Stress Testing in the Investment Process

August 2010

Oleg Ruban
Dimitris Melas

Introduction

Stress tests explore the tails of the loss distribution by looking at the extent of potential large portfolio losses and possible scenarios in which these losses can occur. Stress tests help identify and manage situations that can result in extreme losses (Jorion, 2007). Portfolio risk models typically calculate measures such as volatility, Value at Risk (VaR) or expected shortfall – summary statistics of the forecast return distribution. While these statistics help evaluate potential losses and identify the positions that contribute most to portfolio risk, they do not reveal how the losses might occur. Stress tests complement risk forecasts by attempting to answer questions such as “If oil prices rise by 20%, how much will the value of my portfolio change?” The key advantage of stress tests is that they link a loss to a specific event, which can be more meaningful to portfolio managers than a summary statistic of the loss distribution. By enhancing our understanding of portfolio losses, stress tests can be valuable at all stages of the investment process, including portfolio construction, limit setting, and hedging. While banks increasingly use stress tests, and banking regulators increasingly require them, this paper focuses on the use of stress tests in portfolio risk management, rather than bank balance sheet management.

In 2009, MSCI conducted a global survey of risk practices among large asset owners and asset managers (*The Future of Market Risk Management*, MSCI, 2009), as well as a number of institutional investor roundtables on stress testing and scenario analysis in different countries. We found a strong interest in stress testing and scenario analysis among both asset owners and asset managers, with 74% of asset managers and 27% of plan sponsors reporting they perform stress tests. The survey drew attention to the challenge of interpreting stress test results and designing a course of action around them. While stress testing was mentioned as critical for integrating qualitative and quantitative information, enterprise risk management, and liquidity and counterparty risk analysis, it was noted that integrating stress testing with mainstream risk management practices can be challenging. Riccardo Rebonato[1] recently mirrored this sentiment, pointing out that “stress testing has so far been seen as the acupuncture and herbal remedies corner of risk management, but perceptions are changing”, as quoted in *The Economist*.[2]

Previous research on stress testing has focused on finding ways to come up with relevant shocks. In this paper, we also explore how to use the results of stress tests in the investment process and provide case studies that address issues confronting institutional investors in the current environment.

The sequence of decisions for a stress test is illustrated in Figure 1, which also outlines the structure of the paper. In section 1, we examine the foundations of stress testing, which involve determining the scope of the test. In section 2, we explore how scenarios are formulated, and in section 3 we focus on how scenarios are translated into portfolio losses. Finally, in section 4, we examine a method for integrating the results of stress testing in portfolio construction.

Figure 1: A framework for conducting effective stress tests

[1] http://www.riccardorebonato.co.uk/papers/ShortBio.doc
[2] “Number-crunchers crunched”, 13 February 2010.

1. Determining the Scope of the Stress Test

Before beginning a stress test, we need to consider the investment problem the stress test is addressing. In other words, stress testing begins by specifying the scope of the test. Traditional stress testing involves specifying adverse market movements and the revaluation of the portfolio under these moves (Laubsch, 1999). However, the methodology of stress testing can be applied more broadly and to shocks of different scope. Stress tests of different scope can be relevant at different time horizons for strategic and tactical decisions. Apart from the traditional approach of shocking financial prices, we can also use stress testing to examine changes in expectations, and different ways of constructing portfolios, for example through changes in the degree of leverage. When examining investment decisions that involve long horizons, such as strategic asset allocation, shocks to expectations and portfolio construction methodologies can become particularly relevant.

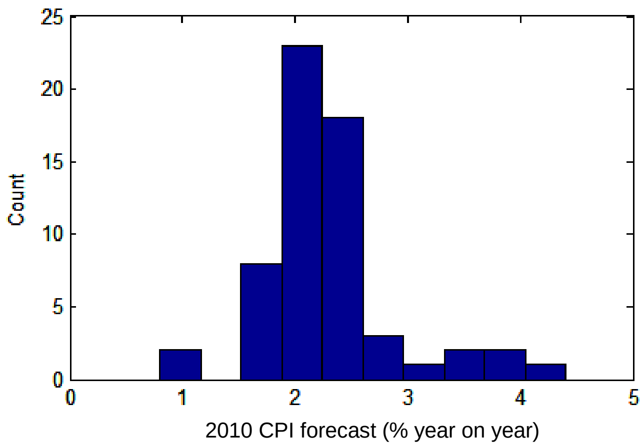
A *systemic* shock is the widest possible shock, which has the potential to affect all markets. An example of a systemic shock is a leverage or liquidity shock. When such shocks occur, as in 2008, it may result in unexpected increases in investment commitments that have no immediate funding source. While it may be difficult to predict the timing of such events, their impact can be analyzed using stress tests.

Macroeconomic shocks and marketwide shocks are more granular. An example of a *macroeconomic* shock is an interest rate shock or an oil price shock. Our risk management survey revealed that both asset managers and plan sponsors pay considerable attention to macroeconomic scenarios. A *market* shock can be viewed as an overall fall in stock market prices proxied by a particular index.

Targeted or factor shocks go into even more detail — such stress tests assess the impact on the portfolio of a shock to a particular market sector or stocks favored by a certain investment style (US