [7.7a] and Eq. [7.7b].

Chart 7.2
Frequency distribution for DEM bond price
1000 trials

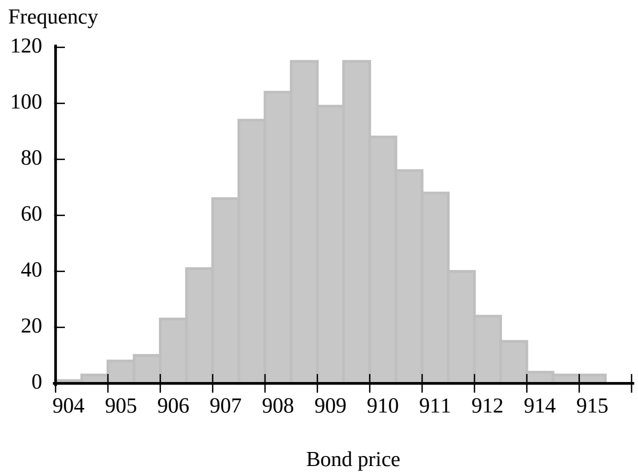
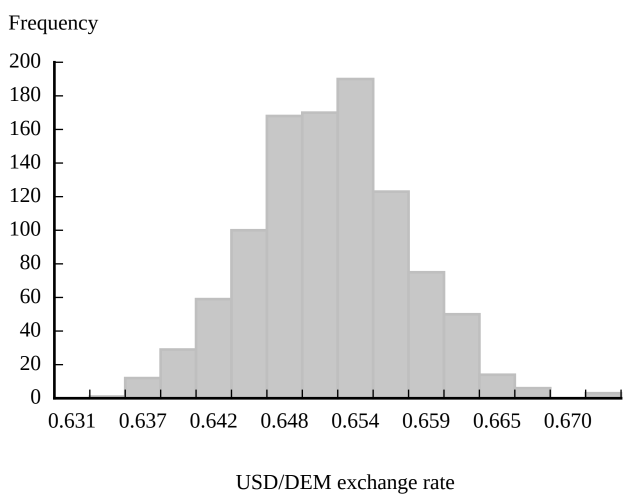


Chart 7.3
Frequency distribution for USD/DEM exchange rate
1000 trials



In Table 7.1, we present the first ten scenarios that we generate.

Table 7.1
Monte Carlo scenarios
1000 trials

USD/DEM	PV of cash flow (DEM)	PV of cash flow (USD)
0.6500	906,663	589,350
0.6540	907,898	593,742
0.6606	911,214	601,935
0.6513	908,004	591,399
0.6707	910,074	610,430
0.6444	908,478	585,460
0.6569	908,860	597,053
0.6559	906,797	594,789
0.6530	906,931	592,267
0.6625	920,768	603,348

Portfolio valuation

In the previous section, we illustrated how to generate scenarios of future prices for the underlying instruments in a portfolio. Here, we take up the next step, how to value the portfolio for each of these scenarios. We will examine three alternatives: full valuation, linear approximation, and higher order approximation. Each of the alternatives is parametric, that is, an approach in which the value of all securities in the portfolio is obtained through the values of its underlying assets, and differ only in their methods for valuing non-linear instruments given underlying prices.

Recall that at the current time, the present value of our cash flow is DEM 909,091, or USD 590,909. The value of the option is USD 10,479.

7.1.1 Full valuation

This is the most straightforward and most accurate alternative, but also the most intense computationally. We assume some type of pricing formula, in our case the Black-Scholes option pricing formula, with which we may value our option in each of the scenarios which we have generated. Say $V(S, K, \tau)$ gives the premium (in USD) associated with the option of selling one DEM given spot USD/DEM rate of S, strike rate of K, and expiration date τ years into the future. (Again, this function will also depend on interest rates and the implied volatility, but we will not model changes in these variables, and so will suppress them in the notation.)

In our example, for a scenario in which the USD/DEM rate has moved to R after five days, our option's value (in USD) moves from $1,000,000 \times V\left(0.65, 0.65, \frac{1}{12}\right)$ to $1,000,000 \times V\left(R, 0.65, \frac{1}{12} - \frac{5}{365}\right)$. The results of applying this method to our scenarios are displayed in Table 7.2. Note that scenarios with moderate changes in the bond price can display significant changes in the value of the option.

Table 7.2
Monte Carlo scenarios—valuation of option
1000 trials

	Value of option (USD)				
USD/DEM	PV of cash flow (USD)	Full	Delta	Delta/Gamma	Delta/Gamma/ Theta
0.6500	589,350	9,558	10,458	10,458	9,597
0.6540	593,742	7,752	8,524	8,644	7,783
0.6606	601,935	5,273	5,272	6,122	5,261
0.6513	591,399	8,945	9,831	9,844	8,893
0.6707	610,430	2,680	273	3,541	2,680
0.6444	585,460	12,575	13,214	13,449	12,588
0.6569	597,053	6,562	7,073	7,437	6,576
0.6559	594,789	6,950	7,565	7,832	6,971
0.6530	592,267	8,156	8,981	9,052	8,190
0.6625	603,348	4,691	4,349	5,528	4,667

7.1.2 Linear approximation

Because utilizing the Black-Scholes formula can be intensive computationally, particularly for a large number of scenarios, it is often desirable to use a simple approximation to the formula. The simplest such approximation is to estimate changes in the option value via a linear model, which is commonly known as the "delta approximation." In this case, given an initial option value V_0 and an initial exchange rate R_0, we approximate a future option value V_1 at a future exchange rate R_1 by

[7.8] $$V_1 = V_0 + \delta (P_1 - P)$$

where

[7.9] $$\delta = \frac{\partial}{\partial R} V(P, S, \tau) \big|_{P_0}$$

is the first derivative of the option price with respect to the spot exchange rate.

For our example, V_0 is USD/DEM 0.0105 and R_0 is USD/DEM 0.65. (To compute the price of our particular option contract, we multiply V_0 by DEM 1M, the notional amount of the contract.) The value of δ for our option is –0.4919. Table 7.2 illustrates the results of the delta approximation for valuing the option's price. Note that for the delta approximation, it is still possible to utilize the standard RiskMetrics methodology without resorting to simulations.

7.1.3 Higher order approximations

It can be seen in Table 7.2 that the delta approximation is reasonably accurate for scenarios where the exchange rate does not change significantly, but less so in the more extreme cases. It is possible to improve this approximation by including the "gamma effect", which accounts for second order effects of changes in the spot rate, and the "theta effect", which accounts for changes in the time to maturity of the option. The two formulas associated with these added effects are

[7.10] $$V_1 = V_0 + \delta(P_1 - P_0) + \frac{1}{2}\Gamma(P_1 - P_0)^2 \text{ and}$$

[7.11] $$V_1 = V_0 + \delta(P_1 - P_0) + \frac{1}{2}\Gamma(P_1 - P_0)^2 - \theta t$$

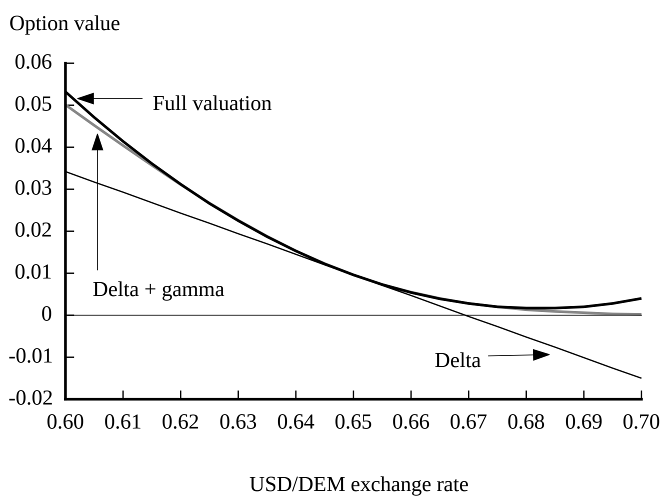
where t is the length of the forecast horizon and Γ and θ are defined by

[7.12a] $$\Gamma = \frac{\partial^2}{\partial R^2} V(P, S, \tau)\,|_{P_0} \text{ and}$$

[7.12b] $$\theta = \frac{\partial}{\partial \tau} V(P, S, \tau)\,|_{\tau_0}$$

Using the values Γ = DEM/USD 15.14 and θ = USD/DEM 0.0629 per year, we value our portfolio for each of our scenarios. The results of these approximations are displayed in Table 7.2. A plot illustrating the differences between the various methods of valuation is displayed in Chart 7.4; the delta/gamma/theta approximation is not plotted since for the values considered, it almost perfectly duplicates the full valuation case. Note that even for these higher order approximations, analytical methods exist for computing percentiles of the portfolio distribution. See, for example, the method outlined in Chapter 6.

Chart 7.4
Value of put option on USD/DEM
strike = 0.65 USD/DEM; Value in USD/DEM



7.2 Summary

Finally, after generating a large number of scenarios and valuing the portfolio under each of them, it is necessary to make some conclusions based on the results. Clearly, one measure which we would like to report is the portfolio's Value-at-Risk. This is done simply by ordering the portfolio return scenarios and picking out the result corresponding to the desired confidence level.

For example, to compute the 5% worst case loss using 1000 trials, we order the scenarios and choose the 50th (5% × 1000) lowest absolute return. The percentiles computed for our example under the various methods for portfolio valuation are reported in Table 7.3.

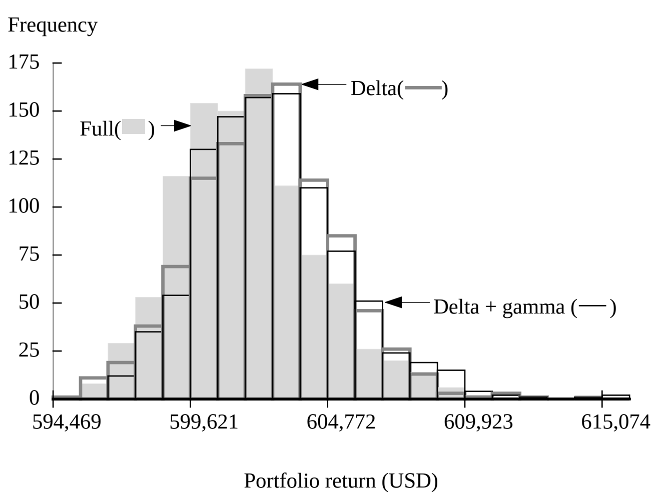
Table 7.3
Value-at-Risk for example portfolio
1000 trials

	Portfolio return (USD)			
Percentile, %	Full	Delta	Delta/Gamma	Delta/Gamma/Theta
1.0	(5,750)	(5,949)	(4,945)	(5,806)
2.5	(5,079)	(5,006)	(4,245)	(5,106)
5.0	(4,559)	(4,392)	(3,708)	(4,569)
10.0	(3,662)	(3,299)	(2,825)	(3,686)
25.0	(2,496)	(1,808)	(1,606)	(2,467)
50.0	(840)	(22)	50	(812)
75.0	915	1,689	1,813	951
90.0	2,801	3,215	3,666	2,805
95.0	4,311	4,331	5,165	4,304
97.5	5,509	5,317	6,350	5,489
99.0	6,652	6,224	7,489	6,628

Thus, at the 5% confidence level and in the full valuation case, we obtain a Value-at-Risk of USD 4,559, or about 0.75% of the current portfolio value.

A particularly nice feature of the Monte Carlo approach is that we obtain an estimate for the entire distribution of portfolio returns. This allows us to compute other risk measures if we desire, and also to examine the shape of the distribution. Chart 7.5 illustrates the return distribution for our example. Note that the distribution is significantly more skewed than the distributions for the underlying assets (see Chart 7.5), which is a result of the non-linearity of the option position.

Chart 7.5
Distribution of portfolio returns
1000 trials



7.3 Comments

In our example, we treated the bond by assuming a lognormal process for its *price*. While this is convenient computationally, it can lead to unrealistic results since the model does not insure positive discount rates. In this case, it is possible to generate a scenario where the individual bond prices are realistic, but where the forward rate implied by the two simulated prices is negative.

We have examined a number of methods to rectify this problem, including decomposing yield curve moves into principal components. In the end, we have concluded that since regularly observed bond prices and volatilities make the problems above quite rare, and since the methods we have investigated only improve the situation slightly, it is not worth the effort to implement a more sophisticated method than what we have outlined in this chapter. We suggest a straight Monte Carlo simulation with our methodology coupled with a check for unrealistic discount or forward rates. Scenarios which yield these unrealistic rates should be rejected from consideration.

Part IV

RiskMetrics Data Sets

Chapter 8. Data and related statistical issues

Chapter 8. Data and related statistical issues

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This chapter covers the RiskMetrics underlying yields and prices that are used in the volatility and correlation calculations. It also discusses the relationship between the number of time series and the amount of historical data available on these series as it relates to the volatility and correlations.

This chapter is organized as follows:

- Section 8.1 explains the basis or construction of the underlying yields and prices for each instrument type.
- Section 8.2 describes the filling in of missing data points, i.e., expectation maximization.
- Section 8.3 investigates the properties of a generic correlation matrix since these determine whether a portfolio's standard deviation is meaningful.
- Section 8.4 provides an algorithm for recomputing the volatilities and correlations when a portfolio is based in a currency other than USD.
- Section 8.5 presents a methodology to calculate correlations when the yields or prices are sampled at different times, i.e., data recording is nonsynchronous.

8.1 Constructing RiskMetrics rates and prices

In this section we explain the construction of the underlying rates and prices that are used in the RiskMetrics calculations. Since the data represent only a subset of the most liquid instruments available in the markets, proxies should be used for the others. Recommendations on how to apply RiskMetrics to specific instruments are outlined in the paragraphs below.

8.1.1 Foreign exchange

RiskMetrics provides estimates of VaR statistics for returns on 31 currencies as measured against the US dollar (e.g., USD/DEM, USD/FRF) as well as correlations between returns. The datasets provided are therefore suited for estimating foreign exchange risk from a US dollar perspective.

The methodology for using the data to measure foreign exchange risk from a currency perspective other than the US dollar is identical to the one described (Section 6.1.2) above but requires the input of revised volatilities and correlations. These modified volatilities and correlations can easily be derived from the original RiskMetrics datasets as described in Section 8.4. Also refer to the examples diskette.

Finally, measuring market exposure to currencies currently not included in the RiskMetrics data set will involve accessing underlying foreign exchange data from other sources or using one of the 31 currencies as a proxy.

8.1.2 Interest rates

In RiskMetrics we describe the fixed income markets in terms of the price dynamics of zero coupon constant maturity instruments. In the interest rate swap market there are quotes for constant maturities (e.g., 10-year swap rate). In the bond markets, constant maturity rates do not exist therefore we must construct them with the aid of a term structure model.

The current data set provides volatilities and correlations for returns on money market deposits, swaps, and zero coupon government bonds in 33 markets. These parameters allow direct calculation of the volatility of cash flows. Correlations are provided between all RiskMetrics vertices and markets.

8.1.2.1 Money market deposits

The volatilities of price returns on money market deposits are to be used to estimate the market risk of all short-term cash flows (from one month to one year). Though they only cover one instrument type at the short end of the yield curve, money market price return volatilities can be applied to measure the market risk of instruments that are highly correlated with money market deposits, such as Treasury bills or instruments that reprice off of rates such as the prime rate in the US or commercial paper rates.[1]

8.1.2.2 Swaps

The volatilities of price returns on zero coupon swaps are to be used to estimate the market risk of interest rate swaps. We construct zero coupon swap prices and rates because they are required for the cashflow mapping methodology described in Section 6.2. We now explain how RiskMetrics constructs zero coupon swap prices (rates) from observed swap prices and rates by the method known as bootstrapping.

Suppose one knows the zero-coupon term structure, i.e., the prices of zero-coupon swaps $P_1, \ldots, P_n$, where each $P_i = 1/(1+z_i)^i$ $i = 1, \ldots,$ n and z_i is the zero-coupon rate for the swap with maturity *i*. Then it is straightforward to find the price of a coupon swap as

[8.1] $$P_{cn} = P_1 S_n + P_2 S_n + \ldots + P_n(1+S_n)$$

where S_n denotes the current swap rate on the n period swap. Now, in practice we observe the coupon term structure, $P_{c1}, \ldots, P_{cn}$ maturing at each coupon payment date. Using the coupon swap prices we can apply Eq. [8.1] to solve for the implied zero coupon term structure, i.e., zero coupon swap prices and rates. Starting with a 1-period zero coupon swap, $P_{c1} = P_1(1+S_1)$ so that $P_1 = P_{c1}/(1+S_1)$ or $z_1 = (1+S_1)/P_{c1} - 1$. Proceeding in an iterative manner, given the discount prices $P_1, \ldots, P_{n-1}$, we can solve for P_n and z_n using the formula

[8.2] $$P_n = \frac{P_{cn} - P_{n-1}S_n - \ldots - P_1 S_n}{1+S_n}$$

The current RiskMetrics datasets do not allow differentiation between interest rate risks of instruments of different credit quality; all market risk due to credit of equal maturity and currency is treated the same.

8.1.2.3 Zero coupon government bonds

The volatilities of price returns on zero coupon government bonds are to be used to estimate the market risk in government bond positions. Zero coupon prices (rates) are used because they are consistent with the cash flow mapping methodology described in Section 6.2. Zero coupon government bond prices can also be used as proxies for estimating the volatility of other securities when the appropriate volatility measure does not exist (corporate issues with maturities longer than 10 years, for example).

[1] See the fourth quarter, 1995 *RiskMetrics Monitor* for details.

Zero coupon government bond yield curves cannot be directly observed, they can only be implied from prices of a collection of liquid bonds in the respective market. Consequently, a term structure model must be used to estimate a synthetic zero coupon yield curve which best fits the collection of observed prices. Such a model generates zero coupon yields for arbitrary points along the yield curve.

8.1.2.4 EMBI+

The J. P. Morgan Emerging Markets Bond Index Plus tracks total returns for traded external debt instruments in the emerging markets. It is constructed as a "composite" of its four markets: Brady bonds, Eurobonds, U.S. dollar local markets, and loans. The EMBI+ provides investors with a definition of the market for emerging markets external-currency debt, a list of the traded instruments, and a compilation of their terms. U.S dollar issues currently make up more than 95% of the index and sovereign issues make up 98%. A fuller description of the EMBI+ can be found in the J. P. Morgan publication *Introducing the Emerging Markets Bond Index Plus (EMBI+)* dated July 12, 1995.

8.1.3 Equities

According to the current RiskMetrics methodology, equities are mapped to their domestic market indices (for example, S&P500 for the US, DAX for Germany, and CAC40 for Canada). That is to say, individual stock betas, along with volatilities on price returns of local market indices are used to construct VaR estimates (see Section 6.3.2.2) of individual stocks. The reason for applying the beta coefficient is that it measures the covariation between the return on the individual stock and the return on the local market index whose volatility and correlation are provided by RiskMetrics.

8.1.4 Commodities

A commodity futures contract is a standardized agreement to buy or sell a commodity. The price to a buyer of a commodity futures contract depends on three factors:

1. the current spot price of the commodity,
2. the carrying costs of the commodity. Money tied up by purchasing and carrying a commodity could have been invested in some risk-free, interest bearing instrument. There may be costs associated with purchasing a product in the spot market (transaction costs) and holding it until or consuming it at some later date (storage costs), and
3. the expected supply and demand for the commodity.

The future price of a commodity differs from its current spot price in a way that is analogous to the difference between 1-year and overnight interest rates for a particular currency. From this perspective we establish a term structure of commodity prices similar to that of interest rates.

The most efficient and liquid markets for most commodities are the futures markets. These markets have the advantage of bringing together not only producers and consumers, but also investors who view commodities as they do any other asset class. Because of the superior liquidity and the transparency of the futures markets, we have decided to use futures prices as the foundation for modeling commodity risk. This applies to all commodities except bullion, as described below.

8.1.4.1 The need for commodity term structures

Futures contracts represent standard terms and conditions for delivery of a product at future dates. Recorded over time, their prices represent instruments with decreasing maturities. That is to say, if the price series of a contract is a sequence of expected values of a single price at a specific date in the future, then each consecutive price implies that the instrument is one day close to expiring.

RiskMetrics constructs constant maturity contracts in the same spirit that it constructs constant maturity instruments for the fixed income market. Compared to the fixed income markets, however, commodity markets are significantly less liquid. This is particularly true for very short and very long maturities. Frequently, volatility of the front month contract may decline when the contract is very close to expiration as it becomes uninteresting to trade for a small absolute gain, difficult to trade (a thin market may exist due to this limited potential gain) and, dangerous to trade because of physical delivery concerns. At the long end of the curve, trading liquidity is limited.

Whenever possible, we have selected the maturities of commodity contracts with the highest liquidity as the vertices for volatility and correlation estimates. These maturities are indicated in Table 9.6 in Section 9.6.

In order to construct constant maturity contracts, we have defined two algorithms to convert observed prices into prices from constant maturity contracts:

- Rolling nearby: we simply use the price of the futures contract that expires closest to a fixed maturity.

- Linear interpolation: we linearly interpolate between the prices of the two futures contracts that straddle the fixed maturity.

8.1.4.2 Rolling nearby futures contracts

Rolling nearby contracts are constructed by concatenating contracts that expire, approximately 1, 6, and 12 months (for instance) in the future. An example of this method is shown in Table 8.1.

Table 8.1
Construction of rolling nearby futures prices for Light Sweet Crude (WTI)

	Rolling nearby			**Actual contracts**					
	1st	6th	12th	Mar-94	Apr-94	Aug-94	Sep-94	Feb-95	Mar-95
17-Feb-94	13.93	15.08	16.17	13.93	14.13	15.08	15.28	16.17	16.3
18-Feb-94	14.23	15.11	16.17	14.23	14.3	15.11	15.3	16.17	16.3
19-Feb-94	14.21	15.06	16.13	14.21	14.24	15.06	15.25	16.13	16.27
23-Feb-94	**14.24**	15.23	16.33	**14.24**	14.39	15.23	15.43	16.33	16.47
24-Feb-94	***14.41***	15.44	16.46		***14.41***	15.24	15.44	16.32	16.46

Note that the price of the front month contract changes from the price of the March to the April contract when the March contract expires. (To conserve space certain active contracts were omitted).

The principal problem with the rolling nearby method is that it may create discontinuous price series when the underlying contract changes: for instance, from February 23 (the March contract) to February 24 (the April contract) in the example above. This discontinuity usually is the largest for very short term contracts and when the term structure of prices is steep.

8.1.4.3 Interpolated futures prices

To address the issue of discontinuous price series, we use the simple rule of linear interpolation to define constant maturity futures prices, P_{cmf} , from quoted futures prices:

[8.3] $$P_{cmf} = \omega_{NB1}P_{NB1} + \omega_{NB2}P_{NB2}$$

where

P_{cmf} = constant maturity futures prices

$\omega_{NB1} = \frac{\delta}{\Delta}$ = ratio of P_{cmf} made up by P_{NB1}

δ = days to expiration of $NB1$

Δ = days to expiration of constant maturity contract

P_{NB1} = price of $NB1$

$\omega_{NB2} = 1 - \omega_{NB1}$

= ratio of P_{cmf} made up by P_{NB2}

P_{NB2} = price of $NB2$

$NB1$ = nearby contract with a maturity < constant maturity contract

$NB2$ = first contract with a maturity < constant maturity contract

The following example illustrates this method using the data for the heating oil futures contract. On April 26, 1994 the 1-month constant maturity equivalent heating oil price is calculated as follows:

[8.4]
$$\begin{aligned} P_{1m,\text{ April 26}} &= \left[\left(\frac{1\text{day}}{30\text{ days}}\right) \times Price_{\text{April}}\right] + \left[\left(\frac{29\text{ days}}{30\text{ days}}\right) \times Price_{\text{May}}\right] \\ &= \left[\left(\frac{1}{30}\right) \times 47.37\right] + \left[\left(\frac{29}{39}\right) \times 47.38\right] \\ &= 47.379 \end{aligned}$$

Table 8.2 illustrates the calculation over successive days. Note that the actual results may vary slightly from the data represented in the table because of numerical rounding.

Table 8.2
Price calculation for 1-month CMF NY Harbor #2 Heating Oil

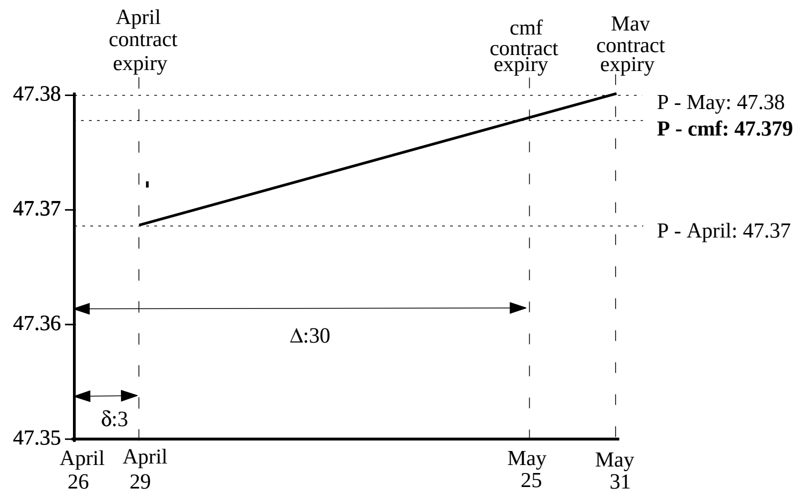
	Contract expiration			**Days to expiration**			**Weights (%)**		**Contract prices**			**cmf†**
Date	1 nb*	1m cmf†	2 nb*	1 nb*	1m cmf	2 nb*	1 nb*	2 nb*	Apr	May	Jun	
22-Apr-94	29-Apr	23-May	31-May	7	30	39	23.33	76.67	47.87	47.86	48.15	47.862
25-Apr-94	29-Apr	25-May	31-May	4	30	36	13.33	86.67	48.23	48.18	48.48	48.187
26-Apr-94	29-Apr	26-May	31-May	3	30	35	10.00	90.00	47.37	47.38	47.78	47.379
28-Apr-94	29-Apr	30-May	31-May	1	30	33	3.33	96.67	46.52	46.57	47.02	47.005
29-Apr-94	29-Apr	31-May	31-May	0	30	32	0.00	100.00	47.05	47.09	47.49	47.490
2-May-94	31-May	1-Jun	30-Jun	29	30	59	96.67	3.33	—	47.57	47.95	47.583
3-May-94	31-May	2-Jun	30-Jun	28	30	58	93.33	6.67	—	46.89	47.29	46.917
4-May-94	31-May	3-Jun	30-Jun	27	30	57	90.00	10.00	—	46.66	47.03	46.697

* 1 nb and 2 nb indicate first and second nearby contracts, respectively.

† cmf means constant maturity future.

Chart 8.1 illustrates the linear interpolation rule graphically.

Chart 8.1
Constant maturity future: price calculation



8.2 Filling in missing data

The preceding section described the types of rates and prices that RiskMetrics uses in its calculations. Throughout the presentation it was implicitly assumed that there were no missing prices. In practice, however, this is often not the case. Because of market closings in a specific location, daily prices are occasionally unavailable for individual instruments and countries. Reasons for the missing data include the occurrence of significant political or social events and technical problems (e.g., machine down time).

Very often, missing data are simply replaced by the preceding day's value. This is frequently the case in the data obtained from specialized vendors. Another common practice has simply been to exclude an entire date from which data were missing from the sample. This results in valuable data being discarded. Simply because one market is closed on a given day should not imply that data from the other countries are not useful. A large number of nonconcurrent missing data points across markets may reduce the validity of a risk measurement process.

Accurately replacing missing data is paramount in obtaining reasonable estimates of volatility and correlation. In this section we describe how missing data points are "filled-in"—by a process known as the EM algorithm—so that we can undertake the analysis set forth in this document. In brief, RiskMetrics applies the following steps to fill in missing rates and prices:

- Assume at any point in time that a data set consisting of a cross-section of returns (that may contain missing data) are multivariate normally distributed with mean μ and covariance matrix Σ.

- Estimate the mean and covariance matrix of this data set using the available, observed data.

- Replace the missing data points by their respective conditional expectations, i.e., use the missing data's expected values given current estimates of μ, Σ and the observed data.

8.2.1 Nature of missing data

We assume throughout the analysis that the presence of missing data occur randomly. Suppose that at a particular point in time, we have K return series and for each of the series we have T historical observations. Let **Z** denote the matrix of raw, observed returns. **Z** has T rows and K columns. Each row of **Z** is a Kx1 vector of returns, observed at any point in time, spanning all K securities. Denote the tth row of **Z** by $\mathbf{z}_t$ for $t = 1,2,...T$. The matrix **Z** may have missing data points.

Define a complete data matrix **R** that consists of all the data points **Z** plus the "filled-in" returns for the missing observations. The tth row of **R** is denoted $\mathbf{r}_t$. Note that if there are no missing observations then $\mathbf{z}_t=\mathbf{r}_t$ for all t=1,...,T. In the case where we have two assets (K=2) and three historical observations (T=3) on each asset, **R** takes the form:

[8.5] $$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \end{bmatrix} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$$

where "T" denotes transpose.

8.2.2 Maximum likelihood estimation

For the purpose of filling in missing data it is assumed that at any period t, the return vector $\mathbf{r}_t$ (Kx1) follows a multivariate normal distribution with mean vector μ and covariance matrix Σ. The probability density function of $\mathbf{r}_t$ is

[8.6] $$p(r_t) = (2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{k}{2}} \exp\left[-\frac{1}{2}(r_t - \mu)^T \Sigma^{-1} (r_t - \mu)\right]$$

It is assumed that this density function holds for all time periods, $t = 1,2,...,T$. Next, under the assumption of statistical independence between time periods, we can write the joint probability density function of returns given the mean and covariance matrix as follows

[8.7] $$\begin{aligned} p(r_1, \ldots r_T | \mu, \Sigma) &= \prod_{t=1}^{T} p(r_t) \\ &= (2\pi)^{-\frac{Tk}{2}} |\Sigma|^{-\frac{T}{2}} \exp\left[-\frac{1}{2}\sum_{t=1}^{T}(r_t - \mu)^T \Sigma^{-1} (r_t - \mu)\right] \end{aligned}$$

The joint probability density function $p(r_1, \ldots r_T | \mu, \Sigma)$ describes the probability density for the data **given** a set of parameter values (i.e., μ and Σ). Define the total parameter vector θ = (μ,Σ). Our task is to estimate θ given the data matrix that contains missing data. To do so, we must derive the likelihood function of θ given the data. The likelihood function $|L(\mu, \Sigma | r_1, \ldots r_T)$ is similar in all respects to $p(r_1, \ldots r_T | \mu, \Sigma)$ except that it considers the parameters as random variables and takes the data as given. Mathematically, the likelihood function is equivalent to the probability density function. Intuitively, the likelihood function embodies the entire set of parameter values for an observed data set.

Now, for a realized sample of, say, exchange rates, we would want to know what set of parameter values most likely generated the observed data set. The solution to this question lies in maximum likelihood estimation. In essence, **the maximum likelihood estimates (*MLE*) θ_{MLE} are the parameter values that most likely generated the observed data matrix.**

θ_{MLE} is found by maximizing the likelihood function $L(\mu, \Sigma | r_1, \ldots r_T)$. In practice it is often easier to maximize natural logarithm of the likelihood function $l(\mu, \Sigma | r_1, \ldots r_T)$ which is given by

[8.8] $$l(\mu, \Sigma | R) = -\frac{1}{2} TK \ln(2\pi) - \frac{T}{2} \ln|\Sigma| - \frac{1}{2} \sum_{t=1}^{T} (r_t - \mu)^T \Sigma^{-1} (r_t - \mu)$$

with respect to θ. This translates into finding solutions to the following first order conditions:

[8.9] $$\frac{\partial}{\partial \mu} l(\mu, \Sigma | r_1, \ldots r_T) = 0, \quad \frac{\partial}{\partial \Sigma} l(\mu, \Sigma | r_1, \ldots r_T) = 0$$

The maximum likelihood estimators for the mean vector, $\hat{\mu}$ and covariance matrix $\hat{\Sigma}$ are

[8.10] $$\hat{\mu} = [\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_k]^T$$

[8.11] $$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^{T} (r_t - \hat{\mu})(r_t - \hat{\mu})^T$$

where $\bar{r}_i$ represents the sample mean taken over T time periods.

8.2.3 Estimating the sample mean and covariance matrix for missing data

When some observations of $\mathbf{r}_t$ are missing, the maximum likelihood estimates θ_{MLE} are not available. This is evident from the fact that the likelihood function is not defined (i.e., it has no value) when it is evaluated at the missing data points. To overcome this problem, we must implement what is known as the EM algorithm.

Since its formal exposition (Dempster, Laird and Rubin, 1977) the expectation maximization or EM algorithm (hereafter referred to as EM) has been on of the most successful methods of estimation when the data under study are incomplete (e.g., when some of the observations are missing). Among its extensive applications, the EM algorithm has been used to resolve missing data problems involving financial time series (Warga, 1992). For a detailed exposition of the EM algorithm and its application in finance see Kempthorne and Vyas (1994).

Intuitively, EM is an iterative algorithm that operates as follows.

- For a given set of (initial) parameter values, instead of evaluating the log likelihood function, (which is impossible, anyway) EM evaluates the conditional expectation of the latent (underlying) log likelihood function. The mathematical conditional expectation of the log-likelihood is taken over the observed data points.

- The expected log likelihood is maximized to yield parameter estimates θ^0_{EM}. (The superscript “0” stands for the initial parameter estimate). This value is then substituted into the log likelihood function and expectations are taken again, and new parameter estimates θ^1_{EM} are found. This iterative process is continued until the algorithm converges at which time final parameter estimates have been generated. For example, if the algorithm is iterated N+1 times then the sequence of parameter estimates $\left(\theta^0_{EM}, \theta^1_{EM}, \ldots, \theta^N_{EM}\right)$ is generated. The algorithm stops when adjacent parameter estimates are sufficiently close to one another, i.e., when θ_{EM}^{N-1} is sufficiently close to θ_{EM}^{N}.

The first step in EM is referred to as the expectation or E-Step. The second step is the maximization or M-step. EM iterates between these two steps, updating the E-Step from the parameter estimates generated in the M-Step. For example, at the *i*th iteration of the algorithm, the following equations are solved in the M-Step:

[8.12a] $$\hat{\mu}^{i+1} = \frac{1}{T}\sum_{t=1}^{T} E\left[r_t \middle| z_t, \theta^i\right] \quad \text{(the sample mean)}$$

[8.12b] $$\hat{\Sigma}^{i+1} = \frac{1}{T}\sum_{t=1}^{T} E\left[r_t r_t^T \middle| z_t, \theta^i\right] - \hat{\mu}^{i+1}\left(\hat{\mu}^{i+1}\right)^T \quad \text{(the sample covariance matrix)}$$

To evaluate the expectations in these expressions ($E\left[r_t \middle| z_t, \theta^i\right]$ and $E\left[r_t r_t^T \middle| z_t, \theta^i\right]$), we make use of standard properties for partitioning a multivariate normal random vector.

[8.13] $$\begin{bmatrix} R \\ \Re \end{bmatrix} \sim NID\left[\begin{bmatrix} \mu_R \\ \mu_{\Re} \end{bmatrix}, \begin{bmatrix} \Sigma_{RR} & \Sigma_{\Re R} \\ \Sigma_{R\Re} & \Sigma_{\Re\Re} \end{bmatrix}\right]$$

Here, one can think of $\Re$ as the sample data with missing values removed and R as the vector of the underlying complete set of observations. Assuming that returns are distributed multivariate normal, the distribution of R conditional on $\Re$ is multivariate normal with mean

[8.14] $$E\left[R|\Re\right] = \mu_R + \Sigma_{R\Re}\Sigma_{\Re\Re}^{-1}\left(\Re - \mu_{\Re}\right)$$

and covariance matrix

[8.15] $$\text{Covariance}\left(R|\Re\right) = \Sigma_{RR} - \Sigma_{R\Re}\Sigma_{\Re\Re}^{-1}\Sigma_{\Re R}$$

Using Eq. [8.14] and Eq. [8.15] we can evaluate the E- and M- steps. The E -Step is given by

[8.16] $$E-Step\begin{bmatrix} E\left[r_t|z_t,\theta\right] = \mu_r + \Sigma_{rz}\Sigma_{zz}^{-1}\left(z_t - \mu_z\right) \\ E\left[r_t r_t^T \middle| z_t, \theta\right] = \text{Covariance}\left(r_t^T \middle| z_t, \theta\right) + \left(E\left[r_t|z_t,\theta\right] E\left[r_t|z_t,\theta\right]^T\right) \end{bmatrix}$$

where

[8.17] $$\text{Covariance}\left(r_t^T \middle| z_t, \theta\right) = \Sigma_{rz} - \Sigma_{rz}\Sigma_{zz}^{-1}\Sigma_{zr}$$

Notice that the expressions in Eq. [8.17] are easily evaluated since they depend on parameters that describe the observed and missing data.

Given the values computed in the E-Step, the M-Step yields updates of the mean vector and covariance matrix.

[8.18] M-Step

$$
\begin{cases}
\hat{\mu}^{i+1} = \dfrac{1}{T}\left(\displaystyle\sum_{\text{Complete}\,t} r_t + \sum_{\text{Incomplete}\,t} E\left[r_t | z_t, \theta^i\right]\right) \\
\hat{\Sigma}^{i+1} = \dfrac{1}{T}\displaystyle\sum_{t=1}^{T} E\left[r_t r_t^T \middle| z_t, \theta^i\right] - \hat{\mu}^{i+1}\left(\hat{\mu}^{i+1}\right)^T \\
\quad = \dfrac{1}{T}\left(\left(\displaystyle\sum_{\text{Complete}\,t} r_t r_t^T\right) + \sum_{\text{Incomplete}\,t} \text{Covariance}\left(r_t^T \middle| z_t, \theta\right) + E\left[r_t | z_t, \theta\right] E\left[r_t | z_t, \theta\right]^T\right)
\end{cases}
$$

Notice that summing over t implies that we are adding “down” the columns of the data matrix **R**. For a practical, detailed example of the EM algorithm see Johnson and Wichern (1992, pp. 203–206).

A powerful result of EM is that when a global optimum exists, the parameter estimates from the EM algorithm converge to the ML estimates. That is, for a sufficiently large number of iterations, EM converges to θ_{MLE}. Thus, the EM algorithm provides a way to calculate the ML estimates of the unknown parameter even if all of the observations are not available.

The assumption that the time series are generated from a multivariate normal distribution is innocuous. Even if the true underlying distribution is not normal, it follows from the theory of pseudo-maximum likelihood estimation that the parameter estimates are asymptotically consistent (White, 1982) although not necessarily asymptotically efficient. That is, it has been shown that the pseudo-MLE obtained by maximizing the unspecified log likelihood as if it were correct produces a consistent estimator despite the misspecification.

8.2.4 An illustrative example

A typical application of the EM algorithm is filling in missing values resulting from a holiday in a given market. We applied the algorithm outlined in the section above to the August 15 Assumption holiday in the Belgian government bond market. While most European bond markets were open on that date, including Germany and the Netherlands which show significant correlation with Belgium, no data was available for Belgium.

A missing data point in an underlying time series generates two missing points in the log change series as shown below (from t–1 to t as well as from t to t + 1). Even though it would be more straightforward to calculate the underlying missing value through the EM algorithm and then generate the two missing log changes, this would be statistically inconsistent with our basic assumptions on the distribution of data.

In order to maintain consistency between the underlying rate data and the return series, the adjustment for missing data is performed in three steps.

1. First the EM algorithm generates the first missing percentage change, or –0.419% in the example below.
2. From that number, we can back out the missing underlying yield from the previous day’s level, which gives us the 8.445% in the example below.
3. Finally, the second missing log change can be calculated from the revised underlying yield series.

Table 8.3 presents the underlying rates on the Belgian franc 10-year zero coupon bond, the corresponding EM forecast, and the adjusted “filled-in” rates and returns.

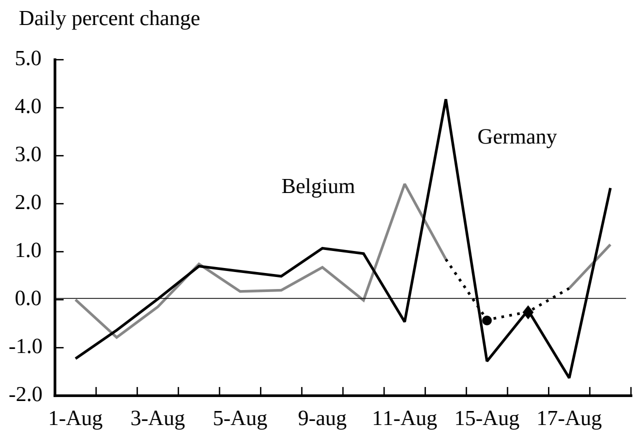
Table 8.3
Belgian franc 10-year zero coupon rate
application of the EM algorithm to the 1994 Assumption holiday in Belgium

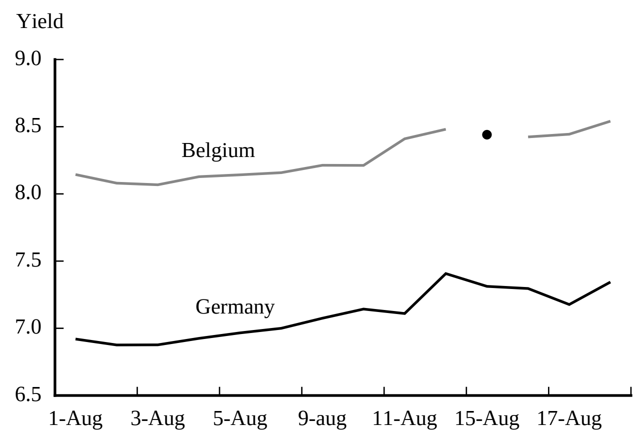
	Observed			Adjusted	
Collection date	10-year rate	Return (%)	EM forecast	10-year rate	Return (%)
11-Aug-94	8.400	2.411		8.410	2.411
12-Aug-94	8.481	0.844		8.481	0.844
15-Aug-94	missing	missing	–0.419	8.445*	–0.419†
16-Aug-94	8.424	missing		8.424	–0.254‡
17-Aug-94	8.444	0.237		8.444	0.237
18-Aug-94	8.541	1.149		8.541	1.149

* Filled-in rate based on EM forecast.
† From EM.
‡ Return now available because prior rate (*) has been filled in.

Chart 8.2 presents a time series of the Belgian franc 10-year rate before and after the missing observation was filled in by the EM algorithm.

Chart 8.2
Graphical representation
10-year zero coupon rates; daily % change



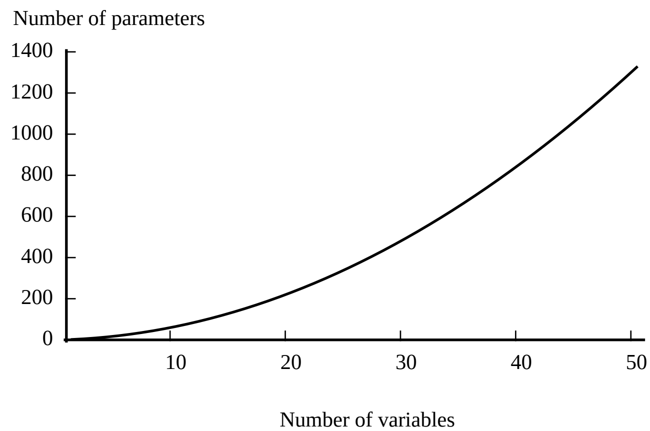


8.2.5 Practical considerations

A major part of implementing the EM algorithm is to devise the appropriate input data matrices for the EM. From both a statistical and practical perspective we do not run EM on our entire time series data set simultaneously. Instead we must partition the original data series into non-overlapping sub-matrices. Our reasons for doing so are highlighted in the following example.

Consider a TxK data matrix where T is the number of observations and K is the number of price vectors. Given this data matrix, the EM must estimate K+K(K+1)/2 parameters. Consequently, to keep the estimation practical K cannot be too large. To get a better understanding of this issue consider Chart 8.3, which plots the number of parameters estimated by EM (K +K(K+1)/2) against the number of variables. As shown, the number of estimated parameters grows rapidly with the number of variables.

Chart 8.3
Number of variables used in EM and parameters required
number of parameters (Y-axis) versus number of variables (X-axis)



The submatrices must be chosen so that vectors within a particular submatrix are highly correlated while those vectors between submatrices are not significantly correlated. If we are allowed to choose the submatrices in this way then EM will perform as if it had the entire original data matrix. This follows from the fact that the accuracy of parameter estimates are not improved by adding uncorrelated vectors.

In order to achieve a logical choice of submatrices, we classify returns into the following categories: (1) foreign exchange, (2) money market, (3) swap, (4) government bond, (5) equity, and (6) commodity.

We further decompose categories 2, 3, 4, and 6 as follows. Each input data matrix corresponds to a particular country or commodity market. The rows of this matrix correspond to time while the columns identify the maturity of the asset. Foreign exchange, equity indices, and bullion are the exceptions: all exchange rates, equity indices, and bullion are grouped into three separate matrices.

8.3 The properties of correlation (covariance) matrices and VaR

In Section 6.3.2 it was shown how RiskMetrics applies a correlation matrix to compute the VaR of an arbitrary portfolio. In particular, the correlation matrix was used to compute the portfolio's standard deviation. VaR was then computed as a multiple of that standard deviation. In this section we investigate the properties of a generic correlation matrix since it is these properties that will determine whether the portfolio’s standard deviation forecast is meaningful.[2] Specifically, we will establish conditions[3] that guarantee the non-negativity of the portfolio’s variance, i.e., $\sigma^2 \geq 0$.

At first glance it may not seem obvious why it is necessary to understand the conditions under which the variance is non-negative. However, the potential sign of the variance, and consequently the VaR number, is directly related to the relationship between (1) the number of individual price return series (i.e., cashflows) per portfolio and (2) the number of historical observations on each of these return series. In practice there is often a trade-off between the two since, on the one hand, large portfolios require the use of many time series, while on the other hand, large amounts of historical data are not available for many time series.

Below, we establish conditions that ensure the non-negativity of a variance that is constructed from correlation matrices based on equally and exponentially weighted schemes. We begin with some basic definitions of covariance and correlation matrices.

8.3.1 Covariance and correlation calculations

In this section we briefly review the covariance and correlation calculations based on equal and exponential moving averages. We do so in order to establish a relationship between the underlying return data matrix and the properties of the corresponding covariance (correlation) matrix.

8.3.1.1 Equal weighting scheme

Let X denote a T x K data matrix, i.e., matrix of returns. X has T rows and K columns.

[8.19] $$X = \begin{bmatrix} r_{11} & \dots & \dots & \dots & r_{1K} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & r_{JJ} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ r_{T1} & \dots & \dots & \dots & r_{TK} \end{bmatrix}$$

Each column of X is a return series corresponding to a particular price/rate (e.g., USD/DEM FX return) while each row corresponds to the time (t = 1,...,T) at which the return was recorded. If we compute standard deviations and covariances around a zero mean, and weigh each observation with probability 1/T, we can define the covariance matrix simply by

[8.20] $$\Sigma = \frac{X^T X}{T}$$

where X^T is the transpose of X.

Consider an example when T = 4 and K = 2.

[2] By properties, we mean specifically whether the correlation matrix is positive definite, positive semidefinite or otherwise (these terms will be defined explicitly below)

[3] All linear algebra propositions stated below can be found in Johnston, J. (1984).

[8.21] $$X = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \\ r_{41} & r_{42} \end{bmatrix} \quad X^T = \begin{bmatrix} r_{11} & r_{21} & r_{31} & r_{41} \\ r_{12} & r_{22} & r_{32} & r_{42} \end{bmatrix}$$

An estimate of the covariance matrix is given by

[8.22] $$\Sigma = \frac{X^T X}{T} = \begin{bmatrix} \frac{1}{4}\sum_{i=1}^{4} r_{i1}^2 & \frac{1}{4}\sum_{i=1}^{4} r_{i1} r_{i2} \\ \frac{1}{4}\sum_{i=1}^{4} r_{i1} r_{i2} & \frac{1}{4}\sum_{i=1}^{4} r_{i2}^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{bmatrix}$$

Next, we show how to compute the correlation matrix R. Suppose we divide each element of the matrix *X* by the standard deviation of the series to which it belongs; i.e., we normalize each series of *X* to have a standard deviation of 1. Call this new matrix with the standardized values *Y.*

The correlation matrix is

[8.23] $$Y = \begin{bmatrix} \frac{r_{11}}{\sigma_1} & \dots & \dots & \dots & \frac{r_{1K}}{\sigma_K} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \frac{r_{JJ}}{\sigma_J} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \frac{r_{T1}}{\sigma_1} & \dots & \dots & \dots & \frac{r_{TK}}{\sigma_K} \end{bmatrix}$$

where

$$\sigma_j = \frac{1}{T}\sqrt{\sum_{i=1}^{T} r^2{}_{ij}} \qquad j = 1, 2, \dots k$$

[8.24] $$R = \frac{Y^T Y}{T}$$

As in the previous example, if we set $T = 4$ and $K = 2$, the correlation matrix is

[8.25] $$R = \frac{Y^T Y}{T} = \begin{bmatrix} \frac{1}{4}\sum_{i=1}^{4} \frac{r_{i1}^2}{\sigma_1} & \frac{1}{4}\sum_{i=1}^{4} \frac{r_{i1} r_{i2}}{\sigma_1 \sigma_2} \\ \frac{1}{4}\sum_{i=1}^{4} \frac{r_{i1} r_{i2}}{\sigma_1 \sigma_2} & \frac{1}{4}\sum_{i=1}^{4} \frac{r_{i2}^2}{\sigma_2} \end{bmatrix} = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix}$$

8.3.1.2 Exponential weighting scheme

We now show how similar results are obtained by using exponential weighting rather than equal weighting. When computing the covariance and correlation matrices, use, instead of the data matrix X, the augmented data matrix $\tilde{X}$ shown in Eq. [8.26].

$$\tilde{X} = \begin{bmatrix} r_{11} & \dots & \dots & \dots & r_{1K} \\ \sqrt{\lambda} r_{21} & \dots & \dots & \dots & \sqrt{\lambda} r_{21} \\ \dots & \dots & \sqrt{\lambda^{J-1}} r_{JJ} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \sqrt{\lambda^{T-1}} r_{T1} & \dots & \dots & \dots & \sqrt{\lambda^{T-1}} r_{TK} \end{bmatrix} \quad [8.26]$$

Now, we can define the covariance matrix simply as

$$\tilde{\Sigma} = \left(\sum_{i=1}^{T} \lambda^{i-1} \right)^{-1} \cdot \tilde{X}^T \tilde{X} \quad [8.27]$$

To see this, consider the example when $T = 4$ and $K = 2$.

$$X = \begin{bmatrix} r_{11} & r_{12} \\ \sqrt{\lambda} r_{21} & \sqrt{\lambda} r_{22} \\ \sqrt{\lambda^2} r_{31} & \sqrt{\lambda^2} r_{32} \\ \sqrt{\lambda^3} r_{41} & \sqrt{\lambda^3} r_{42} \end{bmatrix} \quad X^T = \begin{bmatrix} x_{11} & \sqrt{\lambda} r_{21} & \sqrt{\lambda^2} r_{31} & \sqrt{\lambda^3} r_{41} \\ x_{12} & \sqrt{\lambda} r_{22} & \sqrt{\lambda^2} r_{32} & \sqrt{\lambda^3} r_{42} \end{bmatrix}$$

$$\tilde{\Sigma} = \left(\sum_{i=1}^{T} \lambda^{i-1} \right)^{-1} \tilde{X}^T \tilde{X} \quad [8.28]$$

$$= \left(\sum_{i=1}^{T} \lambda^{i-1} \right)^{-1} \begin{bmatrix} \sum_{i=1}^{4} \lambda^{i-1} r_{i1}^2 & \sum_{i=1}^{4} \lambda^{i-1} r_{i1} r_{i2} \\ \sum_{i=1}^{4} \lambda^{i-1} r_{i1} r_{i2} & \sum_{i=1}^{4} \lambda^{i-1} r_{i2}^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{bmatrix}$$

The exponentially weighted correlation matrix is computed just like the simple correlation matrix. The standardized data matrix and the correlation matrix are given by the following expressions.

$$\tilde{Y} = \begin{bmatrix} \frac{\tilde{r}_{11}}{\sigma_1} & \dots & \dots & \dots & \frac{\tilde{r}_{1K}}{\sigma_K} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \frac{\tilde{r}_{JJ}}{\sigma_J} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\tilde{r}_{T1}}{\sigma_1} & \dots & \dots & \dots & \frac{\tilde{r}_{TK}}{\sigma_K} \end{bmatrix} \quad [8.29]$$

where

$$\sigma_j = \left(\sum_{i=1}^{T} \lambda^{i-1}\right)^{-1} \sqrt{\sum_{i=1}^{T} \lambda^{i-1} \tilde{r}_{ij}^2} \qquad j = 1, 2, \ldots K$$

and the correlation matrix is

$$[8.30] \qquad \tilde{R} = \left(\sum_{i=1}^{T} \lambda^{i-1}\right)^{-1} \cdot \tilde{Y}^T \tilde{Y}$$

which is the exact analogue to Eq. [8.25]. Therefore, all results for the simple correlation matrix carry over to the exponential weighted matrix.

Having shown how to compute the covariance and correlation matrices, the next step is to show how the properties of these matrices relate to the VaR calculations.

We begin with the definition of positive definite and positive semidefinite matrices.

[8.31] If $\vec{z}^T C \vec{z} > (<) \; 0$ for all nonzero vectors $\vec{z}$, then C is said to be positive (negative) definite.

[8.32] If $\vec{z}^T C \vec{z} \geq (\leq) \; 0$ for all nonzero vectors $\vec{z}$, then C is said to be positive semidefinite (nonpositive definite).

Now, referring back to the VaR calculation presented in Section 6.3.2, if we replace the vector $\vec{z}$ by the weight vector $\vec{\sigma}_{t|t-1}$ and C by the correlation matrix, $R_{t|t-1}$, then it should be obvious why we seek to determine whether the correlation matrix is positive definite or not. Specifically,

- If the correlation matrix R is positive definite, then VaR will always be positive.
- If R is positive semidefinite, then VaR could be zero or positive.
- If R is negative definite,[4] then VaR will be negative.

8.3.2 Useful linear algebra results as applied to the VaR calculation

In order to define a relationship between the dimensions of the data matrix X (or $\tilde{X}$) (i.e., the number of rows and columns of the data matrix) and the potential values of the VaR estimates, we must define the rank of X.

The rank of a matrix X, denoted r(X), is the maximum number of linearly independent rows (and columns) of that matrix. The rank of a matrix can be no greater than the minimum number of rows or columns. Therefore, if X is T x K with T > K (i.e., more rows than columns) then $r(X) \leq K$. In general, for an T x K matrix X, $r(X) \leq \min(T,K)$.

[4] We will show below that this is not possible.

A useful result which equates the ranks of different matrices is:

[8.33] $$r(X) = r\left(X^T X\right) = r\left(X X^T\right)$$

As applied to the VaR calculation, the rank of the covariance matrix $\Sigma = X^T X$ is the same as the rank of X.

We now refer to two linear algebra results which establish a relationship between the rank of the data matrix and the range of VaR values.

[8.34] If X is T x K with rank K < T, then $X^T X$ is positive definite and XX^T is positive semidefinite.

[8.35] If X is T x K with rank J < min(T,K) then $X^T X$ and XX^T is positive semidefinite.

Therefore, whether Σ is positive definite or not will depend on the rank of the data matrix X.

Based on the previous discussion, we can provide the following results for RiskMetrics VaR calculations.

- Following from Eq. [8.33], we can deduce the rank of R simply by knowing the rank of Y, the standardized data matrix.
- The rank of the correlation matrix R can be no greater than the number of historical data points used to compute the correlation matrix, and
- Following from Eq. [8.34], if the data matrix of returns has more rows than columns and the columns are independent, then R is positive definite and VaR > 0. If not, then Eq. [8.35] applies, and R is positive semidefinite and $\text{VaR} \geq 0$.

In summary, a covariance matrix, by definition, is **at least** positive semidefinite. Simply put, positive semidefinite is the multi-dimensional analogue to the definition, $\sigma^2 \geq 0$.

8.3.3 How to determine if a covariance matrix is positive semi-definite[5]

Finally, we explain a technique to determine whether a correlation matrix is positive (semi) definite. We would like to note at the beginning that due to a variety of technical issues that are beyond the scope of this document, the suggested approach described below known as the singular value decomposition (SVD) is to serve as a general guideline rather than a strict set of rules for determining the “definiteness” of a correlation matrix.

The singular value decomposition (SVD)

The T x K standardized data matrix Y ($T \geq K$) may be decomposed as[6] $Y = UDV'$ where $U'U = V'V = I_K$ and D is diagonal with non-negative diagonal elements $(\iota_1, \iota_2, \ldots, \iota_K)$, called the singular values of Y. All of the singular values are $\geq (0)$.

---

[5] This section is based on Belsley (1981), Chapter 3.

[6] In this section we work with the mean centered and standardized matrix Y instead of X since Y is the data matrix on which an SVD should be applied.

A useful result is that the number of non-zero singular values is a function by the rank of Y. Specifically, if Y is full rank, then all K singular values will be non zero. If the rank of Y is J=K-2, then there will be J positive singular values and two zero singular values.

In practice, it is difficult to determine the number of zero singular values. This is due to that fact that computers deal with finite, not exact arithmetic. In other words, it is difficult for a computer to know when a singular value is **really** zero. To avoid having to determine the number of zero singular values, it is recommended that practitioners should focus on the condition number of Y which is the ratio of the largest to smallest singular values, i.e.,

[8.36] $$\upsilon = \frac{\iota_{max}}{\iota_{min}} \text{ (condition number)}$$

Large condition numbers point toward 'ill-condition' matrices, i.e., matrices that are nearly not full rank. In other words, a large υ implies that there is a strong degree of collinearity between the columns of Y. More elaborate tests of collinearity can be found in Belsley (1981).

We now apply the SVD to two data matrices. The first data matrix consists of time series of price returns on 10 USD government bonds for the period January 4, 1993–October 14, 1996 (986 observations). The columns of the data matrix correspond to the price returns on the 2yr, 3yr, 4yr, 5yr, 7yr, 9yr, 10yr, 15yr, 20yr, and 30yr USD government bonds. The singular values for this data matrix are given in Table 8.4.

Table 8.4
Singular values for USD yield curve data matrix

3.045	0.051
0.785	0.043
0.271	0.020
0.131	0.017
0.117	0.006

The condition number, υ, is 497.4. We conduct a similar experiment on a data matrix that consists of 14 equity indices.[7] The singular values are shown in Table 8.5. The data set consists of a total number of 790 observations for the period October 5, 1996 through October 14, 1996.

Table 8.5
Singular values for equity indices returns

2.329	0.873	0.696
1.149	0.855	0.639
0.948	0.789	0.553
0.936	0.743	0.554
0.894	0.712	

For this data matrix, the condition number, υ, is 4.28. Notice how much lower the condition number is for equities than it is for the US yield curve. This result should not be surprising since we expect the returns on different bonds along the yield curve to move in a similar fashion to one another relative to equity returns. Alternatively expressed, the relatively large condition number for the USD yield curve is indicative of the near collinearity that exists among returns on US government bonds.

[7] For the countries Austria, Australia, Belgium, Canada, Switzerland, Spain, France, Finland, Great Britain, Hong Kong, Ireland, Italy, Japan and the Netherlands.

The purpose of the preceding exercise was to demonstrate how the interrelatedness of individual time series affects the condition of the resulting correlation matrix. As we have shown with a simple example, highly correlated data (USD yield curve data) leads to high condition numbers relative to less correlated data (equity indices).

In concluding, due to numerical rounding errors it is not unlikely for the theoretical properties of a matrix to differ from its estimated counterpart. For example, covariance matrices are real, symmetric and non-positive definite. However, when estimating a covariance matrix we may find that the positive definite property is violated. More specifically, the matrix may not invert. Singularity may arise because certain prices included in a covariance matrix form linear combinations of other prices. Therefore, if covariance matrices fail to invert they should be checked to determine whether certain prices are linear functions of others. Also, the scale of the matrix elements may be such that it will not invert. While poor scaling may be a source of problems, it should rarely be the case.

8.4 Rebasing RiskMetrics volatilities and correlations

A user's base currency will dictate how RiskMetrics standard deviations and correlations will be used. For example, a DEM-based investor with US dollar exposure is interested in fluctuations in the currency USD/DEM whereas the same investor with an exposure in Belgium francs is interested in fluctuations in BEF/DEM. Currently, RiskMetrics volatility forecasts are expressed in US dollars per foreign currency such as USD/DEM for all currencies. To compute volatilities on cross rates such as BEF/DEM, users must make use of the RiskMetrics provided USD/DEM and USD/BEF volatilities as well as correlations between the two. We now show how to derive the variance (standard deviation) of the BEF/DEM position. Let $r_{1,t}$ and $r_{2,t}$ represent the time t returns on USD/DEM and USD/BEF, respectively, i.e.,

[8.37] $$r_{1t} = \ln\left[\frac{(USD/DEM)_t}{(USD/DEM)_{t-1}}\right] \text{ and } r_{2t} = \ln\left[\frac{(USD/BEF)_t}{(USD/BEF)_{t-1}}\right]$$

The cross rate BEF/DEM is defined as

[8.38] $$r_{3t} = \ln\left[\frac{(BEF/DEM)_t}{(BEF/DEM)_{t-1}}\right] = r_{1t} - r_{2t}$$

The variance of the cross rate r_{3t} is given by

[8.39] $$\sigma^2_{3,t} = \sigma^2_{1,t} + \sigma^2_{2,t} - 2\sigma^2_{12,t}$$

Equation [8.39] holds for any cross rate that can be defined as the arithmetic difference in two other rates.

We can find the correlation between two cross rates as follows. Suppose we want to find the correlation between the currencies BEF/DEM and FRF/DEM. It follows from Eq. [8.38] that we first need to define these cross rates in terms of the returns used in RiskMetrics.

[8.40a] $$r_{1,t} = \ln\left[\frac{(USD/DEM)_t}{(USD/DEM)_{t-1}}\right], \qquad r_{2,t} = \ln\left[\frac{(USD/BEF)_t}{(USD/BEF)_{t-1}}\right],$$

[8.40b] $$r_{3,t} = \ln\left[\frac{(BEF/DEM)_t}{(BEF/DEM)_{t-1}}\right] = r_{1,t} - r_{2,t}, \qquad r_{4,t} = \ln\left[\frac{(USD/FRF)_t}{(USD/FRF)_{t-1}}\right]$$

and

[8.40c] $$r_{5,t} = \ln\left[\frac{(FRF/DEM)_t}{(FRF/DEM)_{t-1}}\right] = r_{1,t} - r_{4,t}$$

The correlation between BEF/DEM and USD/FRF ($r_{3,t}$ and $r_{4,t}$) is the covariance of $r_{3,t}$ and $r_{4,t}$ divided by their respective standard deviations, mathematically,

[8.41]
$$\begin{aligned}\rho_{34,t} &= \frac{\sigma^2_{34,t}}{\sigma_{4,t}\sigma_{3,t}} \\ &= \frac{\sigma^2_{1,t} - \sigma^2_{12,t} - \sigma^2_{14,t} + \sigma^2_{24,t}}{\sqrt{\sigma^2_{1,t} + \sigma^2_{4,t} - 2\sigma^2_{14,t}}\sqrt{\sigma^2_{1,t} + \sigma^2_{2,t} - 2\sigma^2_{12,t}}}\end{aligned}$$

Analogously, the correlation between USD/DEM and FRF/DEM is

[8.42]
$$\begin{aligned}\rho_{35,t} &= \frac{\sigma^2_{15,t}}{\sigma_{5,t}\sigma_{1,t}} \\ &= \frac{\sigma^2_{1,t} - \sigma^2_{14,t}}{\sqrt{\sigma^2_{1,t} + \sigma^2_{4,t} - 2\sigma^2_{14,t}}\sqrt{\sigma^2_{1,t}}}\end{aligned}$$

8.5 Nonsynchronous data collection

Estimating how financial instruments move in relation to each other requires data that are collated, as much as possible, consistently across markets. The point in time when data are recorded is a material issue, particularly when estimating correlations. When data are observed (recorded) at different times they are known to be nonsynchronous.

Table 8.7 (pages 186–187) outlines how the data underlying the time series used by RiskMetrics are recorded during the day. It shows that most of the data are taken around 16:00 GMT. From the asset class perspective, we see that potential problems will most likely lie in statistics relating to the government bond and equity markets.

To demonstrate the effect of nonsynchronous data on correlation forecasts, we estimated the 1-year correlation of daily movements between USD 10-year zero yields collected every day at the close of business in N.Y. with two series of 3-month money market rates, one collected by the British Bankers Association at 11:00 a.m. in London and the other collected by J.P. Morgan at the close of business in London (4:00 p.m.). This data is presented in Table 8.6.

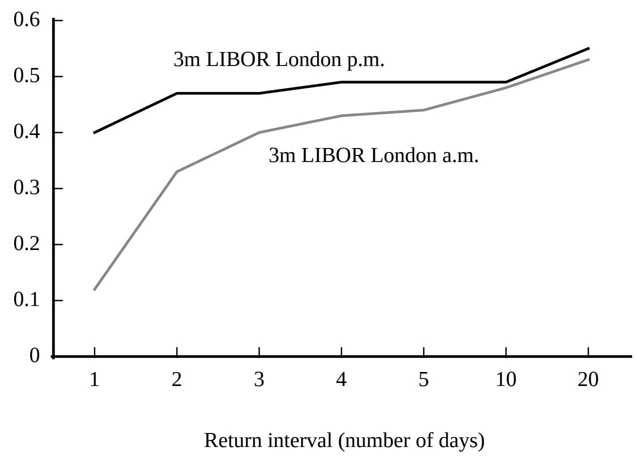
Table 8.6
Correlations of daily percentage changes with USD 10-year
August 1993 to June 1994 – 10-year USD rates collated at N.Y. close

	Correlation at London time:	
LIBOR	**11 a.m.**	**4 p.m.**
1-month	–0.012	0.153
3-month	0.123	0.396
6-month	0.119	0.386
12-month	0.118	0.622

None of the data series are synchronous, but the results show that the money market rates collected at the London close have higher correlation to the USD 10-year rates than those collected in the morning.

Getting a consistent view of how a particular yield curve behaves depends on addressing the timing issue correctly. While this is an important factor in measuring correlations, the effect of timing diminishes as the time horizon becomes longer. Correlating monthly percentage changes may not be dependent on the condition that rates be collected at the same time of day. Chart 8.4 shows how the correlation estimates against USD 10-year zeros evolve for the two money market series mentioned above when the horizon moves from daily changes to monthly changes. Once past the 10-day time interval, the effect of timing differences between the two series becomes negligible.

Chart 8.4
Correlation forecasts vs. return interval
3-month USD LIBOR vs. 10-year USD government bond zero rates



In a perfect world, all rates would be collected simultaneously as all markets would trade at the same time. One may be able to adapt to nonsynchronously recorded data by adjusting either the underlying return series or the forecasts that were computed from the nonsynchronous returns. In this context, data adjustment involves extensive research. The remaining sections of this document present an algorithm to adjust correlations when the data are nonsynchronous.

Table 8.7
Schedule of data collection

		London time, a.m.											
Country	**Instrument summary**	**1:00**	**2:00**	**3:00**	**4:00**	**5:00**	**6:00**	**7:00**	**8:00**	**9:00**	**10:00**	**11:00**	**12:00**
Australia	FX/Eq/LI/Sw/Gv						Eq	Gv					
Hong Kong	FX/Eq/LI/Sw			LI		Eq		Sw					
Indonesia	FX/Eq/LI/Sw								Eq	LI/Sw			
Japan	FX/Eq/LI/Sw/Gv						Gv	Eq					
Korea	FX/Eq								Eq				
Malaysia	FX/Eq/LI/Sw								Eq	LI/Sw			
New Zealand	FX/Eq/LI/Sw/Gv			Eq		LI/Gv	Sw						
Philippines	FX/Eq								Eq				
Singapore	FX/Eq/LI/Sw/Gv									LI/Eq			
Taiwan	FX/Eq/												
Thailand	FX/Eq/LI/Sw								Eq	LI/Sw			
Austria	FX/Eq/LI												Eq
Belgium	FX/Eq/LI/Sw/Gv												
Denmark	FX/Eq/LI/Sw/Gv												
Finland	FX/Eq/LI/Sw/Gv												
France	FX/Eq/LI/Sw/Gv												
Germany	FX/Eq/LI/Sw/Gv												
Ireland	FX/Eq/LI/Sw/Gv												
Italy	FX/Eq/LI/Sw/Gv												
Netherlands	FX/Eq/LI/Sw/Gv												
Norway	FX/Eq/LI/Sw/Gv												
Portugal	FX/Eq/LI/Sw/Gv												
South Africa	FX/Eq/LI//Gv												
Spain	FX/Eq/LI/Sw/Gv												
Sweden	FX/Eq/LI/Sw/Gv												
Switzerland	FX/Eq/LI/Sw/Gv												
U.K.	FX/Eq/LI/Sw/Gv												
ECU	FX/ /LI/Sw/Gv												
Argentina	FX/Eq												
Canada	FX/Eq/LI/Sw/Gv												
Mexico	FX/Eq/LI												
U.S.	FX/Eq/LI/Sw/Gv												
FX = Foreign Exchange, Eq = Equity Index, LI = LIBOR, Sw = Swap, Gv = Government													

Table 8.7 (continued)
Schedule of data collection

London time, p.m.													
1:00	**2:00**	**3:00**	**4:00**	**5:00**	**6:00**	**7:00**	**8:00**	**9:00**	**10:00**	**11:00**	**12:00**	**Instrument summary**	**Country**
			FX/LI/Sw									FX/Eq/LI/Sw/Gv	Australia
			FX									FX/Eq/LI/Sw	Hong Kong
			FX									FX/Eq/LI/Sw	Indonesia
			FX/LI/Sw									FX/Eq/LI/Sw/Gv	Japan
			FX									FX/Eq	Korea
			FX									FX/Eq/LI/Sw	Malaysia
			FX									FX/Eq/LI/Sw/Gv	New Zealand
			FX									FX/Eq	Philippines
			FX									FX/Eq/LI/Sw/Gv	Singapore
			FX									FX/Eq	Taiwan
			FX									FX/Eq/LI/Sw	Thailand
			FX/LI									FX/Eq/LI	Austria
		Eq	FX/LI/Sw/Gv									FX/Eq/LI/Sw/Gv	Belgium
	Eq	Gv	FX/LI/Sw									FX/Eq/LI/Sw/Gv	Denmark
		Eq	FX/LI									FX/Eq/LI/Sw/Gv	Finland
		Gv	FX/LI/Sw/Eq									FX/Eq/LI/Sw/Gv	France
			FX/LI/Sw/Gv/Eq									FX/Eq/LI/Sw/Gv	Germany
			FX/LI/Sw/Gv	Eq								FX/Eq/LI/Sw/Gv	Ireland
			FX/LI/Sw/Gv/Eq									FX/Eq/LI/Sw/Gv	Italy
			FX/LI/Sw/Gv/Eq									FX/Eq/LI/Sw/Gv	Netherlands
	Eq		FX/LI									FX/Eq/LI/Sw/Gv	Norway
			FX/LI/Eq									FX/Eq/LI/Sw/Gv	Portugal
	Eq	Gv	FX/LI									FX/Eq/LI//Gv	South Africa
			FX/LI/Sw	Gv/Eq								FX/Eq/LI/Sw/Gv	Spain
		Gv	FX/LI/Sw/Eq									FX/Eq/LI/Sw/Gv	Sweden
			FX/LI/Sw/Eq									FX/Eq/LI/Sw/Gv	Switzerland
			FX/LI/Sw/Eq	Gv								FX/Eq/LI/Sw/Gv	U.K.
			FX/LI/Sw	Gv								FX/ /LI/Sw/Gv	ECU
			FX					Eq				FX/Eq	Argentina
			FX/LI/Sw				Gv	Eq				FX/Eq/LI/Sw/Gv	Canada
			FX/LI					Eq				FX/Eq/LI	Mexico
			FX/LI/Sw				Gv	Eq				FX/Eq/LI/Sw/Gv	U.S.
FX = Foreign Exchange, Eq = Equity Index, LI = LIBOR, Sw = Swap, Gv = Government													

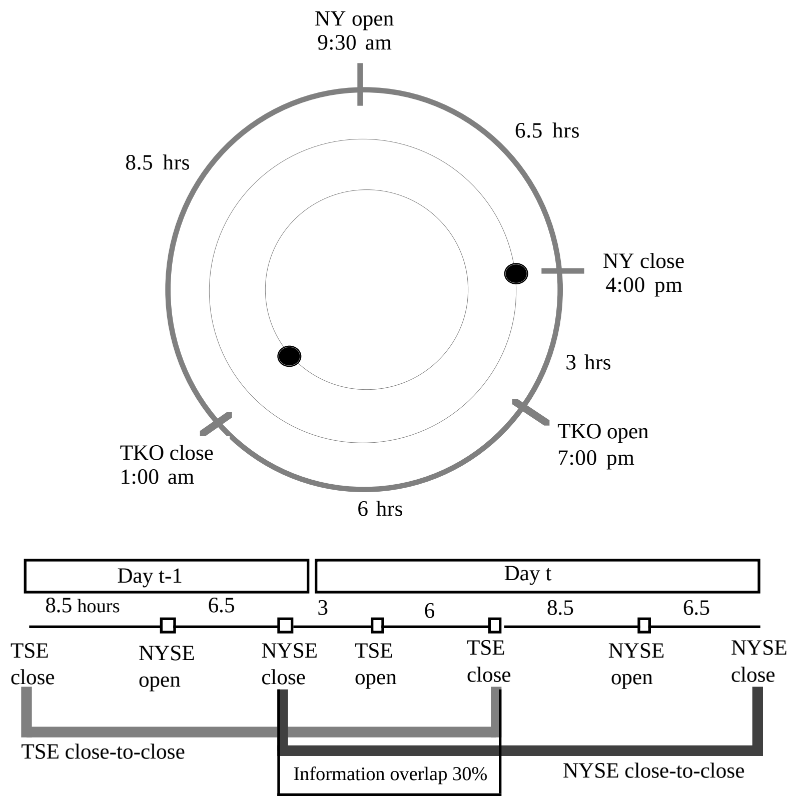
8.5.1 Estimating correlations when the data are nonsynchronous

The expansion of the RiskMetrics data set has increased the amount of underlying prices and rates collected in different time zones. The fundamental problem with nonsynchronous data collection is that correlation estimates based on these prices will be underestimated. And estimating correlations accurately is an important part of the RiskMetrics VaR calculation because standard deviation forecasts used in the VaR calculation depends on correlation estimates.

Internationally diversified portfolios are often composed of assets that trade in different calendar times in different markets. Consider a simple example of a two stock portfolio. Stock 1 trades only on the New York Stock Exchange (NYSE 9:30 am to 4:00 pm EST) while stock 2 trades exclusively on the Tokyo stock exchange (TSE 7:00 pm to 1:00 am EST). Because these two markets are never open at the same time, stocks 1 and 2 cannot trade concurrently. Consequently, their respective daily closing prices are recorded at different times and the return series for assets 1 and 2, which are calculated from daily close-to-close prices, are also nonsynchronous.[8]

Chart 8.5 illustrates the nonsynchronous trading hours of the NYSE and TSE.

Chart 8.5
Time chart
NY and Tokyo stock markets



[8] This terminology began in the nonsynchronous trading literature. See, Fisher, L. (1966) and Sholes, M. and Williams (1977). Nonsynchronous trading is often associated with the situation when some assets trade more frequently than others [see, Perry, P. (1985)]. Lo and MacKinlay (1990) note that “the nonsynchronicity problem results from the assumption that multiple time series are sampled simultaneously when in fact the sampling is nonsynchronous.” For a recent discussion of the nonsynchronous trading issue see Boudoukh, et. al (1994).

We see that the Tokyo exchange opens three hours after the New York close and the New York exchange reopens 81/2 hours after the Tokyo close. Because a new calendar day arrives in Tokyo before New York, the Tokyo time is said to precede New York time by 14 hours (EST).

RiskMetrics computes returns from New York and Tokyo stock markets using daily close-to-close prices. The black orbs in Chart 8.5 mark times when these prices are recorded. Note that the orbs would line up with each other if returns in both markets were recorded at the same time.

The following sections will:

1. Identify the problem and verify whether RiskMetrics really does underestimate certain correlations.
2. Present an algorithm to adjust the correlation estimates.
3. Test the results against actual data.

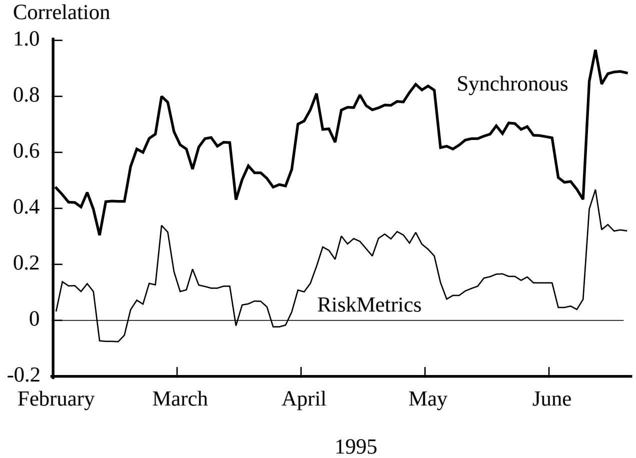
8.5.1.1 Identifying the problem: correlation and nonsynchronous returns

Whether different return series are recorded at the same time or not becomes an issue when these data are used to estimate correlations because the absolute magnitude of correlation (covariance) estimates may be underestimated when calculated from nonsynchronous rather than synchronous data. Therefore, when computing correlations using nonsynchronous data, we would expect the value of observed correlation to be below the true correlation estimate. In the following analysis we first establish the effect that nonsynchronous returns have on correlation estimates and then offer a method for adjusting correlation estimates to account for the nonsynchronicity problem.

The first step in checking for downward bias is estimating what the “true” correlation should be. This is not trivial since these assets do not trade in the same time zone and it is often not possible to obtain synchronous data. For certain instruments, however, it is possible to find limited datasets which can provide a glimpse of the true level of correlation; this data would then become the benchmark against which the methodology for adjusting nonsynchronous returns would be tested.

One of these instruments is the US Treasury which has the advantage of being traded 24 hours a day. While we generally use nonsynchronous close-to-close prices to estimate RiskMetrics correlations, we obtained price data for both the US and Australian markets quoted in the Asian time zone (August 1994 to June 1995). We compared the correlation based on synchronous data with correlation estimates that are produced under the standard RiskMetrics data (using the nonsynchronous US and Australian market close). Plots of the two correlation series are shown in Chart 8.6.

Chart 8.6
10-year Australia/US government bond zero correlation
based on daily RiskMetrics close/close data and 0:00 GMT data



While the changes in correlation estimates follow similar patterns over time (already an interesting result in itself), the correlation estimates obtained from price data taken at the opening of the markets in Asia are substantially higher. One thing worth noting however, is that while the synchronous estimate appears to be a better representation of the "true" level of correlation, it is not necessarily equal to the true correlation. While we have adjusted for the timing issue, we may have introduced other problems in the process, such as the fact that while US Treasuries trade in the Asian time zone, the market is not as liquid as during North American trading hours and the prices may therefore be less representative of "normal trading" volumes. Market segmentation may also affect the results. Most investors, even those based in Asia put on positions in the US market during North American trading hours. U.S. Treasury trading in Asia is often the result of hedging.

Nevertheless, from a risk management perspective, this is an important result. Market participants holding positions in various markets including Australia (and possibly other Asian markets) would be distorting their risk estimates by using correlation estimates generated from close of business prices.

8.5.1.2 An algorithm for adjusting correlations
Correlation is simply the covariance divided by the product of two standard errors. Since the standard deviations are unaffected by nonsynchronous data, correlation is adversely affected by nonsynchronous data through its covariance. This fact simplifies the analysis because under the current RiskMetrics assumptions, long horizon covariance forecasts are simply the 1-day covariance forecasts multiplied by the forecast horizon.

Let us now investigate the effect that nonsynchronous trading has on correlation estimates for historical rate series from the United States (USD), Australian (AUD) and Canadian (CAD) government bond markets. In particular, we focus on 10-year government bond zero rates. Table 8.8 presents the time that RiskMetrics records these rates (closing prices).

Table 8.8
RiskMetrics closing prices
10-year zero bonds

Country	EST	London
USD	3:30 p.m.	8:00 p.m.
CAD	3:30 p.m.	8:00 p.m.
AUD	2:00 a.m.	7:00 a.m.

Note that the USD and CAD rates are synchronous while the USD and AUD, and CAD and AUD rates are nonsynchronous. We chose to analyze rates in these three markets to gain insight as to how covariances (correlations) computed from synchronous and nonsynchronous return series compare with each other. For example, at any time t, the observed return series, $r^{obs}_{USD,t}$ and $r^{obs}_{AUD,t}$ are nonsynchronous, whereas $r^{obs}_{USD,t}$ and $r^{obs}_{CAD,t}$ are synchronous. We are interested in measuring the covariance and autocovariance of these return series.

Table 8.9 provides summary statistics on 1-day covariance and autocovariance forecasts for the period May 1993 to May 1995. The numbers in the table are interpreted as follows: over the sample period, the average covariance between USD and AUD 10-year zero returns, $\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t}\right)$ is 0.16335 while the average covariance between current USD 10-year zero returns and lagged CAD 10-year zero returns (autocovariance) is –0.0039.

Table 8.9
Sample statistics on RiskMetrics daily covariance forecasts
10-year zero rates; May 1993 – May 1995

Daily forecasts	Mean	Median	Std. dev.	Max	Min
$\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t}\right)$	0.1633*	0.0995	0.1973	0.8194	–0.3396
$\mathrm{cov}\left(r^{obs}_{USD,t-1} r^{obs}_{AUD,t}\right)$	0.5685	0.4635	0.3559	1.7053	0.1065
$\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t-1}\right)$	0.0085	–0.0014	0.1806	0.5667	–0.6056
$\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{CAD,t}\right)$	0.6082	0.4912	0.3764	1.9534	0.1356
$\mathrm{cov}\left(r^{obs}_{USD,t-1}, r^{obs}_{CAD,t}\right)$	0.0424	0.0259	0.1474	0.9768	–0.2374
$\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{CAD,t-1}\right)$	–0.0039	–0.0003	0.1814	0.3333	–0.7290

* All numbers are multiplied by 10,000.

The results show that when returns are recorded nonsynchronously, the covariation between lagged 1-day USD returns and current AUD returns (0.5685) is larger, on average, than the covariance (0.1633) that would typically be reported. Conversely, for the USD and CAD returns, the autocovariance estimates are negligible relative to the covariance estimates. This evidence points to a typical finding: first order autocovariances of returns for assets that trade at different times are larger than autocovariances for returns on assets that trade synchronously.[9]

[9] One possible explanation for the large autocovariances has to do with information flows between markets. The literature on information flows between markets include studies analyzing Japanese and US equity markets (Jaffe and Westerfield (1985), Becker, et.al, (1992), Lau and Diltz, (1994)). Papers that focus on many markets include Eun and Shim, (1989).

As a check of the results above and to understand how RiskMetrics correlation forecasts are affected by nonsynchronous returns, we now focus on covariance forecasts for a specific day. We continue to use USD, CAD and AUD 10-year zero rates. Consider the 1-day forecast period May 12 to May 13, 1995. In RiskMetrics, these 1-day forecasts are available at 10 a.m. EST on May 12. The most recent USD (CAD) return is calculated over the period 3:30 pm EST on 5/10 to 3:30 pm EST on 5/11 whereas the most recent AUD return is calculated over the period 1:00 am EST on 5/10 to 1:00 am EST on 5/11. Table 8.10 presents covariance forecasts for May 12 along with their standard errors.

Table 8.10
RiskMetrics daily covariance forecasts
10-year zero rates; May 12, 1995

Return series	Covariance	T-statistic†
$r^{obs}_{USD,\,5/12} r^{obs}_{AUD,\,5/12}$	0.305	-
$r^{obs}_{USD,\,5/11} r^{obs}_{AUD,\,5/12}$	0.629 (0.074)*	8.5
$r^{obs}_{USD,\,5/12} r^{obs}_{AUD,\,5/11}$	0.440 (0.074)	5.9
$r^{obs}_{USD,\,5/11} r^{obs}_{CAD,\,5/12}$	0.530	-
$r^{obs}_{USD,\,5/12} r^{obs}_{CAD,\,5/12}$	0.106 (0.058)	1.8
$r^{obs}_{USD,\,5/12} r^{obs}_{CAD,\,5/11}$	0.126 (0.059)	2.13

* Asymptotic standard errors are reported in parentheses.

† For a discussion on the use of the t-statistic for the autocovariances see Shanken (1987).

In agreement with previous results, we find that while there is strong covariation between lagged USD returns $r^{obs}_{USD,\,5/11}$ and current AUD returns $r^{obs}_{USD,\,5/12}$ (as shown by large t-statistics), the covariation between lagged USD and CAD returns is not nearly as strong. The results also show evidence of covariation between lagged AUD returns and current USD returns.

The preceding analysis describes a situation where the standard covariances calculated from nonsynchronous data do not capture all the covariation between returns. By estimating autocovariances, it is possible to measure the 1-day lead and lag effects across return series. With nonsynchronous data, these lead and lag effects appear quite large. In other words, current and past information in one return series is correlated with current and past information in another series. If we represent information by returns, then following Cohen, Hawawini, Maier, Schwartz and Whitcomb, (CHMSW 1983) we can write observed returns as a function of weighted unobserved current and lag true returns. The weights simply represent how much information in a specific true return appears in the return that is observed. Given this, we can write observed (nonsynchronous) returns for the USD and AUD 10-year zero returns as follows:

[8.43]
$$\begin{aligned} r^{obs}_{USD,\,t} &= \theta_{USD,\,t} R_{USD,\,t} + \theta_{USD,\,t-1} r_{USD,\,t-1} \\ r^{obs}_{AUD,\,t} &= \theta_{AUD,\,t} R_{USD,\,t} + \theta_{AUD,\,t-1} r_{AUD,\,t-1} \end{aligned}$$

The $\theta_{j,\,t-i}$'s are random variables that represent the proportion of the true return of asset *j* generated in period *t*-i that is actually incorporated in observed returns in period *t*. In other words, the $\theta_{j,\,t}$'s are weights that capture how the true return generated in one period impacts on the observed returns in the same period and the next. It is also assumed that:

[8.44]

$$\theta_{AUD,t} \text{ and } \theta_{USD,\tau} \text{ are independent for all t and } \tau$$
$$\theta_{AUD,t} \text{ and } \theta_{USD,\tau} \text{ are independent of } R_{AUD,t} \text{ and } R_{USD,\tau}$$
$$E(\theta_{AUD,t}) = E(\theta_{USD,t}) \text{ for all t and } \tau$$
$$E(\theta_{j,t} + \theta_{j,t-1}) = 1 \text{ for j = AUD, USD and for all t and } \tau$$

Table 8.11 shows, for the example given in the preceding section, the relationship between the date when the true return is calculated and the weight assigned to the true return.

Table 8.11
Relationship between lagged returns and applied weights
observed USD and AUD returns for May 12, 1995

Date	5/9–5/10	5/9–5/10	5/10–5/11	5/10–5/11
Weight	$\theta_{AUD,t-1}$	$\theta_{USD,t-1}$	$\theta_{AUD,t}$	$\theta_{USD,t}$

Earlier we computed the covariance based on observed returns, $\mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t}\right)$ However, we can use Eq. [8.43] to compute the covariance of the true returns $\mathrm{cov}(r_{USD,t}, r_{AUD,t})$, i.e.,

[8.45]

$$\mathrm{cov}(r_{USD,t}, r_{AUD,t}) = \mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t-1}\right) + \mathrm{cov}\left(r^{obs}_{USD,t}, r^{obs}_{AUD,t}\right) + \mathrm{cov}\left(r^{obs}_{USD,t-1}, r^{obs}_{AUD,t}\right)$$

We refer to this estimator as the **"adjusted" covariance. Having established the form of the adjusted covariance estimator, the adjusted correlation estimator for any two return series *j* and *k* is:**

[8.46]

$$\rho_{jk,t} = \frac{\mathrm{cov}\left(r^{obs}_{j,t}, r^{obs}_{k,t-1}\right) + \mathrm{cov}\left(r^{obs}_{j,t}, r^{obs}_{k,t}\right) + \mathrm{cov}\left(r^{obs}_{j,t-1}, r^{obs}_{k,t}\right)}{\mathrm{std}\left(r^{obs}_{j,t}\right)\mathrm{std}\left(r^{obs}_{k,t}\right)}$$

Table 8.12 shows the original and adjusted correlation estimates for USD-AUD and USD-CAD 10-year zero rate returns.

Table 8.12
Original and adjusted correlation forecasts
USD-AUD 10-year zero rates; May 12, 1995

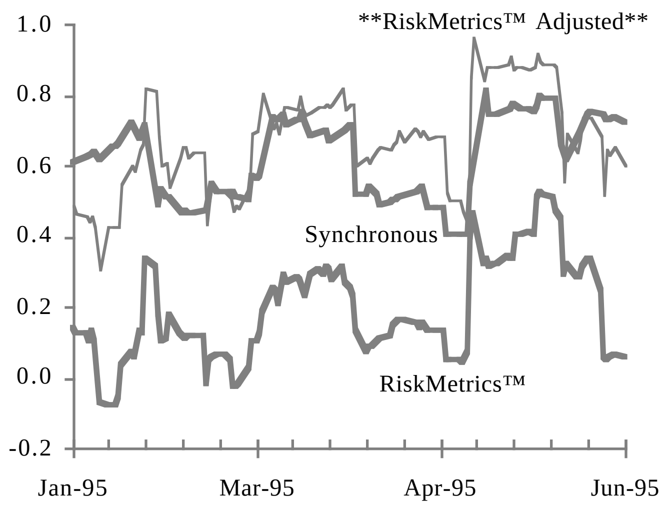
Daily forecasts	**Original**	**Adjusted**	**% change**
$\mathrm{cov}(r_{USD,5/12}, r_{AUD,5/12})$	0.305	0.560	84%
$\mathrm{cov}(r_{USD,5/12}, r_{CAD,5/12})$	0.530	0.573	8%

Note that the USD-AUD adjusted covariance increases the original covariance estimate by 84%. Earlier (see Table 8.10) we found the lead-lag covariation for the USD-AUD series to be statistically significant. Applying the adjusted covariance estimator to the synchronous series USD-CAD, we find only an 8% increase over the original covariance estimate. However, the evidence from Table 8.10 would suggest that this increase is negligible.

8.5.1.3 Checking the results

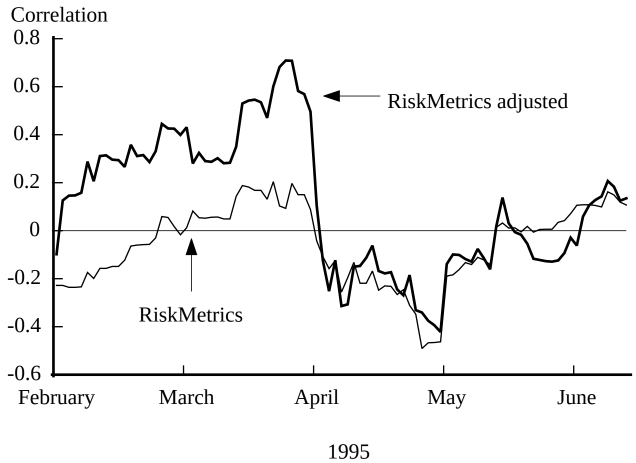
How does the adjustment algorithm perform in practice? Chart 8.7 compares three daily correlation estimates for 10-year zero coupon rates in Australia and the United States: (1) Standard RiskMetrics using nonsynchronous data, (2) estimate correlation using synchronous data collected in Asian trading hours and, (3) RiskMetrics Adjusted using the estimator in Eq. [8.46].

Chart 8.7
Adjusting 10-year USD/AUD bond zero correlation
using daily RiskMetrics close/close data and 0:00 GMT data



The results show that the adjustment factor captures the effects of the timing differences that affect the standard RiskMetrics estimates which use nonsynchronous data. A potential drawback of using this estimator, however, is that the adjusted series displays more volatility than either the unadjusted or the synchronous series. This means that in practice, choices may have to be made as to when to apply the methodology. In the Australian/US case, it is clear that the benefits of the adjustment in terms of increasing the correlation to a level consistent with the one obtained when using synchronous data outweighs the increased volatility. The choice, however, may not always be that clear cut as shown by Chart 8.8 which compares adjusted and unadjusted correlations for the US and Japanese 10-year zero rates. In periods when the underlying correlation between the two markets is significant (Jan-Feb 1995, the algorithm correctly adjusts the estimate). In periods of lower correlation, the algorithm only increases the volatility of the estimate.

Chart 8.8
10-year Japan/US government bond zero correlation
using daily RiskMetrics close/close data and 0:00 GMT data



Also, in practice, estimation of the adjusted correlation is not necessarily straightforward because we must take into account the chance of getting adjusted correlation estimates above 1. This potential problem arises because the numerator in Eq. [8.46] is being adjusted without due consideration of the denominator. An algorithm that allows us to estimate the adjusted correlation without obtaining correlations greater than 1 in absolute value is given in Section 8.5.2.

Table 8.13 on page 196 reports sample statistics for 1-day correlation forecasts estimated over various sample periods for both the original RiskMetrics and adjusted correlation estimators. Correlations between United States and Asia-Pacific are based on non-synchronous data.

8.5.2 Using the algorithm in a multivariate framework

Finally, we explain how to compute the adjusted correlation matrix.

1. Calculate the unadjusted (standard) RiskMetrics covariance matrix, Σ. (Σ is an N x N, positive semi-definite matrix).
2. Compute the nonsynchronous data adjustment matrix K where the elements of K are

[8.47] $$k_{k,j} = \begin{cases} \text{cov}(r_{k,t}, r_{j,t-1}) + \text{cov}(r_{k,t-1}, r_{j,t}) & \text{for } k \neq j \\ 0 & \text{for } k = j \end{cases}$$

3. The adjusted covariance matrix M, is given by $M = \Sigma + fK$ where $0 \leq f \leq 1$. The parameter f that is used in practice is the largest possible f such that M is positive semi-definite.

Table 8.13
Correlations between US and foreign instruments

Correlations between USD 10-year zero rates and JPY, AUD, and NZD 10-year zero rates.*
Sample period: May 1991–May 1995.

	Original			Adjusted		
	JPY	AUD	NZD	JPY	AUD	NZD
mean	0.026	0.166	0.047	0.193	0.458	0.319
median	0.040	0.155	0.036	0.221	0.469	0.367
std dev	0.151	0.151	0.171	0.308	0.221	0.241
max	0.517	0.526	0.613	0.987	0.937	0.921
min	–0.491	–0.172	–0.389	–0.762	–0.164	–0.405

Correlations between USD 2-year swap rates and JPY, AUD, NZD, HKD 2-year swap rates.*
Sample period: May 1993–May 1995.

	Original				Adjusted			
	JPY	AUD	NZD	HKD	JPY	AUD	NZD	HKD
mean	0.018	0.233	0.042	0.139	0.054	0.493	0.249	0.572
median	0.025	0.200	0.020	0.103	0.065	0.502	0.247	0.598
std dev	0.147	0.183	0.179	0.217	0.196	0.181	0.203	0.233
max	0.319	0.647	0.559	0.696	0.558	0.920	0.745	0.945
min	–0.358	–0.148	–0.350	–0.504	–0.456	–0.096	–0.356	–0.411

Correlations between USD equity index and JPY, AUD, NZD, HKD, SGD equity indices.*
Sample period: May 1993–May 1995.

	Original					Adjusted				
	JPY	AUD	NZD	HKD	SGD	JPY	AUD	NZD	HKD	SGD
mean	0.051	0.099	-0.023	0.006	0.038	0.124	0.330	–0.055	–0.013	0.014
median	0.067	0.119	-0.021	-0.001	0.028	0.140	0.348	–0.053	0.056	–0.024
std dev	0.166	0.176	0.128	0.119	0.145	0.199	0.206	0.187	0.226	0.237
max	0.444	0.504	0.283	0.271	0.484	0.653	0.810	0.349	0.645	0.641
min	–0.335	–0.345	–0.455	–0.298	–0.384	–0.395	–0.213	–0.524	–0.527	–0.589

* JPY = Japanese yen, AUD = Australian dollar, NZD = New Zealand dollar, HKD = Hong Kong dollar, SGD = Singapore dollar

Chapter 9. Time series sources

Chapter 9. Time series sources

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Data is one of the cornerstones of any risk management methodology. We examined a number of data providers and decided that the sources detailed in this chapter were the most appropriate for our purposes.

9.1 Foreign exchange

Foreign exchange prices are sourced from WM Company and Reuters. They are mid-spot exchange prices recorded at 4:00 p.m. London time (11:00 a.m. EST). All foreign exchange data used for RiskMetrics is identical to the data used by the J.P. Morgan family of government bond indices. (See Table 9.1.)

Table 9.1
Foreign exchange

Currency Codes					
Americas		**Asia Pacific**		**Europe and Africa**	
ARS	Argentine peso	AUD	Australian dollar	ATS	Austrian shilling
CAD	Canadian dollar	HKD	Hong Kong dollar	BEF	Belgian franc
MXN	Mexican peso	IDR	Indonesian rupiah	CHF	Swiss franc
USD	U.S. dollar	JPY	Japanese yen	DEM	Deutsche mark
EMB	EMBI+*	KRW	Korean won	DKK	Danish kroner
		MYR	Malaysian ringgit	ESP	Spanish peseta
		NZD	New Zealand dollar	FIM	Finnish mark
		PHP	Philippine peso	FRF	French franc
		SGD	Singapore dollar	GBP	Sterling
		THB	Thailand baht	IEP	Irish pound
		TWD	Taiwan dollar	ITL	Italian lira
				NLG	Dutch guilder
				NOK	Norwegian kroner
				PTE	Portuguese escudo
				SEK	Swedish krona
				XEU	ECU
				ZAR	South African rand

* EMBI+ stands for the J.P. Morgan Emerging Markets Bond Index Plus.

9.2 Money market rates

Most 1-, 2-, 3-, 6-, and 12-month money market rates (offered side) are recorded on a daily basis by J.P. Morgan in London at 4:00 p.m. (11:00 a.m. EST). Those obtained from external sources are also shown in Table 9.2.

Table 9.2
Money market rates: sources and term structures

	Source		Time	Term Structure			
Market	J.P. Morgan	Third Party*	U.S. EST	1m	3m	6m	12m
Australia	•		11:00 a.m.	•	•	•	•
Hong Kong		•	10:00 p.m.	•	•	•	•
Indonesia†	•		5:00 a.m.	•	•	•	•
Japan	•		11:00 a.m.	•	•	•	•
Malaysia†	•		5:00 a.m.	•	•	•	•
New Zealand		•	12:00 a.m.	•	•	•	
Singapore		•	4:30 a.m.	•	•	•	•
Thailand†	•		5:00 a.m.	•	•	•	•
Austria		•	11:00 a.m.	•	•	•	•
Belgium	•		11:00 a.m.	•	•	•	•
Denmark	•		11:00 a.m.	•	•	•	•
Finland		•	11:00 a.m.	•	•	•	•
France	•		11:00 a.m.	•	•	•	•
Ireland		•	11:00 a.m.	•	•	•	•
Italy	•		11:00 a.m.	•	•	•	•
Netherlands	•		11:00 a.m.	•	•	•	•
Norway		•	11:00 a.m.	•	•	•	•
Portugal		•	11:00 a.m.	•	•	•	•
South Africa			11:00 a.m.	•	•	•	•
Spain	•		11:00 a.m.	•	•	•	•
Sweden	•		11:00 a.m.	•	•	•	•
Switzerland	•		11:00 a.m.	•	•	•	•
U.K.	•		11:00 a.m.	•	•	•	•
ECU	•		11:00 a.m.	•	•	•	•
Canada	•		11:00 a.m.	•	•	•	•
Mexico‡	•		12:00 p.m.	•	•	•	•
U.S.	•		11:00 a.m.	•	•	•	•

* Third party source data from Reuters Generic except for Hong Kong (Reuters HIBO), Singapore (Reuters MASX), and New Zealand (National Bank of New Zealand).

† Money market rates for Indonesia, Malaysia, and Thailand are calculated using foreign exchange forward-points.

‡ Mexican rates represent secondary trading in Cetes.

9.3 Government bond zero rates

Zero coupon rates ranging in maturity from 2 to 30 years for the government bond markets included in the J.P. Morgan Government Bond Index as well as the Irish, ECU, and New Zealand markets. (See Table 9.3.)

Table 9.3
Government bond zero rates: sources and term structures

	Source		Time	Term structure									
Market	J.P. Morgan	Third Party	U.S. EST	2y	3y	4y	5y	7y	9y	10y	15y	20y	30y
Australia	•		1:30 a.m.	•	•	•	•	•	•	•	•		
Japan	•		1:00 a.m.	•	•	•	•	•	•	•			
New Zealand		•	12:00 a.m.	•	•	•	•	•	•	•	•		
Belgium	•		11:00 a.m.	•	•	•	•	•	•	•	•	•	
Denmark		•	10:30 a.m.	•	•	•	•	•	•	•	•	•	•
France	•		10:30 a.m.	•	•	•	•	•	•	•	•	•	•
Germany	•		11:30 a.m.	•	•	•	•	•	•	•	•	•	•
Ireland		•	10:30 a.m.	•	•	•	•	•	•	•	•	•	
Italy	•		10:45 a.m.	•	•	•	•	•	•	•	•	•	•
Netherlands	•		11:00 a.m.	•	•	•	•	•	•	•	•	•	•
South Africa	•		11:00 a.m.	•	•	•	•	•	•	•	•	•	
Spain	•		11:00 a.m.	•	•	•	•	•	•	•	•		
Sweden		•	10:00 a.m.	•	•	•	•	•	•	•	•		
U.K.	•		11:45 a.m.	•	•	•	•	•	•	•	•	•	•
ECU	•		11:45 a.m.	•	•	•	•	•	•	•			
Canada	•		3:30 p.m.	•	•	•	•	•	•	•	•	•	•
U.S.	•		3:30 a.m.	•	•	•	•	•	•	•	•	•	•
Emerging Mkt.†	•		3:00 p.m.										

* Third party data sourced from Den Danske Bank (Denmark), NCB Stockbrokers (Ireland), National Bank of New Zealand (New Zealand), and SE Banken (Sweden).

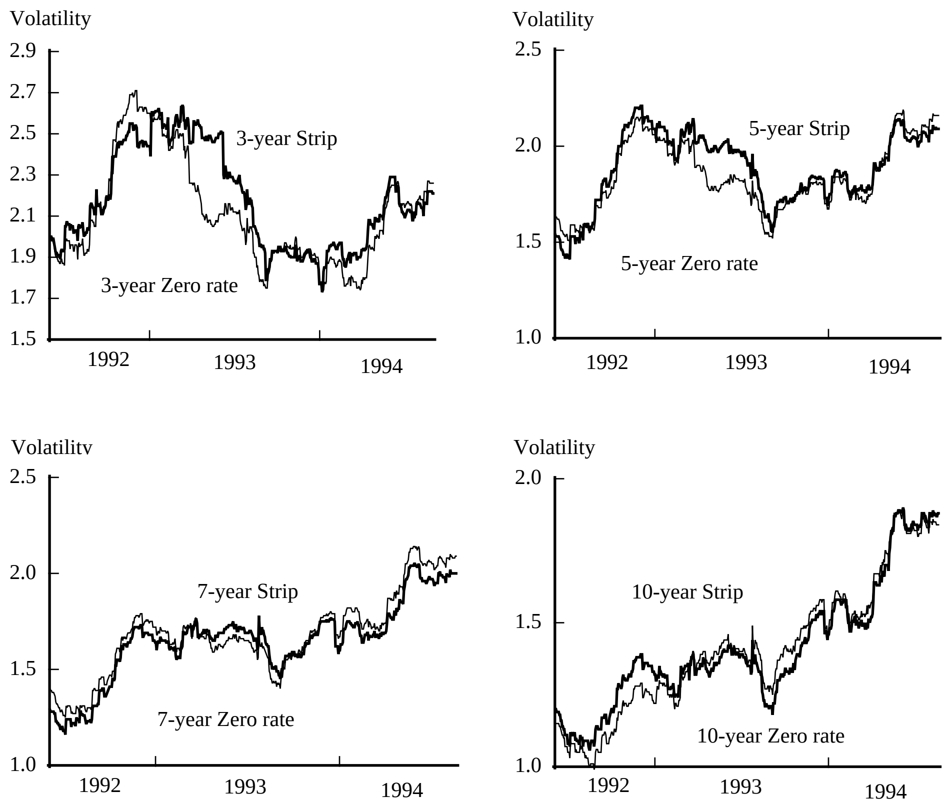
† J. P. Morgan Emerging Markets Bond Index Plus (EMBI+).

If the objective is to measure the volatility of individual cash flows, then one could ask whether it is appropriate to use a term structure model instead of the underlying zero rates which can be directly observed from instruments such as Strips. The selection of a modeled term structure as the basis for calculating market volatilities was motivated by the fact that there are few markets which have observable zero rates in the form of government bond Strips from which to estimate volatilities. In fact, only the U.S. and French markets have reasonably liquid Strips which could form the basis for a statistically solid volatility analysis. Most other markets in the OECD have either no Strip market or a relatively illiquid one.

The one possible problem of the term structure approach is that it would not be unreasonable to assume the volatility of points along the term structure may be lower than the market's real volatility because of the smoothing impact of passing a curve through a universe of real data points.

To see whether there was support for this assumption, we compared the volatility estimates obtained from term structure derived zero rates and actual Strip yields for the U.S. market across four maturities (3, 5, 7, and 10 years). The results of the comparison are shown in Chart 9.1.

Chart 9.1
Volatility estimates: daily horizon
1.65 standard deviation—6-month moving average



The results show that there is no clear bias from using the term structure versus underlying Strips data. The differences between the two measures decline as maturity increases and are partially the result of the lack of liquidity of the short end of the U.S. Strip market. Market movements specific to Strips can also be caused by investor behavior in certain hedging strategies that cause prices to sometimes behave erratically in comparison to the coupon curve from which the term structure is derived.

9.4 Swap rates

Swap par rates from 2 to 10 years are recorded on a daily basis by J.P. Morgan, except for Ireland (provided by NCB Stockbrokers), Hong Kong (Reuters TFHK) and Indonesia, Malaysia and Thailand (Reuters EXOT). (See Table 9.4.) The par rates are then converted to zero coupon equivalents rates for the purpose of inclusion within the RiskMetrics data set. (Refer to Section 8.1 for details).

Table 9.4
Swap zero rates: sources and term structures

	Source		Time	Term structure					
Market	J.P. Morgan	Third Party*	US EST	2y	3y	4y	5y	7y	10y
Australia	•		1:30 a.m.	•	•	•	•	•	•
Hong Kong		•	4:30 a.m.	•	•	•	•	•	•
Indonesia		•	4:00 a.m.	•	•	•	•		
Japan	•		1:00 a.m.	•	•	•	•	•	•
Malaysia		•	4:00 a.m.	•	•	•	•		
New Zealand		•	3:00 p.m.	•	•	•	•	•	
Thailand		•	4:00 a.m.	•	•	•	•		
Belgium	•		10:00 a.m.	•	•	•	•	•	•
Denmark	•		10:00 a.m.	•	•	•	•	•	•
Finland	•		10:00 a.m	•	•	•	•		
France	•		10:00 a.m.	•	•	•	•	•	•
Germany	•		10:00 p.m.	•	•	•	•	•	•
Ireland		•	11:00 a.m.	•	•	•	•		
Italy	•		10:00 a.m.	•	•	•	•	•	•
Netherlands	•		10:00 a.m.	•	•	•	•	•	•
Spain	•		10:00 a.m.	•	•	•	•	•	•
Sweden	•		10:00 a.m.	•	•	•	•	•	•
Switzerland	•		10:00 a.m.	•	•	•	•	•	•
U.K.	•		10:00 a.m.	•	•	•	•	•	•
ECU	•		10:00 a.m.	•	•	•	•	•	•
Canada	•		3:30 p.m.	•	•	•	•	•	•
U.S.	•		3:30 a.m.	•	•	•	•	•	•

* Third party source data from Reuters Generic except for Ireland (NCBI), Hong Kong (TFHK), and Indonesia, Malaysia, Thailand (EXOT).

9.5 Equity indices

The following list of equity indices (Table 9.5) have been selected as benchmarks for measuring the market risk inherent in holding equity positions in their respective markets. The factors that determined the selection of these indices include the existence of index futures that can be used as hedging instruments, sufficient market capitalization in relation to the total market, and low tracking error versus a representation of the total capitalization. All the indices listed below measure principal return except for the DAX which is a total return index.

Table 9.5
Equity indices: sources*

Market	Exchange	Index Name	Weighting	% Mkt. cap.	Time, U.S. EST
Australia	Australian Stock Exchange	All Ordinaries	MC	96	1:10 a.m.
Hong Kong	Hong Kong Stock Exchange	Hang Seng	MC	77	12:30 a.m.
Indonesia	Jakarta Stock Exchange	JSE	MC		4:00 a.m.
Korea	Seoul Stock Exchange	KOPSI	MC		3:30 a.m.
Japan	Tokyo Stock Exchange	Nikei 225	MC	46	1:00 a.m.
Malaysia	Kuala Lumpur Stock Exchange	KLSE	MC		6:00 a.m.
New Zealand	New Zealand Stock Exchange	Capital 40	MC	—	10:30 p.m.
Philippines	Manila Stock Exchange	MSE Com'l &Inustil Price	MC		1:00 a.m.
Singapore	Stock Exchange of Singapore	Sing. All Share	MC	—	4:30 a.m.
Taiwan	Taipei Stock Exchange	TSE	MC		1:00 a.m.
Thailand	Bangkok Stock Exchange	SET	MC		5:00 a.m.
Austria	Vienna Stock Exchange	Creditanstalt	MC	—	7:30 a.m.
Belgium	Brussels Stock Exchange	BEL 20	MC	78	10:00 a.m.
Denmark	Copenhagen Stock Exchange	KFX	MC	44	9:30 a.m.
Finland	Helsinki Stock Exchange	Hex General	MC	—	10:00 a.m.
France	Paris Bourse	CAC 40	MC	55	11:00 a.m.
Germany	Frankfurt Stock Exchange	DAX	MC	57	10:00 a.m.
Ireland	Irish Stock Exchange	Irish SE ISEQ	—	—	12:30 p.m.
Italy	Milan Stock Exchange	MIB 30	MC	65	10:30 a.m.
Japan	Tokyo Stock Exchange	Nikei 225	MC	46	1:00 a.m.
Netherlands	Amsterdam Stock Exchange	AEX	MC	80	10:30 a.m.
Norway	Oslo Stock Exchange	Oslo SE General	—	—	9:00 a.m.
Portugal	Lisbon Stock Exchange	Banco Totta SI	—	—	11:00 a.m.
South Africa	Johannesburg Stock Exchange	JSE	MC		10:00 a.m.
Spain	Madrid Stock Exchange	IBEX 35	MC	80	11:00 a.m.
Sweden	Stockholm Stock Exchange	OMX	MC	61	10:00 a.m.
Switzerland	Zurich Stock Exchange	SMI	MC	56	10:00 a.m.
U.K.	London Stock Exchange	FTSE 100	MC	69	10:00 a.m.
Argentina	Buenos Aires Stock Exchange	Merval	Vol.		5:00 p.m.
Canada	Toronto Stock Exchange	TSE 100	MC	63	4:15 p.m.
Mexico	Mexico Stock Exchange	IPC	MC		3:00 p.m.
U.S.	New York Stock Exchange	Standard and Poor's 100	MC	60	4:15 a.m.

* Data sourced from DRI.

9.6 Commodities

The commodity markets that have been included in RiskMetrics are the same markets as the J.P. Morgan Commodity Index (JPMCI). The data for these markets are shown in Table 9.6.

Table 9.6
Commodities: sources and term structures

Commodity	Source	Time, U.S. EST	Term structure						
			Spot	1m	3m	6m	12m	15m	27m
WTI Light Sweet Crude	NYMEX*	3:10 p.m.		•	•	•	•		
Heating Oil	NYMEX	3:10 p.m.		•	•	•	•		
NY Harbor #2 unleaded gas	NYMEX	3:10 p.m.		•	•	•			
Natural gas	NYMEX	3:10 p.m.		•	•	•	•		
Aluminum	LME†	11:20 a.m.	•		•			•	•
Copper	LME	11:15 a.m.	•		•			•	•
Nickel	LME	11:10 a.m.	•		•			•	
Zinc	LME	11:30 a.m.	•		•			•	•
Gold	LME	11:00 a.m.	•						
Silver	LFOE‡	11:00 a.m.	•						
Platinum	LPPA§	11:00 a.m.	•						

* NYMEX (New York Mercantile Exchange)
† LME (London Metals Exchange)
‡ LFOE (London futures and Options Metal Exchange)
§ LPPA (London Platinum & Palladium Association)

The choice between either the rolling nearby or interpolation (constant maturity) approach is influenced by the characteristics of each contract. We use the interpolation methodology wherever possible, but in certain cases this approach cannot or should not be implemented.

We use interpolation (I) for all energy contracts. (See Table 9.7.)

Table 9.7
Energy maturities

Energy	Maturities					
	1m	3m	6m	12m	15m	27m
Light sweet crude	I*	I	I	I		
Heating Oil	I	I	I	I		
Unleaded Gas	I	I	I			
Natural Gas	I	I	I	I		

* I = Interpolated methodology.

The term structures for base metals are based upon rolling nearby contracts with the exception of the spot (S) and 3-month contracts. Data availability is the issue here. Price data for contracts traded on the London Metals Exchange is available for constant maturity 3-month (A) contracts (prices are quoted on a daily basis for 3 months forward) and rolling 15- and 27- month (N) contracts. Nickel extends out to only 15 months. (See Table 9.8.)

Table 9.8
Base metal maturities

Commodity	Maturities					
	Spot	3m	6m	12m	15m	27m
Aluminum	S*	A†			N‡	N
Copper	S	A			N	N
Nickel	S	A			N	
Zinc	S	A			N	N

* S = Spot contract.
† A = Constant maturity contract.
‡ N = Rolling contract.

Spot prices are the driving factor in the precious metals markets. Volatility curves in the gold, silver, and platinum markets are relatively flat (compared to the energy curves) and spot prices are the main determinant of the future value of instruments: storage costs are negligible and convenience yields such as those associated with the energy markets are not a consideration.

Chapter 10. RiskMetrics volatility and correlation files

Chapter 10. RiskMetrics volatility and correlation files

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This section serves as a guide to understanding the information contained in the RiskMetrics daily and monthly volatility and correlation files. It defines the naming standards we have adopted for the RiskMetrics files and time series, the file formats, and the order in which the data is presented in these files.

10.1 Availability

Volatility and correlation files are updated each U.S. business day and posted on the Internet by 10:30 a.m. EST. They cover data through close-of-business for the previous U.S. business day. Instructions on downloading these files are available in Appendix H.

10.2 File names

To ensure compatibility with MS-DOS, file names use the "8.3" format: 8-character name and 3-character extension (see Table 10.1).

Table 10.1
RiskMetrics file names
"ddmmyy" indicates the date on which the market data was collected

File name format		
Volatility	**Correlation**	**File description**
DVddmmyy.RM3	DCddmmyy.RM3	1-day estimates
MVddmmyy.RM3	MCddmmyy.RM3	25-day estimates
BVddmmyy.RM3	BCddmmyy.RM3	Regulatory data sets
DVddmmyy.vol	DCddmmyy.cor	Add-In 1-day estimates
MVddmmyy.vol	MCddmmyy.cor	Add-In 25-day estimates
BVddmmyy.vol	BCddmmyy.cor	Add-In regulatory

The first two characters designate whether the file is daily (D) or monthly (M), and whether it contains volatility (V) or correlation (C) data. The next six characters identify the collection date of the market data for which the volatilities and correlations are computed. The extension identifies the version of the data set.

10.3 Data series naming standards

In both volatility and correlation files, all series names follow the same naming convention. They start with a three-letter code followed by a period and a suffix, for example, USD.R180.

The three-letter code is either a SWIFT[1] currency code or, in the case of commodities, a commodity code, as shown in Table 10.2. The suffix identifies the asset class (and the maturity for interest-rate and commodity series). Table 10.3 lists instrument suffix codes, followed by an example of how currency, commodity, and suffix codes are used.

[1] The exception is EMB. This represents J. P. Morgan's Emerging Markets Bond Index Plus.

Table 10.2
Currency and commodity identifiers

Currency Codes							
Americas		**Asia Pacific**		**Europe and Africa**		**Commodity Codes**	
ARS	Argentine peso	AUD	Australian dollar	ATS	Austrian shilling	ALU	Aluminum
CAD	Canadian dollar	HKD	Hong Kong dollar	BEF	Belgian franc	COP	Copper
MXN	Mexican peso	IDR	Indonesian rupiah	CHF	Swiss franc	GAS	Natural gas
USD	U.S. dollar	JPY	Japanese yen	DEM	Deutsche mark	GLD	Gold
EMB	EMBI+*	KRW	Korean won	DKK	Danish kroner	HTO	NY Harbor #2 heating oil
		MYR	Malaysian ringgit	ESP	Spanish peseta	NIC	Nickel
		NZD	New Zealand dollar	FIM	Finnish mark	PLA	Platinum
		PHP	Philippine peso	FRF	French franc	SLV	Silver
		SGD	Singapore dollar	GBP	Sterling	UNL	Unleaded gas
		THB	Thailand baht	IEP	Irish pound	WTI	Light Sweet Crude
		TWD	Taiwan dollar	ITL	Italian lira	ZNC	Zinc
				NLG	Dutch guilder		
				NOK	Norwegian kroner		
				PTE	Portuguese escudo		
				SEK	Swedish krona		
				XEU	ECU		
				ZAR	South African rand		

* EMBI+ stands for the J.P. Morgan Emerging Markets Bond Index Plus.

Table 10.3
Maturity and asset class identifiers

	Instrument Suffix Codes					
Maturity	**Foreign exchange**	**Equity indices**	**Money market**	**Swaps**	**Gov't bonds**	**Commodities**
Spot	XS	SE	–	–	–	C00
1m	–	–	R030	–	–	–
3m	–	–	R090	–	–	C03
6m	–	–	R180	–	–	C06
12m	–	–	R360	–	–	C12
15m	–	–	–	–	–	C15
18m	–	–	–	–	–	C18
24m (2y)	–	–	–	S02	Z02	C24
27m	–	–	–	–	–	C27
36m (3y)	–	–	–	S03	Z03	C36
4y	–	–	–	S04	Z04	–
5y	–	–	–	S05	Z05	–
7y	–	–	–	S07	Z07	–
9y	–	–	–	–	Z09	–
10y	–	–	–	S10	Z10	–
15y	–	–	–	–	Z15	–
20y	–	–	–	–	Z20	–
30y	–	–	–	–	Z30	–

For example, we identify the Singapore dollar foreign exchange rate by SGD.XS, the U.S. dollar 6-month money market rate by USD.R180, the CAC 40 index by FRF.SE, the 2-year sterling swap rate by GBP.S02, the 10-year Japanese government bond (JGB) by JPY.Z10, and the 3-month natural gas future by GAS.C03.

10.4 Format of volatility files

Each daily and monthly volatility file starts with a set of header lines that begin with an asterisk (*) and describe the contents of the file. Following the header lines are a set of record lines (without an asterisk) containing the daily or monthly data.

Table 10.4 shows a portion of a daily volatility file.

Table 10.4
Sample volatility file

Line #	Volatility file
1	*Estimate of volatilities for a one day horizon
2	*COLUMNS=2, LINES=418, DATE=11/14/96, VERSION 2.0
3	*RiskMetrics is based on but differs significantly from the market risk management systems
4	*developed by J.P. Morgan for its own use. J.P. Morgan does not warranty any results obtained
5	*from use of the RiskMetrics methodology, documentation or any information derived from
6	*the data (collectively the “Data”) and does not guarantee its sequence, timeliness, accuracy or
7	*completeness. J.P. Morgan may discontinue generating the Data at any time without any prior
8	*notice. The Data is calculated on the basis of the historical observations and should not be relied
9	*upon to predict future market movements. The Data is meant to be used with systems developed
10	*by third parties. J.P. Morgan does not guarantee the accuracy or quality of such systems.
11	*SERIES, PRICE/YIELD,DECAYFCTR,PRICEVOL,YIELDVOL
12	ATS.XS.VOLD,0.094150,0.940,0.554647,ND
13	AUD.XS.VOLD, 0.791600,0.940,0.643127,ND
14	BEF.XS.VOLD, 0.032152,0.940,0.546484,ND

In this table, each line is interpreted as follows:

- Line 1 identifies whether the file is a daily or monthly file.
- Line 2 lists file characteristics in the following order: the number of data columns, the number of record lines, the file creation date, and the version number of the file format.
- Lines 3–10 are a disclaimer.
- Line 11 contains comma-separated column titles under which the volatility data is listed.
- Lines 12 through the last line at the end of file (not shown) represent the record lines, which contain the comma-separated volatility data formatted as shown in Table 10.5.

Table 10.5
Data columns and format in volatility files

Column title (header line)	Data (record lines)	Format of volatility data
SERIES	Series name	See Section 10.3 for series naming conventions. In addition, each series name is given an extension, either ".VOLD" (for daily volatility estimate), or ".VOLM" (for monthly volatility estimate).
PRICE/YIELD	Price/Yield level	#.###### or "NM" if the data cannot be published.
DECAYFCTR	Exponential moving average decay factor	#.###
PRICEVOL	Price volatility estimate	#.###### (% units)
YIELDVOL	Yield volatility estimate	#.###### (% units) or "ND" if the series has no yield volatility (e.g., FX rates).

For example, in Table 10.4, the first value ATS.XS.VOLD in Line 12 corresponds to the SERIES column title, and identifies the series to be a USD/ATS daily volatility series. Similarly, the remaining values are interpreted as follows: The value 0.094150 was used as the price/yield level in the volatility calculation. The value 0.940 was used as the exponential moving average decay factor. The value 0.554647% is the price volatility estimate. The value "ND" indicates that the series has no yield volatility.

10.5 Format of correlation files

Daily and monthly correlation files are formatted similar to the volatility files (see Section 10.4), and contain analogous header and record lines (see Table 10.6). Each file comprises the lower half of the correlation matrix for the series being correlated, including the diagonal, which has a value of "1.000." (The upper half is not shown since the daily and monthly correlation matrices are symmetrical around the diagonal. For example, 3-month USD LIBOR to 3-month DEM LIBOR has the same correlation as 3-month DEM LIBOR to 3-month USD LIBOR.)

Table 10.6
Sample correlation file

Line #	Correlation file
1	*Estimate of correlations for a one day horizon
2	*COLUMNS=2, LINES=087571, DATE=11/14/96, VERSION 2.0
3	*RiskMetrics is based on but differs significantly from the market risk management systems
4	*developed by J.P. Morgan for its own use. J.P. Morgan does not warranty any results obtained
5	*from use of the RiskMetrics methodology, documentation or any information derived from
6	*the data (collectively the "Data") and does not guarantee its sequence, timeliness, accuracy or
7	*completeness. J.P. Morgan may discontinue generating the Data at any time without any prior
8	*notice. The Data is calculated on the basis of the historical observations and should not be relied
9	*upon to predict future market movements. The Data is meant to be used with systems developed
10	*by third parties. J.P. Morgan does not guarantee the accuracy or quality of such systems.
11	*SERIES, CORRELATION
12	ATS.XS.ATS.XS.CORD,1.000000
13	ATS.XS.AUD.XS.CORD, -0.251566
14	ATS.XS.BEF.XS.CORD, 0.985189

In Table 10.6, each line is interpreted as follows:

- Line 1 identifies whether the file is a daily or monthly file.
- Line 2 lists file characteristics in the following order: the number of data columns, the number of record lines, the file creation date, and the version number of the file format.
- Lines 3–10 are a disclaimer.
- Line 11 contains comma-separated column titles under which the correlation data is listed.
- Lines 12 through the last line at the end of the file (not shown) represent the record lines, which contain the comma-separated correlation data formatted as shown in Table 10.7.

Table 10.7
Data columns and format in correlation files

Column title (header line)	Correlation data (record lines)	Format of correlation data
SERIES	Series name	See Section 10.3 for series naming conventions. In addition, each series name is given an extension, either ".CORD" (for daily correlation), or ".CORM" (for monthly correlation).
CORRELATION	Correlation coefficient	#.###### Correlation coefficients are computed by using the same exponential moving average method as in the volatility files (i.e., decay factor of 0.940 for a 1-day horizon, and 0.970 for a 1-month horizon.)

For example, Line 13 in Table 10.6 represents a USD/ATS to USD/AUD daily correlation estimate of –0.251566 measured using an exponential moving average decay factor of 0.940 (the default value for the 1-day horizon).

10.6 Data series order

Data series in the volatility and correlation files are sorted first alphabetically by SWIFT code and commodity class indicator, and then by maturity within the following asset class hierarchy: foreign exchange, money markets, swaps, government bonds, equity indices, and commodities.

10.7 Underlying price/rate availability

Due to legal considerations, not all prices or yields are published in the volatility files. What is published are energy future contract prices and the yields on foreign exchange, swaps, and government bonds. The current level of money market yields can be approximated from Eq. [10.1] by using the published price volatilities and yield volatilities as well as the instruments' modified durations.

[10.1] $$\textit{Current yield} = \sigma_{\mathrm{Price}} / (\sigma_{\mathrm{Yield}} \cdot \textit{Modified Duration})$$

Part V

Backtesting

Chapter 11. Performance assessment

Chapter 11. Performance assessment

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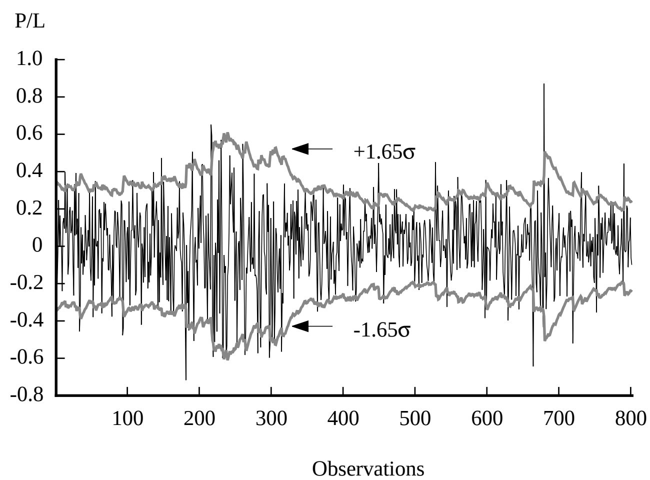
In this chapter we present a process for assessing the accuracy of the RiskMetrics model. We would like to make clear that the purpose of this section is not to offer a review of the quantitative measures for VaR model comparison. There is a growing literature on such measures and we refer the reader to Crnkovic and Drachman (1996) for the latest developments in that area. Instead, we present simple calculations that may prove useful for determining the appropriateness of the RiskMetrics model.

11.1 Sample portfolio

We describe an approach for assessing the RiskMetrics model by analyzing a portfolio consisting of 215 cashflows that include foreign exchange (22), money market deposits (22), zero coupon government bonds (121), equities (12) and commodities (33). Using daily prices for the period April 4, 1990 through March 26, 1996 (a total of 1001 observations), we construct 1-day VaR forecasts over the most recent 801 days of the sample period. We then compare these forecasts to their respective realized profit/loss (P/L) which are represented by 1-day returns.

Chart 11.1 shows the typical presentation of 1-day RiskMetrics VaR forecasts (90% two-tail confidence interval) along with the daily P/L of the portfolio.

Chart 11.1
One-day Profit/Loss and VaR estimates
VaR bands are given by +/– 1.65σ



In Chart 11.1 the black line represents the portfolio return $r_{p,t}$ constructed from the 215 individual returns at time t. The time t portfolio return is defined as follows:

$$[11.1] \qquad r_{p,t} = \sum_{i=1}^{215}\left(\frac{1}{215}\right) r_{i,t}$$

where $r_{i,t}$ represents the log return of the ith underlying cashflow. The Value-at-Risk bands are based on the portfolio's standard deviation. The formula for the portfolio's standard deviation, $\sigma_{p,t|t-1}$ is:

[11.2] $$\sigma_{P,t|t-1} = \sqrt{\sum_{i=1}^{215}\left(\frac{1}{215}\right)^2\sigma_{i,t|t-1}^2 + 2\sum_{i=1}^{215}\sum_{j>i}\left(\frac{1}{215}\right)^2\rho_{ij,t|t-1}\sigma_{i,t|t-1}\sigma_{j,t|t-1}}$$

where $\sigma_{i,t|t-1}^2$ is the variance of the *i*th return series made for time *t* and $\rho_{ij,t|t-1}$ is the correlation between the *i*th and *j*th returns for time *t*.

11.2 Assessing the RiskMetrics model

The first measure of model performance is a simple count the number of times that the VaR estimates "underpredict" future losses (gains). Recall that in RiskMetrics each day it is assumed that there is a 5% chance that the observed loss exceeds the VaR forecast.[1] For the sake of generality, let's define a random variable $X(t)$ on any day *t* such that $X(t) = 1$ if a particular day's observed loss is greater than its corresponding VaR forecast and $X(t)=0$ otherwise. We can write the distribution of $X(t)$ as follows

[11.3] $$f(X(t)|0.05) = \begin{cases} 0.05^{X(t)}(1-0.05)^{1-X(t)} & \text{X(t)=0,1} \\ 0 & \text{otherwise} \end{cases}$$

Now, suppose we observe $X(t)$ for a total of *T* days, $t = 1,2,...,T$, and we assume that the $X(t)$'s are independent over time. In other words, whether a VaR forecast is violated on a particular day is independent of what happened on other days. The random variable $X(t)$ is said to follow a Bernoulli distribution whose expected value is 0.05.The total number of VaR violations over the time period *T* is given by

[11.4] $$X_T = \sum_{t=1}^{T} X(t)$$

The expected value of X_T, i.e., the expected number of VaR violations over *T* days, is *T* times 0.05. For example, if we observe $T = 20$ days of VaR forecasts, then the expected number of VaR violations is 20 x 0.05 = 1; hence one would expect to observe one VaR violation every 20 days. What is convenient about modelling VaR violations according to Eq. [11.3] is that the probability of observing a VaR violation over *T* days is same as the probability of observing a VaR violation at any point in time, *t*. Therefore, we are able to use VaR forecasts constructed over time to assess the appropriateness of the RiskMetrics model for this portfolio of 215 cashflows.

Table 11.1 reports the observed percent of VaR violations for the upper and lower tails of our sample portfolio. For each day the lower and upper VaR limits are defined as $-1.65\sigma_{t|t-1}$ and $1.65\sigma_{t|t-1}$, respectively.

Table 11.1
Realized percentages of VaR violations
True probability of VaR violations = 5%

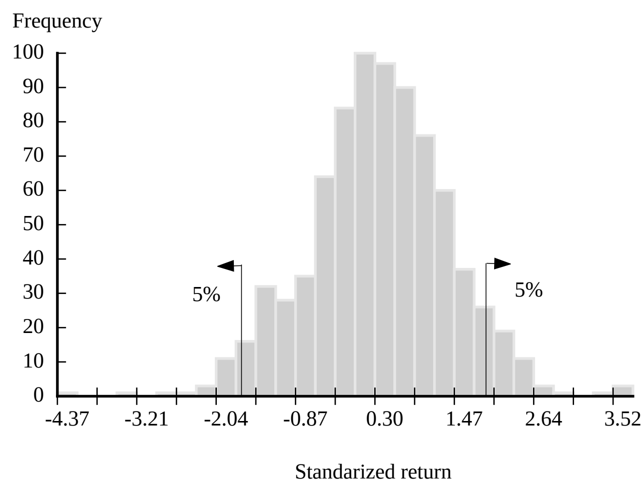
Prob (Loss < $-1.65\sigma_{t|t-1}$)	Prob (Profit > $1.65\sigma_{t|t-1}$)
5.74%	5.87%

A more straightforward approach to derive the preceding results is to apply the maintained assumptions of the RiskMetrics model. Recall that it is assumed that the return distribution of simple portfolios (i.e., those without nonlinear risk) is conditionally normal. In other words, the real-

[1] The focus of this section is on losses. However, the following methodology can also apply to gains.

ized return (P/L) divided by the standard deviation forecast used to construct the VaR estimate is assumed to be normally distributed with mean 0 and variance 1. Chart 11.2 presents a histogram of standardized portfolio returns. We place arrow bars to signify the area where we expect to observe 5% of the observations.

Chart 11.2
Histogram of standardized returns $(r_t/\sigma_{t|t-1})$
Probability that $(r_t/\sigma_{t|t-1})$ *< (>)–1.65 (1.65) = 5%*



A priori, the RiskMetrics model predicts that 5% of the standardized returns fall below (above) –1.65 (1.65). In addition to this prediction, it is possible to derive the expected value (average) of a return **given** that return violates a VaR forecast. For the lower tail, this expected value is defined as follows:

[11.5] $$E[(r_t/\sigma_{t|t-1}) \mid (r_t/\sigma_{t|t-1}) < -1.65] = -\left(\frac{\phi(-1.65)}{\Phi(-1.65)}\right) = -2.63$$

where

$\phi(-1.65)$ = the standard normal density function evaluated at -1.65
$\Phi(-1.65)$ = the standard normal distribution function evaluated at -1.65

It follows from the symmetry of the normal density function that the expected value for upper-tail returns is $E\left[(r_t/\sigma_{t|t-1}) \mid (r_t/\sigma_{t|t-1}) > 1.65\sigma_{t|t-1}\right] = 2.63$.

Table 11.2 reports these realized expected values for our sample portfolio.

Table 11.2
Realized "tail return" averages
Conditional mean tail forecasts of standardized returns

$E[(r_t/\sigma_{t|t-1}) \mid ((r_t/\sigma_{t|t-1}) < -1.65)] = -2.63$	$E[(r_t/\sigma_{t|t-1}) \mid (r_t/\sigma_{t|t-1}) > 1.65] = 2.63$
–1.741	1.828

To get a better understanding of the size of the returns that violate the VaR forecasts, Charts 11.3 and 11.4 plot the observed standardized returns (black circles) that fall in the lower (< −1.65) and upper (> 1.65) tails of the standard normal distribution. The horizontal line in each chart represents the average value predicted by the conditional normal distribution.

Chart 11.3
Standardized lower-tail returns
$r_t / \sigma_{t|t-1} < -1.65$

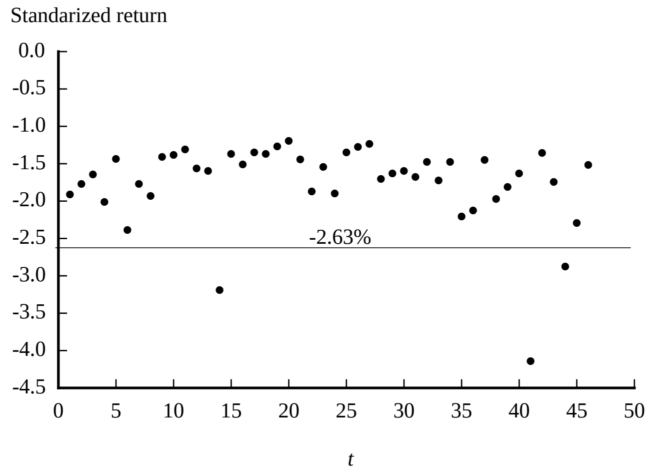
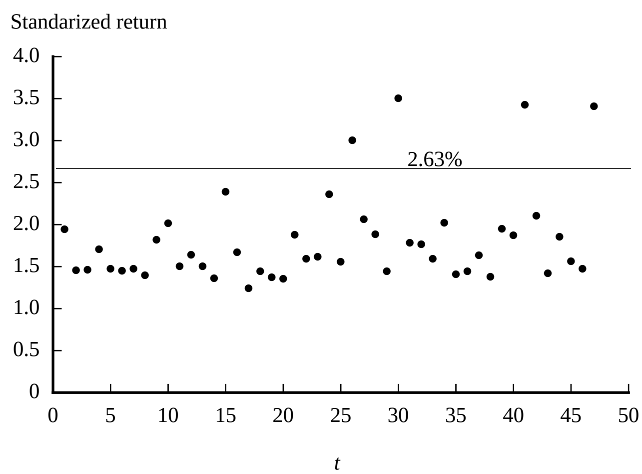


Chart 11.4
Standardized upper-tail returns
$r_t / \sigma_{t|t-1} > 1.65$



Both charts show that the returns that violate the VaR forecasts rarely exceed the expected value predicted by the normal distribution. In fact, we observe about 3 violations out of (approximately) 46/47 tail returns for the upper/lower tails. This is approximately 6.5% of the observations that fall in a particular tail. Note that the normal probability model prediction is 8.5%.[2]

[2] We derive this number from Prob ($X < -2.63$ | $X < -1.65$) = Prob ($X < -2.63$) / Prob ($X < -1.65$).

11.3 Summary

In this chapter we presented a brief process by which risk managers may assess the performance of the RiskMetrics model. We applied these statistics to a sample portfolio that consists of 215 cash-flows covering foreign exchange, fixed income, commodities and equities. Specifically, 1-day VaR forecasts were constructed for an 801-day sample period and for each day the forecast was measured against the portfolio's realized P/L. It was found that overall the RiskMetrics model performs reasonably well.

Appendices

Appendix A. Tests of conditional normality

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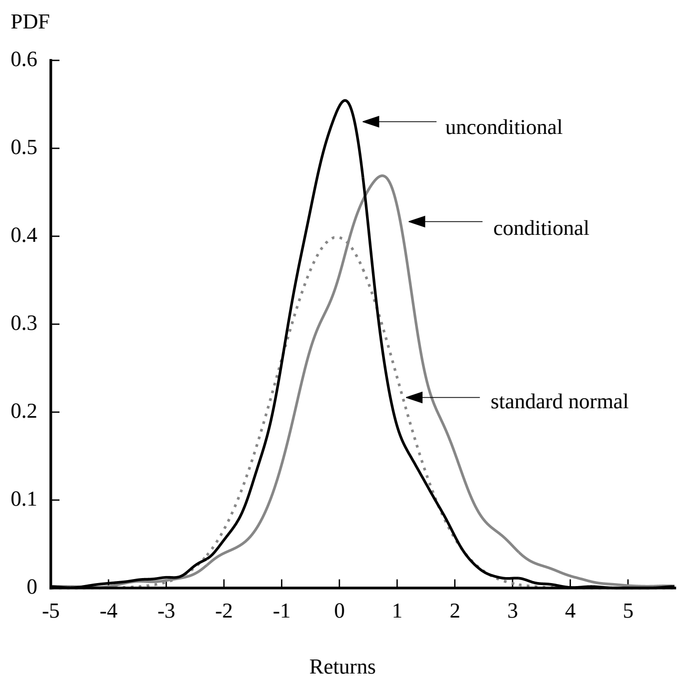
A fundamental assumption in RiskMetrics is that the underlying returns on financial prices are distributed according to the conditional normal distribution. The main implication of this assumption is that while the return distribution at each point in time is normally distributed, the return distribution taken over the entire sample period is **not necessarily** normal. Alternatively expressed, the standardized distribution rather than the observed return is assumed to be normal.

Chart A.1 shows the nontrivial consequence of the conditional normality assumption. The unconditional distribution represents an estimate of the histogram of USD/DEM log price changes that are standardized by the standard deviation taken over the entire sample (i.e., they are standardized by the unconditional standard deviation). As mentioned above, relative to the normal distribution with a constant mean and variance, this series has the typical thin waist, fat tail features. The unconditional distribution represents the distribution of standardized returns which are constructed by dividing each historical return by its corresponding standard deviation forecast[1], i.e., divide every return, r_t, by its standard deviation forecast, $\sigma_{t|t-1}$ (i.e., conditional standard deviation).

Chart A.1
Standard normal distribution and histogram of returns on USD/DEM



The difference between these two lines underscores the importance of distinguishing between conditional and unconditional normality.

[1] The exact construction of this forecast is presented in Chapter 5.

A.1 Numerical methods

We now present some computational tools used to test for normality. We begin by showing how to obtain sample estimates of the two parameters that describe the normal distribution. For a set of returns, r_t, where $t = 1,2\ldots,T$, we obtain estimates of the unconditional mean, $\bar{r}$, and standard deviation, $\hat{\sigma}$, via the following estimators:

[A.1] $$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

[A.2] $$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

Table A.1 presents sample estimates of the mean and standard deviation for the change series presented in Table 4.1.

Table A.1
Sample mean and standard deviation estimates for USD/DEM FX

Parameter estimates	Absolute price change	Relative price change	Log price change
$\bar{r}$, mean (%)	–0.060	–0.089	–0.090
$\hat{\sigma}$, standard deviation, (%)	0.28	0.42	0.42

Several popular tests for normality focus on measuring **skewness** and **kurtosis**. Skewness characterizes the asymmetry of a distribution around its mean. Positive skewness indicates an asymmetric tail extending toward positive values (right skewed). Negative skewness implies asymmetry toward negative values (left skewed). A simple measure of skewness, the coefficient of skewness, $\hat{\gamma}$, is given by

[A.3] $$\hat{\gamma} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^3$$

Computed values of skewness away from 0 point towards non-normality. Kurtosis characterizes the relative peakedness or flatness of a given distribution compared to a normal distribution. The standardized measure of kurtosis, the coefficient of kurtosis, $\hat{\kappa}$, is given by

[A.4] $$\hat{\kappa} = \left\{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^4\right\}$$

The kurtosis for the normal distribution is 3. Often, instead of kurtosis, researchers talk about excess kurtosis which is defined as kurtosis minus 3 so that in a normal distribution excess kurtosis is zero. Distributions with an excess kurtosis value greater than 0 are frequently referred to as having fat tails.

One popular test for normality that is based on skewness and kurtosis is presented in Kiefer and Salmon (1983). Shapiro and Wilk (1965) and Bera and Jarcque (1980) offer more computationally intensive tests. To give some idea about the values of the mean, standard deviation, skewness and kurtosis coefficients that are observed in practice, Table A.2 on page 230 presents estimates of these statistics as well as two other measures—tail probability and tail values, to 48 foreign exchange series. For each of the 48 time series we used 86 historical weekly prices for the period July 1, 1994 through March 1, 1996. (Note that many of the time series presented in Table A.2 are not part of the RiskMetrics data set). Each return used in the analysis is standardized by its corresponding 1-week standard deviation forecast. Interpretations of each of the estimated statistics are provided in the table footnotes.

When large data samples are available, specific statistics can be constructed to test whether a given sample is skewed or has excess kurtosis. This allows for formal hypothesis testing. The large sample skewness and kurtosis measures and their distributions are given below:

[A.5] Skewness measure $$\sqrt{T}\gamma \equiv \sqrt{T}\frac{\frac{1}{T}\sum_{t=1}^{T}(r_t-\bar{r})^3}{\left[\frac{1}{T}\sum_{t=1}^{T}(r_t-\bar{r})^2\right]^{\frac{3}{2}}} \sim N(0,6)$$

[A.6] Kurtosis measure $$\sqrt{T}\kappa \equiv \sqrt{T}\left\{\frac{\frac{1}{T}\sum_{t=1}^{T}(r_t-\bar{r})^4}{\left[\frac{1}{T}\sum_{t=1}^{T}(r_t-\bar{r})^2\right]^{2}}-3\right\} \sim N(0,24)$$

Table A.2
Testing for univariate conditional normality[1]
normalized return series; 85 total observations

	Skewness[2]	Kurtosis[3]	Mean[4]	Std. Dev.[5]	Tail Probability (%)[8] < –1.65	Tail Probability (%)[8] > 1.65	Tail value[9] < –1.65	Tail value[9] > 1.65
Normal	**0.000**	**0.000**	**-**	**1.000**	**5.000**	**5.000**	**–2.067**	**2.067**
OECD								
Australia	0.314	3.397	0.120	0.943	2.900	5.700	–2.586	2.306
Austria	0.369	0.673	–0.085	1.037	8.600	5.700	–1.975	2.499
Belgium	0.157	2.961	–0.089	0.866	8.600	2.900	–1.859	2.493
Denmark	0.650	4.399	–0.077	0.903	11.400	2.900	–1.915	2.576
France	0.068	3.557	–0.063	0.969	8.600	2.900	–2.140	2.852
Germany	0.096	4.453	–0.085	0.872	5.700	2.900	–1.821	2.703
Greece	0.098	2.259	–0.154	0.943	11.400	2.900	–1.971	2.658
Holland	0.067	4.567	–0.086	0.865	5.700	2.900	–1.834	2.671
Italy	0.480	0.019	0.101	0.763	0	2.900	0	1.853
New Zealand	1.746	7.829	0.068	1.075	2.900	2.900	–2.739	3.633
Portugal	1.747	0.533	–0.062	0.889	11.400	2.900	–1.909	2.188
Spain	6.995	1.680	–0.044	0.957	8.600	2.900	–2.293	1.845
Turkey	30.566	118.749	–0.761	1.162	11.400	0	–2.944	0
UK	7.035	2.762	–0.137	0.955	8.600	2.900	–2.516	1.811
Switzerland	0.009	0.001	–0.001	0.995	2.900	5.700	–2.415	2.110
Latin Amer. Econ. System								
Brazil	0.880	1.549	–0.224	0.282	0	0	0	0
Chile	1.049	0.512	–0.291	0.904	8.600	0	–2.057	0
Colombia	2.010	4.231	–0.536	1.289	11.400	2.900	–3.305	2.958
Costa Rica	0.093	33.360	–0.865	0.425	5.700	0	–2.011	0
Dominican Rep	0.026	41.011	0.050	1.183	5.700	5.700	–3.053	3.013
El Salvador	2.708	49.717	0.014	0.504	0	2.900	0	1.776
Equador	0.002	50.097	0.085	1.162	5.700	5.700	–3.053	3.013
Guatemala	0.026	1.946	–0.280	1.036	8.600	5.700	–2.365	2.237
Honduras	42.420	77.277	–0.575	1.415	14.300	0	–3.529	0
Jamaica	81.596	451.212	–0.301	1.137	2.900	2.900	–6.163	1.869
Mexico	13.71	30.237	–0.158	0.597	2.900	0	–2.500	0
Nicaragua	0.051	2.847	–0.508	0.117	0	0	0	0
Peru	122.807	672.453	–0.278	1.365	5.700	0	–5.069	0
Trinidad	0.813	0.339	0.146	1.063	8.600	11.400	–2.171	1.915
Uruguay	0.724	0.106	–0.625	0.371	0	0	0	0

Table A.2 (continued)
Testing for univariate conditional normality[1]
normalized return series; 85 total observations

	Skewness[2]	Kurtosis[3]	Mean[4]	Std. Dev.[5]	Tail Probability (%)[8] < –1.65	> 1.65	Tail value[9] < –1.65	> 1.65
ASEAN								
Malaysia	1.495	0.265	–0.318	0.926	8.600	0	–2.366	0
Philippines	1.654	0.494	–0.082	0.393	0	0	0	0
Thailand	0.077	0.069	–0.269	0.936	8.600	2.900	–2.184	1.955
Fiji	4.073	6.471	–0.129	0.868	2.900	2.900	–3.102	1.737
Hong Kong	5.360	29.084	0.032	1.001	5.700	5.700	–2.233	2.726
Reunion Island	0.068	3.558	–0.063	0.969	8.600	2.900	–2.140	2.853
Southern African Dev. Comm.								
Malawi	0.157	9.454	–0.001	0.250	0	0	0	0
South Africa	34.464	58.844	–0.333	1.555	8.600	0	–4.480	0
Zambia	22.686	39.073	–0.007	0.011	0	0	0	0
Zimbabwe	20.831	29.234	–0.487	0.762	5.700	0	–2.682	0
Ivory Coast	0.068	3.564	–0.064	0.970	8.600	2.900	–2.144	2.857
Uganda	40.815	80.115	–0.203	1.399	8.600	2.900	–4.092	1.953
Others								
China	80.314	567.012	0.107	1.521	2.900	2.900	–3.616	8.092
Czech Repub	0.167	12.516	–0.108	0.824	5.700	2.900	–2.088	2.619
Hungary	1.961	0.006	–0.342	0.741	5.700	0	–2.135	0
India	5.633	3.622	–0.462	1.336	17.100	5.700	–2.715	1.980
Romania	89.973	452.501	–1.249	1.721	14.300	0	–4.078	0
Russia	0.248	2.819	–0.120	0.369	0	0	0	0

1 Countries are grouped by major economic groupings as defined in *Political Handbook of the World: 1995–1996*. New York: CSA Publishing, State University of New York, 1996. Countries not formally part of an economic group are listed in their respective geographic areas.

2 If returns are conditionally normal, the skewness value is zero.

3 If returns are conditionally normal, the excess kurtosis value is zero.

4 Sample mean of the return series.

5 Sample standard deviation of the normalized return series.

8 Tail probabilities give the observed probabilities of normalized returns falling below −1.65 and above +1.65. Under conditional normality, these values are 5%.

9 Tail values give the observed average value of normalized returns falling below −1.65 and above +1.65. Under conditional normality, these values are −2.067 and +2.067, respectively.

A.2 Graphical methods

Q-Q (quantile-quantile) charts offer a visual assessment of the deviations from normality. Recall that the *q*th quantile is the number that exceeds *q* percent of the observations. A Q-Q chart plots the quantiles of the standardized distribution of observed returns (observed quantiles) against the quantiles of the standard normal distribution (normal quantiles). Consider the sample of observed returns, $r_t, t = 1, \ldots, T$. Denote the jth observed quantile by q_j so that for all T observed quantiles we have

[A.7] $$\text{Probability}\ (\tilde{r}_t < q_j) \cong p_j$$

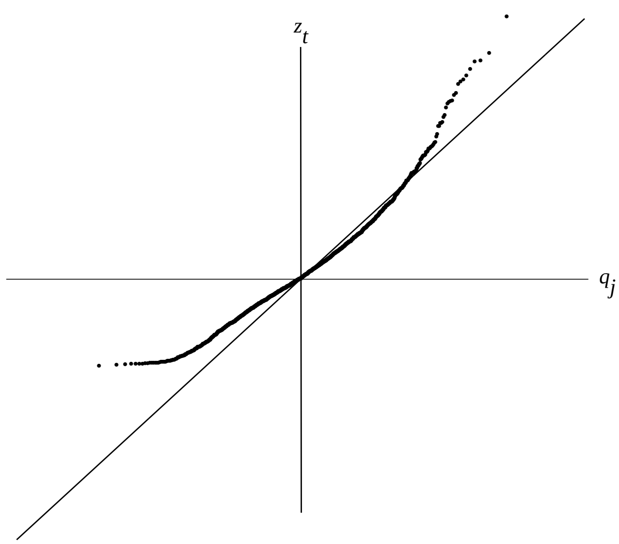
where $p_j = \dfrac{j - 0.5}{T}$

Denote the *j*th standard normal quantile by z_j for $j = 1\ldots,T$. For example, if $T = 100$, then $z_5 = -1.645$. In practice, the five steps to compute the Q-Q plot are given below:[2]

1. Standardize the daily returns by their corresponding standard deviation forecast, i.e., compute $\tilde{r}_t$ from r_t for $t = 1,\ldots,T$.
2. Order $\tilde{r}_t$ and compute their percentiles q_j, $j = 1,\ldots,T$.
3. Calculate the probabilities p_j corresponding to each q_j.
4. Calculate the standard normal quantiles, z_j that correspond to each p_j.
5. Plot the pairs $(z_1, q_1), (z_2, q_2), \ldots (z_T, q_T)$.

Chart A.2 shows an example of a Q-Q plot for USD/DEM daily standardized returns for the period January 1988 through September 1996.

Chart A.2
Quantile-quantile plot of USD/DEM
standardized returns



[2] For a complete description of this test see Johnson and Wichern (1992, pp. 153-158).

The straighter the plot, the closer the distribution of returns is to a normal distribution. If all points were to lie on a straight line, then the distribution of returns would be normal. As the chart above shows, there is some deviation from normality in the distribution of daily returns of USD/DEM over the last 7 years.

A good way to measure how much deviation from normality occurs is to calculate the correlation coefficient of the Q-Q plot,

[A.8]
$$\rho_Q = \frac{\sum_{j=1}^{T} (q_j - \bar{q})(z_j - \bar{z})}{\sqrt{\sum_{j=1}^{T} (q_j - \bar{q})^2} \sqrt{\sum_{j=1}^{T} (z_j - \bar{z})^2}}$$

For large sample sizes as in the USD/DEM example, ρ_Q needs to be at least 0.999 to pass a test of normality at the 5% significant.[3] In this example, $\rho_Q = 0.987$. The returns are not normal according to this test.

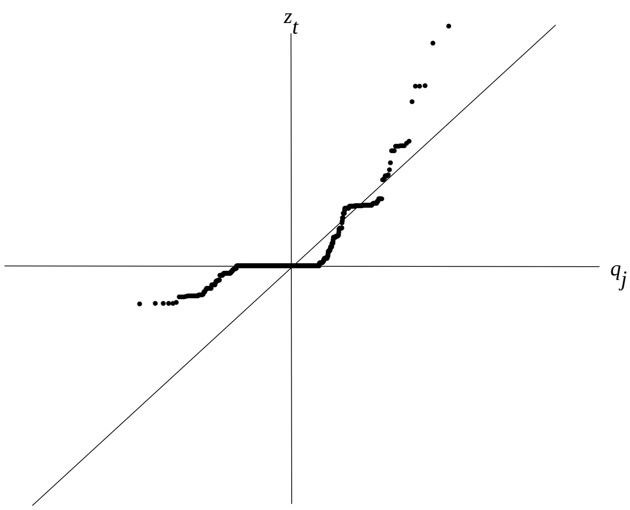
Used across asset classes, ρ_Q can provide useful information as to how good the univariate normality assumption approximates reality. In the example above, while the returns on the USD/DEM exchange rate are not normal, their deviation is slight.

Deviations from normality can be much more significant among other time series, especially money market rates. This is intuitively easy to understand. Short-term interest rates move in a discretionary fashion as a result of actions by central banks. Countries with exchange rate policies that have deviated significantly from economic fundamentals for some period often show money market rate distributions that are clearly not normal. As a result they either change very little when monetary policy remains unchanged (most of the time), or more significantly when central banks change policy, or the markets force them to do so. Therefore, the shape of the distribution results from discrete "jumps" in the underlying returns.

A typical example of this phenomenon can be seen from the Q-Q chart of standardized price returns on the 3-month sterling over the period 3-Jan-91 to 1-Sep-94. The ρ_Q calculated for that particular series is 0.907.

[3] See Johnson and Wichern (1992, p 158) for a table of critical values required to perform this test.

Chart A.3
Quantile-quantile plot of 3-month sterling
standardized returns



The Q-Q charts are useful because they allow the researcher a visual depiction of departures from normality. However, as stated before, there are several other tests for normality. It is important to remember that when applied directly to financial returns, conventional tests of normality should be used with caution. A reason is that the assumptions that underlie these tests (e.g., constant variance, nonautocorrelated returns) are often violated. For example, if a test for normality assumes that the data is not autocorrelated over the sample period when, in fact, the data are autocorrelated, then the test may incorrectly lead one to reject normality (Heuts and Rens, 1986).

The tests presented above are tests for univariate normality and not multivariate normality. In finance, tests of multivariate normality are often most relevant since the focus is on the return distribution of a portfolio that consists of a number of underlying securities. If each return series in a portfolio is found to be univariate normal, then the set of returns taken as a whole are still not necessarily multivariate normal. Conversely, if any one return series is found not to be univariate normal then multivariate normality can be ruled out. Recently, Richardson and Smith (1993) propose a direct test for multivariate normality in stock returns. Also, Looney (1995) describes test for univariate normality that can be used to determine to whether a data sample is multivariate normality.

Appendix B. Relaxing the assumption of conditional normality

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Since its release in October 1994, RiskMetrics has inspired an important discussion on VaR methodologies. A focal point of this discussion has been the assumption that returns follow a conditional normal distribution. Since the distributions of many observed financial return series have tails that are "fatter" than those implied by conditional normality, risk managers may underestimate the risk of their positions if they assume returns follow a conditional normal distribution. In other words, large financial returns are observed to occur more frequently than predicted by the conditional normal distribution. Therefore, it is important to be able to modify the current RiskMetrics model to account for the possibility of such large returns.

The purpose of this appendix is to describe two probability distributions that allow for a more realistic model of financial return tail distributions. It is organized as follows:

- Section B.1 reviews the fundamental assumptions behind the current RiskMetrics calculations, in particular, the assumption that returns follow a conditional normal distribution.

- Section B.2 presents the RiskMetrics model of returns under the assumption that the returns are conditionally normally distributed and two alternative models (distributions) where the probability of observing a return far away from the mean is relatively larger than the probability implied by the conditional normal distribution.

- Section B.3 explains how we estimate each of the three models and then presents results on forecasting the 1st and 99th percentiles of 15 return series representing 9 emerging markets.

B.1 A review of the implications of the conditional normality assumption

In a normal market environment RiskMetrics VaR forecasts are given by the bands of a confidence interval that is symmetric around zero. These bands represent the maximum change in the value of a portfolio with a specified level of probability. For example, the VaR bands associated with a 90% confidence interval are given by $\{-1.65\sigma_p, 1.65\sigma_p\}$ where –/+1.65 are the 5th/95th percentiles of the standardized normal distribution, and σ_p is the portfolio standard deviation which may depend on correlations between returns on individual instruments. The scale factors –/+ 1.65 result from the assumption that standardized returns (i.e., a mean centered return divided by its standard deviation) are normally distributed. When this assumption is true we expect 5% of the (standardized) realized returns to lie below –1.65 and 5% to lie above +1.65.

Often, whether complying with regulatory requirements or internal policy, risk managers compute VaR at different probability levels such as 95% and 98%. Under the assumption that returns are conditionally normal, the scale factors associated with these confidence intervals are –/+1.96 and –/+2.33, respectively. It is our experience that while RiskMetrics VaR estimates provide reasonable results for the 90% confidence interval, the methodology does not do as well at the 95% and 98% confidence levels.[1] Therefore, our goal is to extend the RiskMetrics model to provide better VaR estimates at these larger confidence levels.

Before we can build on the current RiskMetrics methodology, it is important to understand exactly what RiskMetrics assumes about the distribution of financial returns. RiskMetrics assumes that returns follow a conditional normal distribution. This means that while returns themselves are not normal, returns divided by their respective forecasted standard deviations are normally distributed with mean 0 and variance 1. For example, let r_t, denote the time t return, i.e., the return on an asset over a one-day period. Further, let σ_t denote the forecast of the standard deviation of returns for

[1] See Darryl Hendricks, "Evaluation of Value-at-Risk Models Using Historical Data," *FRBNY Economic Policy Review*, April, 1996.

time t based on historical data. It then follows from our assumptions that while r_t is not necessarily normal, the standardized return, r_t/σ_t, is normally distributed.

To summarize, RiskMetrics assumes that financial returns divided by their respective volatility forecasts are normally distributed with mean 0 and variance 1. This assumption is crucial because it recognizes that volatility changes over time.

B.2 Three models to produce daily VaR forecasts

In this section we present three models to forecast the distribution of one-day returns from which a VaR estimate will be derived.

- The first model that is discussed is referred to as standard RiskMetrics. This model is the basis for VaR calculations that are presented in the current *RiskMetrics—Technical Document*.
- The second model that we analyze was introduced in the 2nd quarter 1996 *RiskMetrics Monitor.* It is referred to in this appendix as the normal mixture model. The name "normal mixture" refers to the idea that returns are assumed to be generated from a mixture of two different normal distributions. Each day's return is assumed to be a draw from one of the two normal distributions with a particular probability.
- The third, and most sophisticated model that we present is known as RiskMetrics-GED. This model is the same as standard RiskMetrics except the returns in this model are assumed to follow a conditional generalized error distribution (GED). The GED is a very flexible distribution in that it can take on various shapes, including the normal distribution.

B.2.1 Standard RiskMetrics

The standard RiskMetrics model assumes that returns are generated as follows

[B.1]
$$\begin{aligned} r_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \end{aligned}$$

where

ε_t is a normally distributed random variable with mean 0 and variance 1

σ_t and $\left(\sigma_t^2\right)$, respectively, are the time *t* standard deviation and variance of returns (r_t)

λ is a parameter (decay factor) that regulates the weighting on past variances. For one-day variance forecasts, RiskMetrics sets λ =0.94.

In summary, the standard RiskMetrics model assumes that returns follow a conditional normal distribution—conditional on the standard deviation—where the variance of returns is a function of the previous day's variance forecast and squared return.

B.2.2 Normal mixture

In the second quarter 1996 *RiskMetrics Monitor* we introduced the normal mixture model of returns that was found to more effectively measure the tails of selected return distributions. In essence, this model allows for a larger probability of observing very large returns (positive or negative) than the conditional normal distribution.

The normal mixture model assumes that returns are generated as follows

[B.2] $$r_t = \sigma_{1,t} \cdot \varepsilon_{1,t} + \sigma_{1,t} \cdot \delta_t \cdot \varepsilon_{2,t}$$

where

r_t is the time t continuously compounded return

$\varepsilon_{1,t}$ is a normally distributed random variable with mean 0 and variance 1

$\varepsilon_{2,t}$ is a normally distributed random variable with mean $\mu_{2,t}$ and variance $\sigma^2_{2,t}$

δ_t is a 0/1 variable that takes the value 1 with probability *p* and 0 with probability 1–*p*

$\sigma_{1,t}$ is the standard deviation given in the RiskMetrics model

Alternatively stated, the normal mixture model assumes that daily returns standardized by the RiskMetrics volatility forecasts, $\tilde{r}_t$, are generated according to the model

[B.3] $$\tilde{r}_t = \varepsilon_{1,t} + \delta_t \cdot \varepsilon_{2,t}$$

Intuitively, we can think of Eq. [B.3] as representing a model where each day's standardized return is generated from one of two distributions:

1. If $\delta_t = 0$ then the standardized return is generated from a standard normal distribution, that is, a normal distribution with mean 0 and variance 1.
2. If $\delta_t = 1$ then the return is generated from a normal distribution with mean $\mu_{2,t}$ and variance $1 + \sigma^2_{2,t}$.

We can think of δ_t as a variable that signifies whether a return that is inconsistent with the standard normal distribution has occurred. The parameter p is the probability of observing such a return. It is important to remember that although the assumed mixture distribution is composed of normal distributions, the mixture distribution itself is not normal. Also, note that when constructing a VaR forecast, the normal mixture model applies the standard RiskMetrics volatility.

Chart B.1 shows the tails of two normal mixture models (and the standard normal distribution) for different values of $\mu_{2,t}$, and $\sigma_{2,t}$. Mixture(1) is the normal mixture model with parameter values set at $\mu_{2,t}$ =*-4,* $\sigma_{2,t}$ =*1,* p=*2%,* $\mu_{1,t}$ =0 $\sigma_{1,t}$ =1. Mixture(2) is the normal mixture model with the same parameter values as mixture(1) except now $\mu_{2,t}$ =*0,* $\sigma_{2,t}$ =*10.*

Chart B.1
Tails of normal mixture densities
Mixture(1) $\mu_{2,t} = -4$, $\sigma_{2,t} = 1$, $p = 2\%$, $\mu_{1,t} = 0$ $\sigma_{1,t} = 1$;
Mixture(2) $\mu_{2,t} = 0$, $\sigma_{2,t} = 10$, $p = 2\%$, $\mu_{1,t} = 0$ $\sigma_{1,t} = 1$

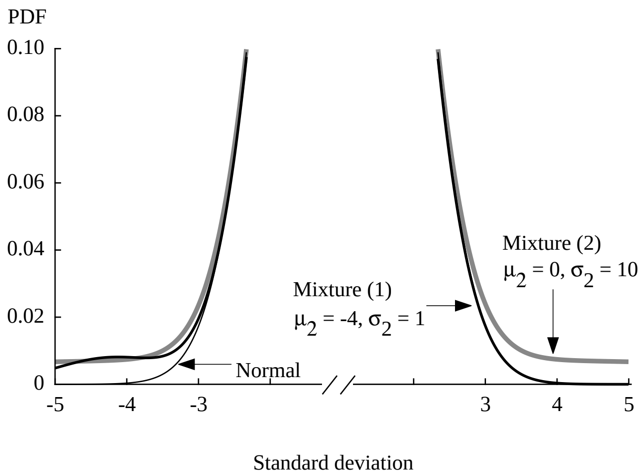


Chart B.1 shows that when there is a large negative mean for one of the normal distributions as in mixture(1), this translates into a larger probability of observing a large negative return relative to the standard normal distribution. Also, as in the case of mixture (2) we can construct a probability distribution with thicker tails than the standard normal distribution by mixing the standard normal with a normal distribution with a large standard deviation.

B.2.3 RiskMetrics-GED

According to this model, returns are generated as follows

$$\text{[B.4]} \qquad \begin{aligned} r_t &= \sigma_t \xi_t \\ \sigma_t^2 &= \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \end{aligned}$$

where

r_t is the time t continuously compounded return

ξ_t is a random variable distributed according to the GED (generalized error distribution) with parameter ν. As will be shown below, ν regulates the shape of the GED distribution.

σ_t^2 is the time t variance of returns (r_t)

The random variable (ξ_t) in Eq. [B.4] is assumed to follow a generalized error distribution (GED). This distribution is quite popular among researchers in finance because of the variety of shapes the GED can take. The probability density function for the GED is

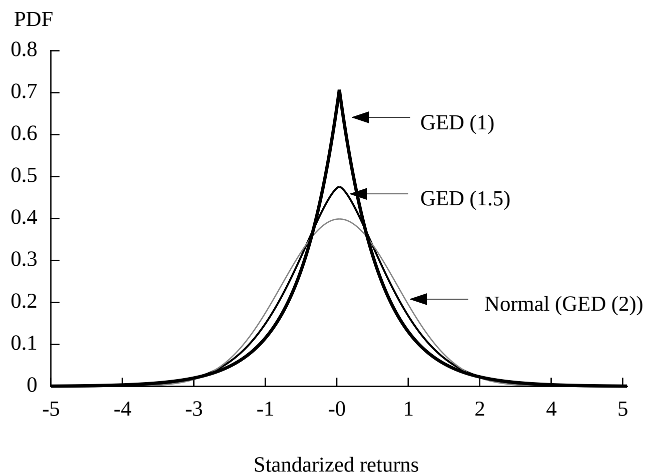
[B.5] $$f(\xi_t) = \frac{\nu \exp\left(-\frac{1}{2}|\xi_t/\lambda|^{\nu}\right)}{\lambda 2^{(1+\nu^{-1})}\Gamma\left(\nu^{-1}\right)}$$

where Γ is the gamma function and

[B.6] $$\lambda = \left[2^{-(2/\nu)}\Gamma(1/\nu)/(3/\nu)\right]^{1/2}$$

When $\nu = 2$ this produces a normal density while ν >(<)2 is more thin (flat) tailed than a normal. Chart B.2 shows the shape of the GED distribution for values of ν = 1, 1.5 and 2.

Chart B.2
GED distribution
ν *=1, 1.5 and 2*



Notice that when the parameter of the GED distribution is below 2 (normal), the result is a distribution with greater likelihood of very small returns (around 0) and a relatively large probability of returns far away from the mean. To better understand the effect that the parameter ν has on the tails of the GED distribution, Chart B.3 plots the left (lower) tail of the GED distribution when ν =1, 1.5 and 2.

Chart B.3
Left tail of GED (ν) distribution
ν = 1, 1.5, and 2

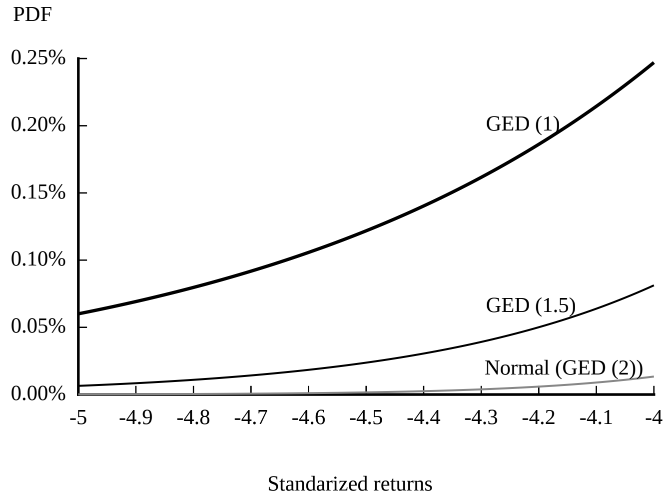


Chart B.3 shows that as ν becomes smaller, away from 2 (normal), there is more probability placed on relatively large negative returns.

B.3 Applying the models to emerging market currencies and equity indices

We applied the three models described above to 15 time series representing 9 emerging market countries to determine how well each model performs at estimating the 1st and 99th percentiles of the return distributions. The time series cover foreign exchange and equity indices. In order to facilitate our exposition of the process by which we fit each of the models and tabulate the results on forecasting the percentiles, we focus on one specific time series, the South African rand.

B.3.1 Model estimation and assessment

We first fit each model to 1152 returns on each of the 15 time series for the period May 25, 1992 through October 23, 1996. Table B.1 shows the parameter estimates from each of the three models for the South African rand.

Table B.1
Parameter estimates for the South African rand

Normal Mixture		**Standard RiskMetrics**		**RiskMetrics-GED**	
Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
$\mu_{2,t}$	−5.086	λ	0.94	ν	0.927
$\sigma_{2,t}$	9.087				
p	0.010				
$\sigma_{1,t}$	1.288				

Table B.1 points to some interesting results:

- In the RiskMetrics-GED model, the estimate of ν implies that the distribution of returns on the rand are much thicker than the normal distribution (recall that ν =2 is a normal distribution). In other words, we are much more likely to observe a return that is far away from the mean return than is implied by the normal distribution.

- In the normal mixture model there is a 1% chance of observing a normally distributed return with a mean –5 and standard deviation 9 and a 99% chance of observing a normally distributed return with mean 0 and standard deviation 1.288.

- The RiskMetrics optimal decay factor for the South African rand is 0.940. This decay factor was found by minimizing the root mean squared error of volatility forecasts. Coincidentally, this happens to be the same decay factor applied to all times series in RiskMetrics when estimating one-day volatility.

If a volatility model such as RiskMetrics fits the data well its standardized returns (i.e., the returns divided by their volatility forecast) should have a volatility of 1. Table B.2 presents four sample statistics—mean, standard deviation, skewness and kurtosis—for the standard RiskMetrics model and estimates of ν for the RiskMetrics-GED model. Recall that skewness is a measure of a distribution's symmetry. A value of 0 implies that the distribution is symmetric. Kurtosis measures a distribution's "tail thickness". For example, since the kurtosis for a normal distribution is 3, values of kurtosis greater than 3 indicate that there is a greater likelihood of observing returns that are far away from the mean return than implied by the normal distribution.

Table B.2
Sample statistics on standardized returns
Standard RiskMetrics model

Instrument type	Source	Mean	Std dev	Skewness	Kurtosis	GED parameter, ν
Foreign exchange	Mexico	0.033	3.520	–21.744	553.035	0.749
	Philippines	–0.061	1.725	–13.865	327.377	0.368
	Taiwan	0.069	1.720	8.200	162.234	0.492
	Argentina	0.028	1.177	5.672	112.230	0.219
	Indonesia	–0.013	1.081	–1.410	12.314	0.460
	Korea	–0.013	1.106	–1.142	10.188	0.778
	Malaysia	0.029	1.210	–0.589	12.488	0.908
	South Africa	0.040	1.291	–6.514	116.452	0.927
	Thailand	–0.004	1.003	0.168	4.865	1.101
Equity	Argentina	0.043	1.007	–0.376	3.817	1.221
	Indonesia	0.020	1.085	1.069	12.436	0.868
	Malaysia	0.002	1.130	0.346	5.966	1.023
	Mexico	0.007	1.046	0.042	4.389	0.798
	South Africa	0.027	1.023	0.081	5.412	1.136
	Thailand	–0.019	1.056	–0.008	5.014	0.999

Under the maintained assumption of the RiskMetrics model the statistics of the standardized returns should be as follows; mean = 0, standard deviation = 1, skewness = 0, kurtosis = 3. Table B.2 shows that except for Mexico, Philippines and Taiwan foreign exchange, standard RiskMetrics does a good job at recovering the standard deviation. The fact that kurtosis for many of the time series are well above three signifies that the tails of these return distributions are much larger than the normal distribution.

Also, note the estimates of ν produced from RiskMetrics-GED. Remember that if the distribution of the standardized returns is normal, $\nu = 2$ and values of $\nu < 2$ signify that the distribution has thicker tails than that implied by the normal distribution. The fact that all of the estimates of ν are well below 2 indicate that these series contain a relatively large number of returns (negative and positive).

B.3.2 VaR analysis

In this section we report the results of an experiment to determine how well each of the models described above can predict the 1st and 99th percentiles of the 15 return distribution. These results are provided in Table B.3.

Our analysis consisted of the following steps:

- First, we estimate the parameters in each of the three models using price data from May, 25, 1992 through October 23, 1996. This sample consists of 1152 historical returns on each of the 15 time series.
- Second, we construct one-day volatility estimates for each of the three models using the most recent 952 returns.
- Third, we use the 952 volatility estimates and the three probability distributions (normal, mixture normal and GED) evaluated at the parameter estimates to construct VaR forecasts at the 1st and 99th percentiles.
- Fourth, we count the number of times the next day realized return exceeds each of the VaR forecasts. This number is then converted to a percentage by dividing it by the total number of trials—952 in this experiment. The "ideal" model would yield percentages of 1%.

Table B.3 presents these percentages for the three models.

Table B.3
VaR statistics (in %) for the 1st and 99th percentiles
RGD = RiskMetrics-GED; RM = RiskMetrics; MX = Normal mixture

		1st percentile (1%)			99th percentile (99%)		
Instrument type	**Source**	**RGD**	**RM**	**MX**	**RGD**	**RM**	**MX**
Foreign exchange	Mexico	1.477	2.346	0.434	1.043	1.998	0.434
	Philippines	1.390	2.520	1.043	1.216	1.998	1.043
	Taiwan	0.956	1.651	0.782	1.043	1.911	0.782
	Argentina	1.998	1.998	1.303	2.172	2.172	1.39
	Indonesia	1.651	3.562	1.651	1.129	1.998	1.411
	Korea	1.216	2.433	1.303	0.521	1.303	0.956
	Malaysia	1.564	2.433	1.477	1.911	3.215	1.911
	South Africa	1.390	1.998	1.216	1.129	1.998	1.303
	Thailand	0.695	1.129	1.043	1.651	2.172	1.072
Equity	Argentina	1.825	2.520	1.646	0.869	1.129	1.129
	Indonesia	0.608	1.998	1.564	1.564	2.520	1.564
	Malaysia	1.651	2.433	1.698	1.651	2.693	1.738
	Mexico	0.956	2.259	1.738	1.043	1.651	1.303
	South Africa	1.216	2.172	1.651	1.477	2.085	1.738
	Thailand	1.129	1.564	1.190	0.869	2.346	1.611
	Column average	1.315	2.201	1.396	1.286	2.060	1.270

Table B.3 shows that for the RiskMetrics-GED model the VaR forecasts at the 1st percentile are exceeded 1.315 percent of the time whereas the VaR forecasts at the 99th percentile are exceeded 1.286% of the time. Similarly, the VaR forecasts produced from the mixture model are exceeded at the 1st and 99th percentiles by 1.396% and 1.270% of the realized returns, respectively. Both models are marked improvements over the standard RiskMetrics model that assumes conditional normality.

Appendix C. Methods for determining the optimal decay factor

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In this appendix we present alternative measures to assess forecast accuracy of volatility and correlation forecasts.

C.1 Normal likelihood (LKHD) criterion

Under the assumption that returns are conditionally normal, the objective here is to specify the joint probability density of returns given a value of the decay factor. For the return on day t this can be written as:

[C.1] $$f(r_t|\lambda) = \left(\frac{1}{\sqrt{2\pi}\sigma_{t|t-1}(\lambda)}\right)\exp-\left[\frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right]$$

Combining the conditional distributions from all the days in history for which we have data, we get:

[C.2] $$f(r_1, \ldots, r_T|\lambda) = \left\{\prod_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi}\sigma_{t|t-1}(\lambda)}\right)\exp\left[-\frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right]\right\}$$

Equation [C.2] is known as the normal likelihood function. Its value depends on λ. In practice, it is often easier to work with the log-likelihood function which is simply the natural logarithm of the likelihood function.

The maximum likelihood (ML) principle stipulates that the optimal value of the decay factor λ is one which maximizes the likelihood function Eq. [C.2]. With some algebra, it can be shown that this is equivalent to finding the value of λ that minimizes the following function:

[C.3] $$LKLHD_v = \sum_{t=1}^{T}\left\{\ln\left[\sigma_{t|t-1}(\lambda)\right] + \frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right\}$$

Notice that the criterion Eq. [C.3] imposes the assumption that returns are distributed conditionally normal when determining the optimal value of λ. The RMSE criterion, on the other hand, does not impose any probability assumptions in the determination of the optimal value of λ.

C.2 Other measures

In addition to the RMSE and Normal likelihood measures alternative measures could also be applied such as the mean absolute error measure for the variance

[C.4] $$MAE_v = \frac{1}{T}\sum_{t=1}^{T}\left|r_{t+1}^2 - \hat{\sigma}^2_{t+1|t}\right|$$

For individual cashflows, RiskMetrics VaR forecasts are based on standard deviations. Therefore, we may wish to measure the error in the standard deviation forecast rather than the variance forecast. If we take as a proxy for the one period ahead standard deviation, $|r_t|$, then we can define the RMSE of the standard deviation forecast as

[C.5] $$RMSE_{\sigma} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\left|r_{t+1}\right| - \hat{\sigma}_{t+1|t}\right)^2}$$

Notice in Eq. [C.5] that $E_t\left|r_{t+1}\right| \neq \sigma_{t+1}$. In fact for the normal distribution, the following equation holds: $E_t\left|r_{t+1}\right| = (2/\pi)^{-1/2}\sigma_{t+1}$.

Other ways of choosing optimal λ include the Q-statistic described by Crnkovic and Drachman (RISK, September, 1996) and, under the assumption that returns are normally distributed, a likelihood ratio test that is based on the normal probability density likelihood function.

C.3 Measures for choosing an optimal decay factor for multiple time series.

In Chapter 5, we explained how an optimal decay factor for the 480 RiskMetrics time series was chosen. This method involved finding optimal decay factors for each series, and then taking a weighted average of these factors, with those factors which provided superior performance in forecasting volatility receiving the greatest weight. In this section, we briefly describe some alternative methods which account for the performance of the correlation forecasts as well.

The first such method is an extension of the likelihood criterion to a multivariate setting. If we consider a collection of *n* assets whose returns on day *t* are represented by the vector $\vec{r}_t$, then the joint probability density for these returns is

[C.6] $$f\left(\vec{r}_t\middle|\lambda\right) = \left(\frac{1}{(2\pi)^{\frac{n}{2}}\left|\Sigma_{t|t-1}(\lambda)\right|^{\frac{1}{2}}}\right)\exp-\left[\frac{1}{2}\left(\vec{r}^{\,T}_t\Sigma_{t|t-1}(\lambda)^{-1}\vec{r}_t\right)\right],$$

where $\Sigma_{t|t-1}(\lambda)$ is the matrix representing the forecasted covariance of returns on day *t* using decay factor λ. The likelihood for the returns for all of the days in our data set may be constructed analogously to Eq. [C.2]. Using the same reasoning as above, it can be shown that the value of λ which maximizes this likelihood is the one which also maximizes

[C.7] $$LKLHD_v = \sum_{t=1}^{T}\left\{\ln\left[\left|\Sigma_{t|t-1}(\lambda)\right|\right] + \vec{r}^{\,T}_t\Sigma_{t|t-1}(\lambda)^{-1}\vec{r}_t\right\}.$$

As noted before, choosing the decay factor according to this criterion imposes the assumption of conditional normality. In addition, to evaluate the likelihood function in Eq. [C.7], it is necessary at each time to invert the estimated covariance matrix $\Sigma_{t|t-1}(\lambda)$. In theory, this matrix will always be invertible, although in practice, due to limited precision calculations, there will likely be cases where the inversion is impossible, and the likelihood function cannot be computed.

A second approach is a generalization of the RMSE criterion for the covariance forecasts. Recall from Chapter 5 that the covariance forecast error on day *t* for the *i*th and *j*th returns is

[C.8] $$\varepsilon_{ij,t|t-1}(\lambda) = r_{i,t}r_{j,t} - \Sigma_{ij,t|t-1}(\lambda).$$

(Recall also that under the RiskMetrics assumptions, $E_{t-1}\left[\varepsilon_{ij,t|t-1}\right] = 0$.) The total squared error for day *t* is then obtained by summing the above over all pairs (i, j), and the mean total squared error (MTSE) for the entire data set is then

[C.9] $$MTSE = \frac{1}{T}\sum_{t=1}^{T}\sum_{i,j}\varepsilon_{ij,\,t|t-1}(\lambda)^2.$$

The value of λ which minimizes the MTSE above can be thought of as the decay factor which historically has given the best covariance forecasts across all of the data series.

The above description presents a myriad of choices faced by the researcher when determining "optimal λ". The simple answer is that there is no clear-cut, simple way of choosing the optimal prediction criterion. There has been an extensive discussion among academics and practitioners on what error measure to use when assessing post-sample prediction.[1] Ultimately, the forecasting criterion should be motivated by the modeler's objective. For example, West, Edison and Cho (1993) note "an appropriate measure of performance depends on the use to which one puts the estimates of volatility…." Recently, Diebold and Mariano (1995) remind us, "of great importance and almost always ignored, is the fact that the economic loss associated with a forecast may be poorly assessed by the usual statistical measures. That is, forecasts are used to guide decisions, and the loss associated with a forecast error of a particular sign and size induced directly by the nature of the decision problem at hand." In fact, Leitch and Tanner (1991) use profitability rather than size of the forecast error or its squared value as a test of forecast accuracy.

[1] For a comprehensive discussion on various statistical error measures (including the RMSE) to assess forecasting methods, see the following:

Ahlburg, D.
Armstrong, J. S., and Collopy, F.
Fildes, R.

in the *International Journal of Forecasting*, 8, 1992, pp. 69–111.

Appendix D. Assessing the accuracy of the delta-gamma approach

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In this appendix we compare the VaR forecasts of the delta-gamma approach to those produced by full simulation. Before doing so, however, we investigate briefly when the delta-gamma approach is expected to perform poorly in relation to full simulation.

The accuracy of the delta-gamma approach depends on the accuracy of the approximation used to derive the return on the option. The expression for the option's return is derived using what is known as a "Taylor series expansion." We now present the derivation.

[D.1] $$V_{t+n} \approx V_t + \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

This expression can be rewritten as follows:

[D.2] $$V_{t+n} - V_t \approx \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

We now express the changes in the value of the option and the underlying in relative terms:

[D.3] $$V_t \cdot \left(\frac{V_{t+n} - V_t}{V_t}\right) = \delta \cdot P_t \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right) + 0.5 \cdot \Gamma \cdot P_t^2 \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

Dividing Eq. [D.3] by P_t, we get

[D.4] $$\left(\frac{V_t}{P_t}\right) \cdot \left(\frac{V_{t+n} - V_t}{V_t}\right) = \delta \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right) + 0.5 \cdot \Gamma \cdot P_t \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right)^2 + \left(\frac{\theta}{P_t}\right) \cdot (\tau_{t+n} - \tau_t)$$

and define the following terms:

$$R_V = \left(\frac{V_{t+n} - V_t}{V_t}\right),\ R_P = \left(\frac{P_{t+n} - P_t}{P_t}\right),\ n = (\tau_{t+n} - \tau_t)\text{ , and }\eta = \left(\frac{P_t}{V_t}\right)$$

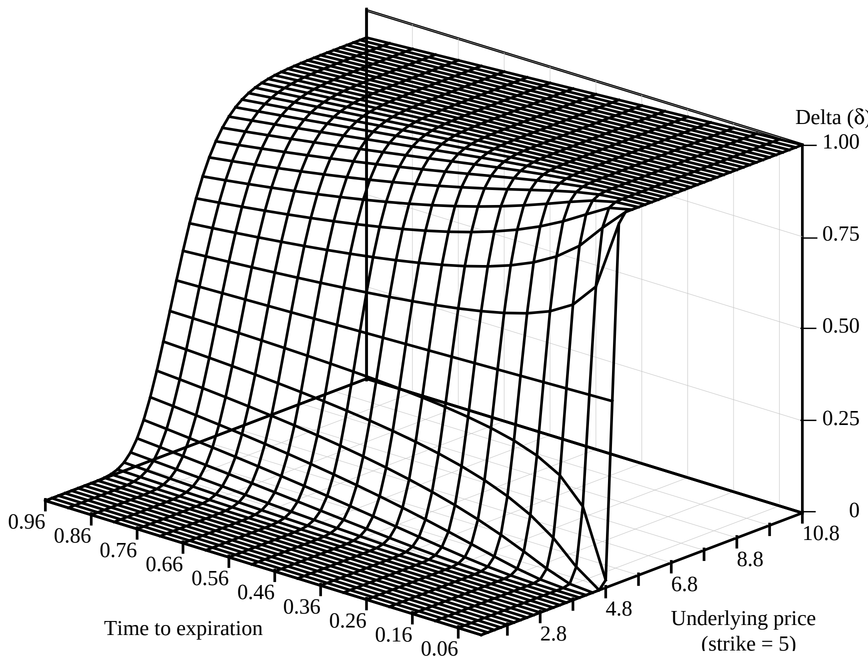
We can now write the return on the option as follows:

[D.5] $$\begin{aligned} R_V &= \eta \delta R_P + 0.5\,(\alpha \Gamma P_t)\,(R_P)^2 + \left(\frac{\theta}{V_t}\right) n \\ &= \tilde{\delta} R_P + 0.5 \tilde{\Gamma}\,(R_P)^2 + \tilde{\theta}\,(\tau_{t+n} - \tau_t) \end{aligned}$$

This expansion is a reasonable approximation when the "greeks" δ and Γ are stable as the underlying price changes. In our example, the underlying price is the US dollar/deutschemark exchange rate. If changes in the underlying price causes large changes in these parameters then we should not expect the delta-gamma approach to perform well.

Chart D.1 shows the changes in the value of delta (δ) when the underlying price and the time to the option's expiry both change. This example assumes that the option has a strike price of 5.

Chart D.1
Delta vs. time to expiration and underlying price

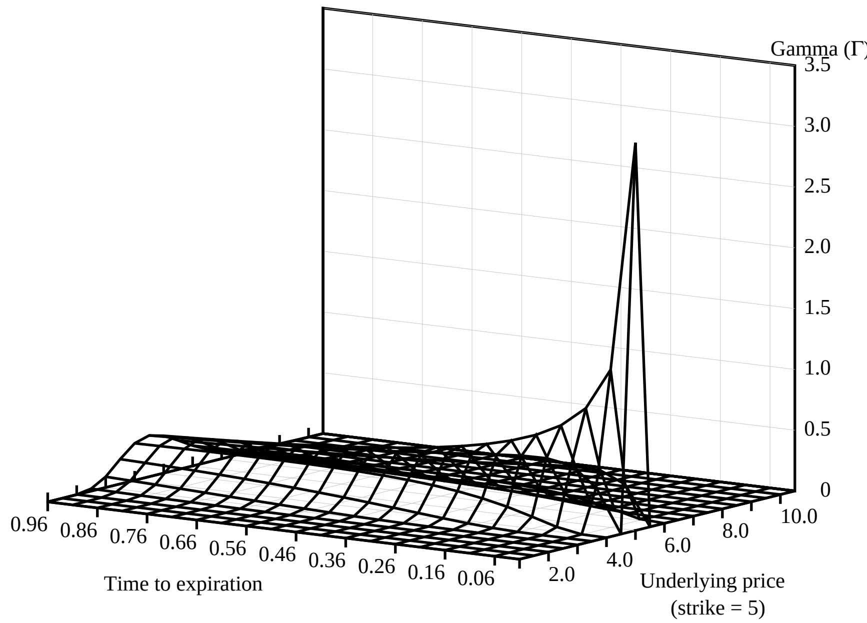


Notice that large changes in delta occur when the current price in the underlying instrument is near the strike. In other words, we should expect to see large changes in delta for small changes in the underlying price when the option is exactly, or close to being, an at-the-money option.

Since the delta and gamma components of an option are closely related, we should expect a similar relationship between the current underlying price and the gamma of the option. For the same option, Chart D.2 presents values of gamma as the underlying price and the time to expiry both change.

The chart shows that gamma changes abruptly when the option is near to being an at-the-money option and the time to expiry is close to zero.

Chart D.2
Gamma vs. time to expiration and underlying price



Together, Charts D.1 and D.2 demonstrate that we should expect the delta-gamma method to do most poorly when portfolios contain options that are close to being at-the-money and the time to expiry is short (about one week or less).

D.1 Comparing full simulation and delta-gamma: A Monte Carlo study

In this section we describe an experiment undertaken to determine the difference in VaR forecasts produced by the full simulation and delta-gamma methodologies. The study focuses on one call option. (For more complete results, see the third quarter 1996 *RiskMetrics Monitor.*) VaR forecasts, defined as the 5th percentile of the distribution of future changes in the value of the option, were made over horizons of one day. The Black-Scholes formula was used to both revalue the option and to derive the "greeks."

We set the parameters used to value the option, determine the "greeks", and generate future prices (for full simulation) as shown in Table D.1.

Table D.1
Parameters used in option valuation

Parameter	Value
Strike price (K)	5.0
Standard deviation (annualized)	23.0%
Risk-free interest rate	8.0%

Given these parameter settings we generate a series of underlying spot prices, P_t, with values 4.5, 4.6, 4.7,..., 5.6. Here the time t subscript denotes the time the VaR forecast is made. These spot prices imply a set of ratios of spot-to-strike price, P_t/K, that define the "moneyness" of the option. The values of P_t/K are 0.90, 0.92,0.94,...,1.12. In addition, we generate a set of time to expirations, τ, (expressed in years) for the option. Values of τ range from 1 day (0.004) to 1 year (1.0).

In full simulation, we are required to simulate future prices of the underlying instrument. Denote the future price of the underlying instrument by P_{t+n} where n denotes the VaR forecast horizon (i.e., n = 1 day, 1 week, 1 month and 3 months). We simulate underlying prices at time $t+n$, P_{t+n}, according to the density for a lognormal random variable

[D.6] $$P_{t+n} = P_t e^{((r - \sigma^2/2) + z\sigma\sqrt{n})}$$

where z is a standard unit normal random variable.

In full simulation, VaR is defined as the difference between the value of the option at time $t + n$ (the forecast horizon) and today, time t. This means that all instruments are revalued.

[D.7] $$\text{Exact} = BS(P_{t+n}, t+n) - BS(P_t, t) ,$$

where *BS()* stands for the Black-Scholes formula.

We use the term "Exact" to represent the fact that the option is being revalued using its exact option pricing formula. In the delta-gamma approach, VaR is approximated in terms of the Taylor series expansion discussed earlier:

[D.8] $$\text{Approx} = \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot n$$

Here, the term "Approx" denotes the approximation involved in using only the delta, gamma and theta components of the option. To compare VaR forecasts we define the statistics VaR_E and VaR_A as follows:

VaR_E = the 5th percentile of the Exact distribution which represents full simulation.

VaR_A = the 5th percentile of the Approx distribution which represents delta-gamma.

For a given spot price, P_t, time to expiration, τ, and VaR forecast horizon, n, we generate 5,000 future prices, P_{t+n}, and calculate VaR_E and VaR_A. This experiment is then repeated 50 times to produce 50 VaR_E's and VaR_A's. We then measure the difference in these VaR forecasts by computing two metrics:

[D.9] $$MAPE = \frac{1}{50}\sum_{i=1}^{50}\left|\frac{\left(VaR_A^i - VaR_E^i\right)}{VaR_E^i}\right| \quad \text{(Mean Absolute Percentage Error)}$$

[D.10] $$ME = \frac{1}{50}\sum_{i=1}^{50}\left(VaR_A^i - VaR_E^i\right) \quad \text{(Mean Error)}$$

Tables D.2 and D.3 report the results of this experiment. Specifically, Tables D.2 and D.3 show, respectively, the mean absolute percentage error (MAPE) and the mean error (ME) for a call option for a one-day forecast horizon. Each row of a table corresponds to a different time to expiration (maturity). Time to expiration is measured as a fraction of a year (e.g., 1 day = 1/250 or 0.004) and cannot be less than the VaR forecast horizon which is one day. Each column represents a ratio of the price of the underlying when the VaR forecast was made (spot) to the option's strike price, P_t/K. This ratio represents the option's "moneyness" at the time VaR was computed. All entries greater than or equal to 10 percent are reported without decimal places.

Table D.2
MAPE (%) for call, 1-day forecast horizon

Time to maturity, (years)	Spot/Strike											
	0.90	**0.92**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**	**1.08**	**1.10**	**1.12**
0.004	3838	2455	1350	633	187	28	21	1.7463	0.004	0.004	0.004	0.0036
0.054	11	7.12	3.960	1.561	0.042	0.762	0.946	0.733	0.372	0.082	0.052	0.069
0.104	3.226	2.061	1.180	0.486	0.021	0.271	0.395	0.399	0.330	0.229	0.133	0.061
0.154	1.592	1.039	0.615	0.272	0.028	0.130	0.216	0.245	0.232	0.195	0.148	0.102
0.204	0.983	0.655	0.400	0.190	0.036	0.069	0.133	0.163	0.168	0.155	0.131	0.104
0.254	0.685	0.465	0.293	0.148	0.040	0.037	0.087	0.115	0.125	0.122	0.111	0.095
0.304	0.515	0.355	0.230	0.123	0.041	0.018	0.059	0.084	0.096	0.098	0.093	0.084
0.354	0.407	0.285	0.189	0.106	0.042	0.007	0.041	0.063	0.075	0.079	0.078	0.073
0.404	0.333	0.237	0.160	0.094	0.041	0.003	0.028	0.047	0.059	0.065	0.066	0.063
0.454	0.28	0.202	0.139	0.084	0.041	0.007	0.019	0.036	0.048	0.054	0.056	0.055
0.504	0.241	0.175	0.123	0.077	0.040	0.010	0.012	0.028	0.038	0.045	0.048	0.048
0.554	0.211	0.155	0.111	0.071	0.039	0.013	0.007	0.021	0.031	0.038	0.041	0.042
0.604	0.187	0.139	0.101	0.066	0.038	0.015	0.003	0.016	0.025	0.032	0.035	0.037
0.654	0.167	0.125	0.092	0.062	0.037	0.016	0.001	0.012	0.021	0.027	0.031	0.033
0.704	0.151	0.114	0.085	0.058	0.035	0.017	0.003	0.008	0.017	0.023	0.026	0.029
0.754	0.138	0.105	0.079	0.055	0.034	0.018	0.005	0.006	0.013	0.019	0.023	0.025
0.804	0.126	0.097	0.074	0.052	0.033	0.018	0.006	0.003	0.011	0.016	0.020	0.022
0.854	0.117	0.09	0.069	0.049	0.033	0.019	0.007	0.002	0.008	0.014	0.017	0.020
0.904	0.108	0.084	0.065	0.047	0.032	0.019	0.008	0.001	0.006	0.011	0.015	0.018

Table D.3
ME (%) for call, 1-day forecast horizons

Time to maturity, (years)	Spot/Strike											
	0.90	**0.92**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**	**1.08**	**1.10**	**1.12**
0.004	0.000	0.000	0.000	–0.003	–0.186	0.569	–0.180	–0.010	0.000	0.000	0.000	0.000
0.054	–0.004	–0.006	–0.008	–0.005	0.000	0.005	0.007	0.005	0.003	0.001	0.000	0.000
0.104	–0.003	–0.004	–0.003	–0.002	0.000	0.001	0.002	0.003	0.002	0.002	0.001	0.000
0.154	–0.002	–0.002	–0.002	–0.001	0.000	0.001	0.001	0.002	0.002	0.001	0.001	0.001
0.204	–0.002	–0.002	–0.001	–0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
0.254	–0.001	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
0.304	–0.001	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
0.354	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.404	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.454	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.504	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.554	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.604	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.654	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.704	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.754	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.804	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.854	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.904	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

D.2 Conclusions

The results reported in this appendix show that the relative error between delta-gamma and full simulation is reasonably low, but becomes large as the option nears expiration and is at-the-money. Note that the extremely large errors in the case where the option is out-of-the-money reflects the fact that the option is valueless. Refer to Tables C.10 and C.19 in the *RiskMetrics Monitor* (third quarter, 1996) to see the value of the option at various spot prices and time to expirations. Therefore, aside from the case where the option is near expiration and at-the-money, the delta-gamma methodology seems to perform well in comparison to full simulation.

Overall, the usefulness of the delta-gamma method depends on how users view the trade-off between computational speed and accuracy. For risk managers seeking a quick, efficient means of computing VaR that measures gamma risk, delta-gamma offers an attractive method for doing so.

Appendix E. Routines to simulate correlated normal random variables

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In Section E.1 of this appendix we briefly introduce three algorithms for simulating correlated normal random variables from a specified covariance matrix Σ (Σ is square and symmetric). In Section E.2 we present the details of the Cholesky decomposition.

E.1 Three algorithms to simulate correlated normal random variables

This section describes the Cholesky decomposition (CD), eigenvalue decomposition (ED) and the singular value decomposition (SVD). CD is efficient when Σ is positive definite. However, CD is not applicable for positive semi-definite matrices. ED and SVD, while computationally more intensive, are useful when Σ is positive semi-definite.

- Cholesky decomposition

  We begin by decomposing the covariance matrix as follows:

  [E.1] $$\Sigma = P^T P$$

  where P is an upper triangular matrix. To simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

  1. Find the upper triangular matrix P.
  2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix *I* (identity matrix).
  3. Compute the vector $y = P^T\varepsilon$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

  Step 3 follows from the fact that

  [E.2] $$V(y) = P^T E\left(\varepsilon\varepsilon^T\right)P = P^T IP = P^T P = \Sigma$$

  where *V*( ) and *E*( ) represent the variance and mathematical expectation, respectively.

- Eigenvalue decomposition

  Applying spectral decomposition to Σ yields

  [E.3] $$\Sigma = C\Delta C^T = Q^T Q$$

  where *C* is an *N*x*N* orthogonal matrix of eigenvectors, i.e., $C^T C = I$

  Δ is an *N*x*N* matrix with the *N*-eigenvalues of *X* along its diagonal and zeros elsewhere

  [E.4] $$Q = \Delta^{1/2} C^T$$

To simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

1. Find the eigenvectors and eigenvalues of Σ.
2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix I (identity matrix).
3. Compute the vector $y = Q^T\varepsilon$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

Step 3 follows from the fact that

[E.5] $$V(y) = Q^T E\left(\varepsilon\varepsilon^T\right)Q = Q^T IQ = Q^T Q = C\Delta^{1/2}\Delta^{1/2}C^T = C\Delta C^T = \Sigma$$

The final algorithm that is proposed is known as the singular value decomposition.

- Singular Value decomposition

We begin with the following representation of the covariance matrix

[E.6] $$\Sigma = UDV^T$$

where U and V are NxN orthogonal matrices, i.e., $V^T V = U^T U = I$, and D is an NxN matrix with the N singular values of Σ along its diagonal and zeros elsewhere.

It follows directly from Takagi's decomposition that for any square, symmetric matrix, $\Sigma = VDV^T$. Therefore, to simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

1. Apply the singular value decomposition to Σ to get V and D.
2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix I (identity matrix).
3. Compute the vector $y = Q^T\varepsilon$ where $Q = D^{1/2}V^T$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

Step 3 follows from the fact that

[E.7] $$V(y) = Q^T E\left(\varepsilon\varepsilon^T\right)Q = Q^T IQ = Q^T Q = VD^{1/2}D^{1/2}V^T = VDV^T = \Sigma$$

E.2 Applying the Cholesky decomposition

In this section we explain exactly how to create the A matrix which is necessary for simulating multivariate normal random variables from the covariance matrix Σ. In particular, Σ can be decomposed as:

[E.8] $$\Sigma = A^T A$$

If we simulate a vector of independent normal random variables X then we can create a vector of normal random variables with covariance matrix Σ by using the transformation Y=A'X. To show how to obtain the elements of the matrix A, we describe the Cholesky decomposition when the dimension of the covariance matrix is 3 x 3. After, we give the general recursive equations used to derive the elements of A from Σ.

Consider the following definitions:

[E.9] $$\Sigma = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Then we have

[E.10] $$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

equivalent to

[E.11] $$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{32}a_{22} \\ a_{11}a_{31} & a_{21}a_{31} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix}$$

Now we can use the elements of Σ to solve for the $a_{i,j}$'s – the elements of A. This is done recursively as follows:

[E.12] $$\begin{aligned} s_{11} &= a_{11}^2 \Rightarrow a_{11} = \sqrt{s_{11}} \\ s_{21} &= a_{11}a_{21} \Rightarrow a_{21} = \frac{s_{21}}{a_{11}} \\ s_{22} &= a_{21}^2 + a_{22}^2 \Rightarrow a_{22} = \sqrt{s_{22} - a_{21}^2} \\ s_{31} &= a_{11}a_{31} \Rightarrow a_{31} = \frac{s_{31}}{a_{11}} \\ s_{32} &= a_{21}a_{31} + a_{32}a_{22} \Rightarrow a_{32} = \frac{1}{a_{22}}(s_{32} - a_{21}a_{31}) \\ s_{33} &= a_{11}^2 + a_{22}^2 + a_{33}^2 \Rightarrow a_{33} = \sqrt{a_{33} - a_{11}^2 - a_{22}^2} \end{aligned}$$

Having shown how to solve recursively for the elements in A we now give a more general result. Let *i* and *j* index the row and column of an *N* x *N* matrix. Then the elements of A can be solved for using

[E.13] $$a_{ii} = \left(s_{ii} - \sum_{k=1}^{i-1} a_{ik}^2 \right)^{1/2}$$

and

[E.14] $$a_{ij} = \frac{1}{a_{ii}}\left(s_{ij} - \sum_{k=1}^{i-1} a_{ik}a_{jk} \right)^{1/2} \qquad j = i+1, i+2, \ldots, N$$

Appendix F. BIS regulatory requirements

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The Basel Committee on Banking Supervision under the auspices of BIS issued in January 1996 a final Amendment to the 1988 Capital Accord that requires capital charges to cover market risks in addition to the existing framework covering credit risk. The framework covers risks of losses in on- and off- balance sheet positions arising from movements in market prices.

Banks' minimum capital charges will be calculated as the sum of credit risk requirements under the 1988 Capital Accord, excluding debt and equity securities in the trading book and all positions in commodities, but including the credit counterparty risk on all OTC derivatives, and capital charges for market risks. The proposal sets forth guidelines for the measurement of market risks and the calculation of a capital charge for market risks.

I. Measurement of market risk

Market risk may be measured using banks' internal models (subject to approval by the national supervisor) and incorporates the following:

1. Market risk in the trading account (i.e., debt and equity securities and derivatives):

   Standardized method — uses a “building block” approach where charges for general risk and issuer specific risk for debt and equities risks are calculated separately.

   Internal model — must include a set of risk factors corresponding to interest rates in each currency in which the bank has interest sensitive on- and off-balance sheet positions and corresponding to each of the equity markets in which the bank holds significant positions.

2. Foreign exchange risk across the firm (including gold:

   Standardized method — uses the shorthand method of calculating the capital requirement.

   Internal model — must include FX risk factors of the bank’s exposures.

3. Commodities risk across the firm (including precious metals but excluding gold)

   Standardized method — risk can be measured using the standardized approach or the simplified approach.

   Internal model — must include commodity risk factors of the bank’s exposures.

4. Options risk across the firm:

   Standardized method — banks using only purchased options should use a simplified approach and banks using written options should, at a minimum, use one of the intermediate approaches (“delta plus” or simulation method).

   Internal model — must include risk factors (interest rate, equity, FX, commodity) of the bank’s exposures.

II. Capital charge for market risk

Standardized method —simple sum of measured risk for all factors (i.e., debt/equity/FX/commodities/options)

Internal model —

- Higher of the previous day's VaR (calculated in accordance with specific quantitative standards) or average of daily VaR on each of the preceding 60 days times a multiplication factor, subject to a minimum of 3.

- A separate capital charge to cover the specific risk of traded debt and equity securities if not incorporated in model.

- A "plus" will be added that is directly related to the ex-post performance of the model (derived from "back-testing" outcome)

- Among other qualitative factors, stress testing should be in place as a supplement to the risk-analysis based on the day-to-day output of the model.

III. Methods of measuring market risks

A choice between a Standardized Methodology and an Alternative Methodology (i.e., use of banks' internal models) will be permitted for the measurement of market risks subject to the approval of the national supervisor.

1. The standardized methodology

This method uses a "building block" approach for debt and equity positions, where issuer-specific risk and general risk are calculated separately. The capital charge under the standardized method will be the arithmetic sum of the measures of each market risk (i.e., debt/equity/foreign exchange/commodities/options).

Debt securities
Instruments covered include: debt securities (and instruments that behave like them including non-convertible preferred shares) and interest rate derivatives in the trading account. Matched positions in identical issues (e.g., same issuer, coupon rates, liquidity, call features) as well as closely matched swaps, forwards, futures and FRAs which meet additional conditions are permitted to be offset. The capital charge for debt securities is the sum of the specific risk charge and general risk charge.

- Specific risk

  The specific risk charge is designed to protect against an adverse movement in the price of an individual security owing to factors related to the individual issuer. Debt securities and derivatives are classified into broad categories (government, qualified, and other) with a varying capital charge applied to gross long positions in each category. Capital charges range from 0% for the government category to 8% for the Other category.

- General market risk

  The general risk charge is designed to capture the risk of loss arising from changes in market interest rates. A general risk charge would be calculated separately for each currency in which the bank has a significant position. There are two principal methods to choose from:

1. **Maturity method —** long and short positions in debt securities and derivatives are slotted into a maturity ladder with 13 time bands (15 for deep discount securities). The net position in each time band is risk weighted by a factor designed to reflect the price sensitivity of the positions to changes in interest rates.

2. **Duration method** — achieves a more accurate measure of general market risk by calculating the price sensitivity of each position separately.

The general risk charge is the sum of the risk-weighted net short or long position in the whole trading book, a small proportion of the matched positions in each time-band (vertical disallowance 10% for maturity method; 5% for duration method), and a larger proportion of the matched positions across different time bands (horizontal disallowance).

Equities
Instruments covered include: common stocks, convertible securities that behave like equities, commitments to buy or sell equities, and equity derivatives. Matched positions in each identical equity in each market may be fully offset, resulting in a single net short or long position to which the specific and general market risk charges apply. The capital charge for equities is the sum of the specific risk charge and general risk charge.

- Specific risk

  Specific risk is the risk of holding a long or short position in an individual equity, i.e., the bank's absolute equity positions (the sum of all long and short equity positions). The specific risk charge is 8% (or 4% if the portfolio is liquid and diversified). A specific risk charge of 2% will apply to the net long or net short position in an index comprising a diversified portfolio of equities.

- General market risk

  General market risk is the risk of holding a long or short position in the market as a whole, i.e., the difference between the sum of the longs and the sum of the shorts (the overall net position in an equity market). The general market risk charge is 8% and is calculated on a market by market basis.

Foreign exchange risk (including gold)
The shorthand method of calculating the capital requirement for foreign exchange risk is performed by measuring the net position in each foreign currency and gold at the spot rate and applying an 8% capital charge to the net open position (i.e., the higher of net long or net short positions in foreign currency and 8% of the net position in gold).

Commodities risk
Commodities risk including precious metals, but excluding gold, can be measured using the standardized approach or the simplified approach for banks which conduct only a limited amount of commodities business. Under the standardized approach, net long and short spot and forward positions in each commodity will be entered into a maturity ladder. The capital charge will be calculated by applying a 1.5% spread rate to matched positions (to capture maturity mismatches) and a capital charge applied to the net position in each bucket. Under the simplified method, a 15% capital charge will be applied to the net position in each commodity.

Treatment of options
Banks that solely use purchased options are permitted to use a simplified approach; however, banks that also write options will be expected to use one of the intermediate approaches or a comprehensive risk management model. Under the standardized approach, options should be “carved out” and become subject to separately calculated capital charges on particular trades to be added to other capital charges assessed. Intermediate approaches are the “delta plus approach” and scenario

analysis. Under the delta plus approach, delta-weighted options would be included in the standardized methodology for each risk type.

2. Alternative methodology: internal models

This method allows banks to use risk measures derived from their own internal risk management models, subject to a general set of standards and conditions. Approval by the supervisory authority will only be granted if there are sufficient numbers of staff (including trading, risk control, audit and back office areas) skilled in using the models, the model has a proven track record of accuracy in predicting losses, and the bank regularly conducts stress tests.

- Calculation of capital charge under the internal model approach
  - Each bank must meet on a daily basis a capital requirement expressed as the higher of its previous day's value at risk number measured according to the parameters specified or an average of the daily value at risk measures on each of the preceding sixty business days, multiplied by a multiplication factor.
  - The multiplication factor will be set by supervisors on the basis of their assessment of the quality of the bank's risk management system, subject to a minimum of 3. The plus factor will range from 0 to 1 based on backtesting results and that banks that meet all of the qualitative standards with satisfactory backtesting results will have a plus factor of zero.The extent to which banks meet the qualitative criteria may influence the level at which supervisors will set the multiplication factor.
  - Banks using models will be subject to a separate capital charge to cover the specific risk of traded debt and equity securities to the extent that this risk is not incorporated into their models. However, for banks using models, the specific risk charge applied to debt securities or equities should not be less than half the specific risk charge calculated under the standardized methodology.
  - Any elements of market risk not captured by the internal model will remain subject to the standardized measurement framework.
  - Capital charges assessed under the standardized approach and the internal model approach will be aggregated according to the simple sum method.
- Requirements for the use of internal models:

Qualitative standards

- Existence of an independent risk control unit with active involvement of senior management
- Model must be closely integrated into day-to-day risk management and should be used in conjunction with internal trading and exposure limits
- Routine and rigorous programs of stress testing and back-testing should be in place
- A routine for ensuring compliance and an independent review of both risk management and risk measurement should be carried out at regular intervals
- Procedures are prescribed for internal and external validation of the risk measurement process

Specification of market risk factors
The risk factors contained in a risk measurement system should be sufficient to capture the risk inherent in the bank's portfolio, i.e., interest rates, exchange rates, equity prices, commodity prices.

Quantitative standards

- Value at risk should be computed daily using a 99th percentile, one-tailed confidence interval and a minimum holding period of 10 trading days. Banks are allowed to scale up their 1-day VaR measure for options by the square root of 10 for a certain period of time after the internal models approach takes effect at the end of 1997.

- Historical observation period will be subject to a minimum length of one year. For banks that use a weighting scheme or other methods for the historical observation period, the "effective" observation period must be at least one year.

- Banks will have discretion to recognize empirical correlations within broad risk categories. Use of correlation estimates across broad risk categories is subject to regulatory approval of the estimation methodology used.

- Banks should update their data sets no less frequently than once every three months and should reassess them whenever market prices are subject to material change

- Models must accurately capture the unique risks associated with options within the broad risk categories (using delta/gamma factors if analytical approach is chosen)

IV. Calculation of the capital ratio

- The minimum capital ratio representing capital available to meet credit and market risks is 8%.

- The denominator of the ratio is calculated by multiplying the measure of market risk by 12.5 (reciprocal of the 8% ratio) and adding the results to credit risk-weighted assets. The numerator is eligible capital, i.e., sum of the bank's Tier 1 capital, Tier 2 capital under the limits permitted by the 1988 Accord, and Tier 3 capital, consisting of short-term subordinated debt. Tier 3 capital is permitted to be used for the sole purpose of meeting capital requirements for market risks and is subject to certain quantitative limitations.

- Although regular reporting will in principle take place only at intervals (in most countries quarterly), banks are expected to manage the market risk in their trading portfolios in such a way that the capital requirements are being met on a continuous basis, i.e., at the close of each business day.

V. Supervisory framework for the use of backtesting

Backtesting represents the comparison of daily profits and losses with model-generated risk measures to gauge the quality and accuracy of banks' risk measurement systems. The backtests to be applied compare whether the observed percentage of outcomes covered by the risk measure is consistent with a 99% level of confidence. The backtesting framework should use risk measures calibrated to a 1-day holding period. The Committee urges banks to develop the capability to perform backtests using both hypothetical (based on the changes in portfolio value that would occur were end-of-day positions to remain unchanged) and actual trading outcomes.

The framework adopted by the Committee calculates the number of times that the trading outcomes are not covered by the risk measures (exceptions) on a quarterly basis using the most recent 12 months of data. The framework encompasses a range of possible responses which are classified into 3 zones. The boundaries are based on a sample of 250 observations.

- **Green zone —** the backtesting results do not suggest a problem with the quality or accuracy of a bank's model (only four exceptions are allowed here).

- **Yellow zone —** the backtesting results do raise questions, but such a conclusion is not definitive (only 9 exceptions are allowed here). Outcomes in this range are plausible for both accurate and inaccurate models. The number of exceptions will guide the size of potential supervisory increases in a firm's capital requirement. The purpose of the increase in the multiplication factor is to return the model to a 99th percentile standard. Backtesting results in the yellow zone will generally be presumed to imply an increase in the multiplication factor unless the bank can demonstrate that such an increase is not warranted. The burden of proof in these situations should not be on the supervisor to prove that a problem exists, but rather should be on the bank to prove that their model is fundamentally sound.

- **Red zone** — the backtesting results almost certainly indicate a problem with a bank's risk model (10 or more exceptions). If a bank's model falls here, the supervisor will automatically increase the multiplication factor by one and begin investigation.

Appendix G. Using the RiskMetrics examples diskette

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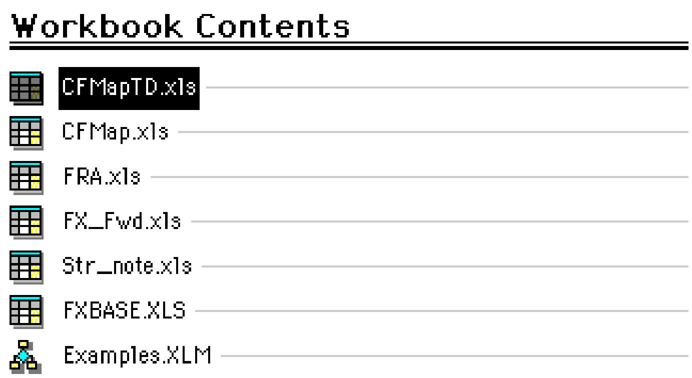
A number of the examples in this *Technical Document*, are included on the enclosed examples diskette. This diskette contains a Microsoft Excel workbook file containing six spreadsheets and one macro file. The workbook can be used under Excel Version 4.0 or higher.

Some of the spreadsheets allow the user to modify inputs in order to investigate different scenarios. Other spreadsheets are non-interactive. In the latter case, the objective is to provide the user with a detailed illustration of the calculations. This workbook and user guide is presented to the experienced user of Microsoft Excel, although we hope the material is meaningful to less experienced users. Please make a duplicate of the Examples.XLW workbook and save at least one copy on your hard drive and at least one copy on a floppy disk. This will allow you to manipulate the enclosed spreadsheets without sacrificing their original format.

Opening the Examples.XLW workbook will show the following file structure:



The files listed above are described as follows:

File	Section, page	Description
CFMapTD.xls	Section 6.4, page 134	Decomposition of the 10-year benchmark OAT into RiskMetrics vertices
CFMap.xls	Section 6.4, page 135	Generic Excel cash flow mapping spreadsheet (users are given flexibility to map standard bullet bonds)
FRA.xls	Section 6.4, page 136	Mapping and VaR calculation of a 6x12 French franc FRA
FX_Fwd.xls	Section 6.4, page 143	Mapping and VaR calculation of a DEM/USD 1-year forward
Str_note.xls	Section 6.4, page 139	Mapping and VaR calculation of a 1-year Note indexed to 2-year DEM swap rates
FXBase.xls	Section 8.4, page 183	Generic calculator to convert U.S. dollar based volatilities and correlations to another base currency
Examples.XLM		Macro sheet that links to buttons on the various spreadsheets

CFMapTD.xls & CFMap.xls

These two spreadsheets are similar, although CFMap.xls allows the user to change more of the inputs in order to investigate different scenarios, or to perform sensitivity analysis. CFMap.xls allows provides more vertices to which to map the cash flows. Note that only data in red is changeable on all spreadsheets.

In CFMapTD.xls, Example Part 1 illustrates the mapping of a single cash flow, while Example Part II illustrates the mapping of the entire bond.

To begin mapping on either spreadsheet, enter your chosen data in all cells that display red font. Then click the "Create cash flows" button. Wait for the macro to execute, then click the "Map the cash flows to vertices" button to initiate the second macro, which executes for the final output of Diversified Value at Risk, Market Value, and Percentage of market value. If you wish to print the cash flow mapping output, simply click the "Print Mapping" macro button.

FRA.xls

This Forward Rate Agreement example is for illustrative purposes only. We encourage the user, however, to manipulate the spreadsheet is such a way as to increase it's functionality. Changing any spreadsheet, of course, should be done after creating a duplicate workbook.

In this spreadsheet, cells are named so that formulae show the inputs to their respective calculations. This naming convention, we hope, increases user friendliness. For example, looking at cell C21 shows the calculation for the FRA rate utilizing the data in 1. Basic Contract Data and data under the Maturity column under 2. FRA Mapping and VaR on 6-Jan-95.

Cells are named according to the heading under which they fall, or the cell to their left that best describes the data. For example, cell B30 is named Maturity_1, while cell K32 is named Divers_VaR_1. Also note that the RiskMetrics Correlations are named in two-dimensional arrays: cells L30:M31 are named Corr_Matrx_1, while cells L40:N42 are named Corr_Matrx_2.

If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

The cells containing the individual VaR calculations (K30, K31, K40, K41, K42) contain the absolute value of the value at risk. In order to calculate the Diversified VaR, however, in cells K32 and K43, we have placed the actual VaR values to the right of the correlation matrices. If you go to cell K32, you will see that the formula makes use of VaR_Array_1, which refers to cells O31:O32. This VaR array contains the actual values of VaR_1 and VaR_2, which are essential to calculating the Divers_VaR_1. Cells O31 and O32 are formatted in white font in order to maintain the clarity of the spreadsheet. Similarly, the calculation in cell K43 utilizes VaR_Array_2, found in cells O40:O42.

FX_Fwd.xls

This spreadsheet offers some interaction whereby the user can enter data in all red cells.

Before examining this spreadsheet, please review the names of the cells in the 1. Basic contract data section in order to better understand the essential calculations. If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

Please note that the Diversified Value at Risk calculation utilizes the var_array input, which refers to cells I33:I35. These cells are formatted in white font in order to maintain the clarity of the worksheet.

Str_note.xls

This spreadsheet is for illustrative purposes only. Again, we encourage the user to format the spreadsheet for custom use.

Please notice that the Diversified VaR calculations make use of VaR_Array1 and VaR_Array2. VaR_Array1 references cells N26:N28, while VaR_Array2 references cells O37:O40. These two arrays are formatted in white font in order to maintain the clarity of the worksheet.

If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

Appendix H. RiskMetrics on the Internet

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RiskMetrics home pages on the Internet are currently located at

http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html

and

http://www.riskmetrics.reuters.com

The RiskMetrics home page on the Reuters Web is located at:

http://riskmetrics.session.rservices.com

The Internet can be accessed through such services as CompuServe®, Prodigy®, or America® Online, or through service providers by using browsers such as Netscape™ Navigator, Microsoft® Internet Explorer, Mosaic or their equivalents. The Reuters Web is available with the Reuters 3000 series.

RiskMetrics data sets can be downloaded from the Internet and from the Reuters Web. RiskMetrics documentation and a listing of third parties, both consultants and software developers who incorporate RiskMetrics methodology and/or data sets, are also freely available from these sites or from local Reuters offices. Users can receive e-mail notification of new publications or other information relevant to RiskMetrics by registering at the following address:

http://www.jpmorgan.com/RiskManagement/RiskMetrics/rmform.html

Note that URL addresses are subject to change.

H.1 Data sets

RiskMetrics data sets are updated daily on the Internet at *http://www.riskmetrics.reuters.com* and on the Reuters Web at *http://riskmetrics.session.rservices.com.*

The data sets are available by 10:30 a.m. U. S. Eastern Standard Time. They are based on the previous day's data through close of business, and provide the latest estimates of volatilities and correlations for daily and monthly horizons, as well as for regulatory requirements.

The data sets are not updated on official U.S. holidays. For these holidays, foreign market data is included in the following business day's data sets; U.S. market data is adjusted according to the Expectation Maximization (EM) algorithm described in Section 8.25. EM is also used in a consistent fashion for filling in missing data in other markets.

The data sets are supplied in compressed file format for DOS, Macintosh, and UNIX platforms. The DOS and Macintosh files are auto-extracting, i.e., the decompression software is enclosed in the file.

On the same page as the data sets is the Excel add-in, which enables users to perform DEaR/VaR calculations on other than a US dollar currency basis. The add-in allows users full access to the data sets when building customized spreadsheets. Current rate, price volatility, and correlation of specified pairs can be returned. The add-in was compiled in 16 bit and runs under Excel 4.0 and 5.0. It does not run under Windows NT. The name of the add-in is JPMVAR for the Mac and JPMVAR.XLL for the PC. It has an expiration date of November 1, 1997.

H.2 Publications

http://www.jpmorgan.com/RiskManagement/riskMetrics/pubs.html

The annual *RiskMetrics—Technical Document*, the quarterly *Monitor* and all other RiskMetrics documents are available for downloading in Adobe Acrobat pdf file format. Adobe Acrobat Reader is required to view these files. It can be downloaded from *http://www.adobe.com*.

RiskMetrics documents are also available from your local Reuters office.

H.3 Third parties

http://www.jpmorgan.com/RiskManagement/RiskMetrics/Third_party_directory.html

Setting up a risk management framework within an organization requires more than a quantitative methodology. A listing of several consulting firms who have capital advisory practices to help ensure the implementation of effective risk management and system developers who have integrated RiskMetrics methodology and/or data sets is available for viewing or saving as a file.

Users should be able to choose from a number of applications that will achieve different goals, offer various levels of performance, and run on a number of different platforms. Clients should review the capabilities of these systems thoroughly before committing to their implementation. **J. P. Morgan and Reuters do not endorse the products of these third parties nor do they warrant their accuracy in the application of the RiskMetrics methodology** and in the use of the underlying data accompanying it.

Reference

Glossary of terms

absolute market risk. Risk associated with the change in value of a position or a portfolio resulting from changes in market conditions i.e., yield levels or prices.

adverse move X. Defined in RiskMetrics as 1.65 times the standard error of returns. It is a measure of the most the return will move over a specified time period.

ARCH. Autoregressive Conditional Heteroskedascticity. A time series process which models volatility as dependent on past returns. GARCH—Generalized ARCH, models volatility as a function of past returns and past values of volatility. EGARCH—Exponential GARCH, IGARCH—Integrated GARCH. SWARCH—Switching Regime ARCH.

autocorrelation (serial correlation). When observations are correlated over time. In other words, the covariance between data recorded on the same series sequentially in time is non-zero.

beta. A volatility measure relating the rate of return on a security with that of its market over time. It is defined as the covariance between a security's return and the return on the market portfolio divided by the variance of the return of the market portfolio.

bootstrapping. A method to generate random samples from the observed data's underlying, possibly unknown, distribution by randomly resampling the observed data. The generated samples can be used to compute summary statistics such as the median. In this document, bootstrapping is used to show monthly returns can be generated from data which are sampled daily.

CAPM. Capital Asset Pricing Model. A model which relates the expected return on an asset to the expected return on the market portfolio.

Cholesky factorization/decomposition. A method to simulation of multivariate normal returns based on the assumption that the covariance matrix is symmetric and positive-definite.

constant maturity. The process of inducing fixed maturities on a time series of bonds. This is done to account for bonds "rolling down" the yield curve.

decision horizon. The time period between entering and unwinding or revaluing a position. Currently, RiskMetrics offers statistics for 1-day and 1-month horizons.

decay factor. See lambda.

delta equivalent cash flow. In situations when the underlying cash flows are uncertain (e.g. option), the delta equivalent cash flow is defined as the change in an instrument's fair market value when its respective discount factor changes. These cash flows are used to find the net present value of an instrument.

delta neutral cash flows. These are cash flows that exactly replicate a callable bond's sensitivity to shifts in the yield curve. A single delta neutral cash flow is the change in the price of the callable bond divided by the change in the value of the discount factor.

duration (Macaulay). The weighted average term of a security's cash flow.

EM algorithm. A statistical algorithm that can estimate parameters of a function in the presence of incomplete data (e.g. missing data). EM stands for Expectation Maximization. Simply put, the missing values are replaced by their expected values given the observed data.

exponential moving average. Applying weights to a set of data points with the weights declining exponentially over time. In a time series context, this results in weighing recent data more than the distant past.

GAAP. Generally Accepted Accounting Principles.

historical simulation. A non-parametric method of using past data to make inferences about the future. One application of this technique is to take today's portfolio and revalue it using past historical price and rates data.

kurtosis. Characterizes relative peakedness or flatness of a given distribution compared to a normal distribution.[1]

$$K_x = \left\{ \frac{N^2 - 2N + 3}{(N-1)\ (N-2)\ (N-3)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_x} \right)^4 \right\} - 3 \frac{(N-1)\ (2N-3)}{N\,(N-2)\ (N-3)}$$

Since the unconditional normal distribution has a kurtosis of 3, excess kurtosis is defined as K_x-3.

λ **lambda (decay factor).** The weight applied in the exponential moving average. It takes a value between 0 and 1. In the RiskMetrics lambda is 0.94 in the calculation of volatilities and correlations for a 1-day horizons and 0.97 for 1-month horizon.

leptokurtosis (fat tails). The situation where there are more occurrences far away from the mean than predicted by a standard normal distribution.

linear risk (nonlinear). For a given portfolio, when the underlying prices/rates change, the incremental change in the payoff of the portfolio remains constant for all values of the underlying prices/rates. When this does not occur, the risk is said to be nonlinear.

log vs. change returns. For any price or rate P_t, log return is defined as ln (P_t/P_{t-1}) whereas the change return is defined by $(P_t - P_{t-1})/P_{t-1}$. For small values of $(P_t - P_{t-1})$, these two types of returns give very similar results. Also, both expressions can be converted to percentage returns/changes by simply multiplying them by 100.

mapping. The process of translating the cash flow of actual positions into standardized position (vertices). Duration, Principal, and cash flow.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

mean. A Measure of central tendency. Sum of daily rate changes divided by count

mean reversion. When short rates will tend over time return to a long-run value.

modified duration. An indication of price sensitivity. It is equal to a security's Macaulay duration divided by one plus the yield.

outliers. Sudden, unexpectedly large rate or price returns.

[1] We would like to thank Steven Hellinger of the New York State Banking Department for pointing this formula out for us.

overlapping data. Consecutive returns that share common data points. An example would be monthly returns (25-day horizon) computed on a daily basis. In this instance adjacent returns share 24 data points.

nonparametric. Potential market movements are described by assumed scenarios, not statistical parameters.

parametric. When a functional form for the distribution a set of data points is assumed. For example, when the normal distribution is used to characterize a set of returns.

principle of expected return. The expected total change in market value of the portfolio over the evaluation period.

relative market risk. Risk measured relative to an index or benchmark

residual risk. The risk in a position that is issue specific.

skewness. Characterizes the degree of asymmetry of the distribution around its mean. Positive skews indicate asymmetric tail extending toward positive values (right-hand side). Negative skewness implies asymmetry toward negative values (left-hand side).

$$S_x = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}^2}{\sigma_x} \right)$$

speed of adjustment. A parameter used in modelling forward rates. It is estimated from past data on short rates. A fast speed of adjustment will result in a forward curve that approaches the long-run rate at a relatively short maturity.

stochastic volatility. Applied in time series models that take volatility as an unobservable random process. Volatility is often modeled as a first order autoregressive process.

standard deviation. Indication of the width of the distribution of changes around the mean.

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{x})^2}$$

Structured Monte Carlo. Using the RiskMetrics covariance matrix to generate random normal variates to simulate future price scenarios.

total variance. The variance of the market portfolio plus the variance of the return on an individual asset.

zero mean. When computing sample statistics such as a variance or covariance, setting the mean to a prior value of zero. This is often done because it is difficult to get a good estimate of the true mean.

Bibliography

Ahlburg, Dennis A. "A commentary on error measures," *International Journal of Forecasting*, 8, (1992), pp. 99–111.

Armstrong, J. Scott and Fred Collopy. "Error measures for generalizing about forecasting methods: Empirical comparisons,"*International Journal of Forecasting*, 8, (1992), pp. 69–80.

Bachelier, L. *Theorie de la Speculation* (Paris: Gauthier-Villars, 1900).

Becker, K., Finnerty, J., and A. Tucker. "The intraday interdependence structure between U.S. and Japanese Equity markets," *Journal of Financial Research*, 1, (1992), pp. 27–37.

Belsley, D.A. *Conditioning Diagnostics: Collinearity and Weak Data in Regression* (New York: John Wiley & Sons, 1980).

Bera, A.K. and C.M. Jarque. "Efficient tests for Normality, Heteroscedasticity, and Serial Independence in Regression Residuals," *Economics Letters*, 6, (1980), pp. 255–259.

Blattberg, R. and N. Gonedes. "A comparison of Stable and Student Distributions as Statistical Models for Stock Prices," *Journal of Business*, 47, (1974), pp. 244–280.

Bollersev, T. "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 31, (1986), pp. 307–327.

Bollersev, T. "A Conditional Heteroskedastic Model for Speculative Prices and Rates of Return," *Review of Economics and Statistics*, 69, (1987), pp. 542–547.

Boudoukh, J., Richardson, M., and R. F. Whitelaw. "A Tale of Three Schools: Insights on Autocorrelations of Short-Horizon Stock Returns," *The Review of Financial Studies*, 3, (1994), pp. 539–573.

Braun, P.A., Nelson, D.B., and Alain M. Sunier. "Good News, Bad News, Volatility and Betas," Working Paper 90-93, Graduate School of Business, University of Chicago, (August 1991).

Campbell, B. and J. Dufour. "Exact NonParametric Orthogonality and Random Walk Tests," *Review of Economics and Statistics*, 1, (February 1995), pp. 1–15.

Campbell, J.Y., Lo, A.W., and A.C. MacKinley. "The Econometrics of Financial Markets," manuscript, June 1995.

Cohen, K., et al. "Friction in the trading process and the estimation of systematic risk," *Journal of Financial Economics*, (1983), pp. 263–278.

Crnkovic, Cedomir and Jordan Drachman. "Model Risk Quality Control," *RISK*, 9, (September 1996), pp. 138–143

Davidian, M. and R. J. Carroll. "Variance Function Estimation," *Journal of the American Statistical Association*, 400, (1987), pp. 1079–1091.

DeGroot, M. *Probability and Statistics*, 2nd edition, (Reading, MA: Addison-Wesley Publishing Company, 1989), Chapter 8.

Dempster, A.P., Laird, N.M., and D. B. Rubin. “Maximum likelihood from incomplete data via the EM algorithm,” *Journal of the Royal Statistical Society,* 39, (1977), pp. 1–38.

Diebold, F.X. and R.S. Mariano. “Comparing predictive accuracy,” *Journal of Business and Economic Statistics*, 13, (1995), pp. 253–263.

Engle, R.F. “Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation,” *Econometrica*, 50, (1982), pp. 987–1007.

Engle, R.F. and T. Bollerslev. “Modelling the Persistence of Conditional Variances,” *Econometric Reviews*, 5, (1986), pp. 1–50.

Engle, R.F. and Ken Kroner. “Multivariate Simultaneous Generalized ARCH,” *Econometric Theory*, (1995), pp. 122–150.

Eun, C.S. and S. Shim. “International Transmission of Stock Market Movements,” *Journal of Financial and Quantitative Analysis*, 24, (1989), pp. 241–256.

Fabozzi, Frank (editor), *The Handbook of Fixed Income Options*, (Chicago, IL: Probus Publishing, 1989), Chapter 3.

Fama, E. “The Behavior of Stock Market Prices,” *Journal of Business,* 38, (1965), pp. 34–105.

Fama, E. and K. French. “Permanent and Temporary Components of Stock Prices,” *Journal of Political Economy*, 96, (1988), pp. 246–273.

Figlewski, Stephen. “Forecasting Volatility using Historical Data,” New York University Working Paper, S-94-13, (1994).

Fildes, Robert. “The evaluation of extrapolative forecasting methods,” *International Journal of Forecasting*, 8, (1992), pp. 81–98.

Finger, Christopher C. “Accounting for the “pull to par” and “roll down” for RiskMetrics cashflows,” *RiskMetrics™ Monitor*, (September 16, 1996).

Fisher, L. “Some New Stock-Market Indexes,” *Journal of Business*, 39, (1966), pp. 191–225.

Ghose, Dev and Ken Kroner. “The Relationship between GARCH and Stable Processes: Finding the source of fat-tails in financial data,” University of Arizona Working Paper, 93-1, (June 1993).

Government Bond Outlines, 8th edition, C. Dulligan, 1995.

Greene, William. *Econometric Analysis*, 2nd edition. (New York: Macmillan Publishing Company, 1993).

Harvey, A.C. *Time Series Models*, 2nd edition. (Cambridge, MA: MIT Press, 1993).

Harvey, A.C., E. Ruiz and N.G. Shepard. “Multivariate Stochastic Variance Models,” *Review of Economic Studies*, 61, (1994), pp. 247–264.

Hendricks, Darryl. “Evaluation of Value-at-Risk Models Using Historical Data,” Federal Reserve Bank of New York *Economic Policy Review*, (April 1996).

Heuts, R.M.J. and S. Rens. “Testing Normality When Observations Satisfy a Certain Low Order ARMA-Scheme,” *Computational Statistics Quarterly,* 1, (1986), pp. 49–60.

Heynen, R. and H. Kat. “Volatility prediction: A comparison of GARCH(1,1), EGARCH(1,1) and Stochastic Volatility model.” Mimeograph, (1993).

Hill, I.D., Hill, R. and R.L. Holder. “Fitting Johnson Curves by Moments (Algorithm AS 99),” *Applied Statistics*, (1976), pp. 180–189.

Introducing the Emerging Markets Bond Index Plus, J.P. Morgan publication, July 12, 1995.

Jaffe and Westerfield. “Patterns in Japanese Common Stock Returns: Day of the Week and Turn of the Year Effects,” *Journal of Financial and Quantitative Analysis*, 20, (1985), pp. 261–272.

Jarrow, Robert and Donald van Deventer. “Disease or Cure?” *RISK*, 9, (February 1996), pp. 54–57.

Johnson, N.L. “Systems of frequency curves generated by methods of translation,” *Biometrika*, (1949), pp. 149–175.

Johnson, R.A. and D.W. Wichern. *Applied Multivariate Statistical Analysis*, 3rd edition (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1992).

Johnston, J. *Econometric Methods*. (New York: McGraw-Hill, Inc., 1984).

Jorion, Phillipe. “Predicting Volatility in the Foreign Exchange Market,” *Journal of Finance*, 2, (June 1995), pp. 507–528.

Jorion, P. “On Jump Processes in the Foreign Exchange and Stock Markets,” *Review of Financial Studies*, 62, (1988), pp. 281–300.

Karolyi, G.A. “A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: The Case of the United States and Canada,” *Journal of Business and Economic Statistics*, (January 1995), pp. 11–25.

Kempthorne, J. and M. Vyas. “Risk Measurement in Global Financial Markets with Asynchronous, Partially Missing Price Data.” IFSRC Working Paper No. 281–94, (1994).

Kiefer, N. and M. Salmon. “Testing Normality in Econometric Models,” *Economic Letters*, 11, (1983), pp. 123–127.

King, M., Sentena, E., and S. Wadhwani. “Volatility and links between national stock markets,” *Econometrica*, (July 1994), pp. 901–933.

Kon, S.J. “Models of Stock Returns: A comparison,” *Journal of Finance*, 39, (1988), pp. 147–165.

Kroner, K., Kneafsey, K.P., and S. Claessens. “Forecasting volatility in commodity markets,” forthcoming, *International Journal of Forecasting*, (1995).

Lau, S.T. and J.D. Diltz. “Stock returns and the transfer of information between the New York and Tokyo stock exchanges,” *Journal of International Money and Finance*, 13, (1994), pp. 211–222.

LeBaron, B. “Chaos and Nonlinear Forecastability in Economics and Finance.” Working Paper University of Wisconsin - Madison, (February 1994).

Leitch, G. and J.E. Tanner. “Economic Forecast Evaluation: Profit Versus The Conventional Error Measures,” *American Economic Review*, 3, (1991), pp. 580–590.

Li, W.K. and T.K. Mak. “On the squared residual autocorrelations in non-linear time series with conditional heteroskedasticity,” *Journal of Time Series Analysis*, 6, (1994), pp. 627–635.

Lo and MacKinlay. “An econometric analysis of nonsynchronous trading,” *Journal of Econometrics*, 45, (1990), pp. 181.

Looney, Stephen. “How to use test for univariate normality to assess multivariate normality,” *The American Statistician*, 49, (1995), pp. 64–70.

Lumsdaine, R.L. “Finite-Sample properties of the maximum likelihood estimator in GARCH(1,1) and IGARCH(1,1) Models: A Monte Carlo Investigation,” *Journal of Business and Economic Statistics*, 1, (January 1995), pp. 1–9.

Maomoni, Hakim. “Valuing and Using FRAs,” J.P. Morgan publication, October 1994.

Mandlebrot, B. “The Variations of Certain Speculative Prices,” *Journal of Business*, 36, (1963), pp. 394–419.

McLeod, A.I. and W.K. Li. “Diagnostic checking ARMA time series models using squared-residual autocorrelations,” *Journal of Time Series Analysis*, 4, (1983), pp. 269–273.

Meese, Richard A. and Kenneth S. Rogoff. “Empirical exchange rate models of the seventies: Do they fit out of sample?” *Journal of International Economics*, 14, (1983), pp. 3–24.

J.P. Morgan & Co., Inc., Arthur Andersen & Co. SC, and Financial Engineering Limited, *The J.P. Morgan/Arthur Andersen Guide to Corporate Exposure Management*, published by RISK magazine, (August 1994).

Nelson, D. B. “Stationarity and persistence in the GARCH (1,1) model,” *Econometric Theory*, 6, (1990), pp. 318–334.

Pagan, A. and G. W. Schwert. “Alternative Models for Conditional Stock Volatility,” *Journal of Econometrics*, 45, (1990), pp. 267–290.

Parkinson, M. “The Extreme Value Method for Estimating the Variance of the Rate of Return,” *Journal of Business*, 1, (1980), pp. 61–65.

Perry, P. “Portfolio Serial Correlation and Nonsynchronous Trading,” *Journal of Financial and Quantitative Analysis*, 4, (1985), pp. 517–523.

Political Handbook of the World: 1995–1996. (New York: CSA Publishing, State University of New York, 1996).

Richardson, M. and T. Smith. “A Test for Multivariate Normality in Stock Returns,” *Journal of Business*, 2, (1993), pp. 295–321.

Ruiz, Esther. “Stochastic Volatility verses Autoregressive Conditional Heteroskedasticity,” Working Paper 93-44, Universidad Carlos III de Madrid (1993).

Ruiz, Esther. “Quasi-maximum likelihood estimation of stochastic volatility models,” *Journal of Econometrics*, 63, (1994), pp. 289–306.

Schwert, W.G. “Why does Stock Market Volatility Change over Time?” *Journal of Finance*, 44, (1989), pp. 1115–1153.

Shanken, J. “Nonsynchronous data and covariance-factor structure of returns,” *Journal of Finance*, 42, (1987), pp. 221–231.

Shapiro, S.S. and M.B. Wilk. “An Analysis of Variance Test for Normality,” *Biometrika*, 52, (1965), pp. 591–611.

Sholes, M. and J. Williams. “Estimating beta from non-synchronous data,” *Journal of Financial Economics*, 5, (1977), pp. 309–327.

Silverman, B.W. *Density Estimation for Statistics and Data Analysis*, (New York: Chapman and Hall, 1986).

Taylor, S.J. *Modeling Financial Time Series* (Chichester, UK: John Wiley and Sons, 1986).

Tucker, A. L. “A Reexamination of Finite- and Infinite-Variance Distributions as Models of Daily Stock Returns,” *Journal of Business and Economic Statistics*, 1, (1992), pp. 73–81.

Warga, Arthur. “Bond Returns, Liquidity, and Missing Data,” *Journal of Financial and Quantitative Analysis*, 27, (1992), pp. 605–617.

West, Ken and D. Cho. “The predictive ability of several models of exchange rate volatility,” *Journal of Econometrics*, 69, (1995), pp. 367–391.

West, K.D., Edison, H.J., and D. Cho. “A utility-based comparison of some models of exchange rate volatility,” *Journal of International Economics*, 35, (1993), pp. 23–45.

White, H. “Maximum likelihood estimation of unspecified models,” *Econometrica,* (1982), pp. 1–16.

Xu, X. and S. Taylor. “Conditional volatility and the information efficiency of the PHLX currency options market,” *Journal of Banking and Finance*, 19, (1995), pp. 803–821.

Look to the J.P. Morgan site on the Internet for updates of these examples or useful new tools.

If there is not a diskette in the pocket below, the examples it contains are available from the Internet web pages.

http://www.jpmorgan.com/RiskManagement/RiskMetrics/pubs.html

The enclosed Excel workbook is intended as a demontration of the RiskMetrics market risk management methodology and volatility and correlation datasets. The worksheets have been designed as an educational tool and should not be used for the risk estimation of acutual portfolios. Clients should contact firms specialized int he design of risk management software for the implementation of a market risk estimation system. If you have any questions about the use of this workbook contact your local J.P.Morgan representative or:

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RiskMetrics™ products

Introduction to RiskMetrics™: An eight-page document that broadly describes the RiskMetrics™ methodology for measuring market risks.

RiskMetrics™ Directory: Available exclusively on-line, a list of consulting practices and software products that incorporate the RiskMetrics™ methodology and data sets.

RiskMetrics™—Technical Document: A manual describing the RiskMetrics™ methodology for estimating market risks. It specifies how financial instruments should be mapped and describes how volatilities and correlations are estimated in order to compute market risks for trading and investment horizons. The manual also describes the format of the volatility and correlation data and the sources from which daily updates can be downloaded.

RiskMetrics™ Monitor: A quarterly publication that discusses broad market risk management issues and statistical questions as well as new software products built by third-party vendors to support RiskMetrics™.

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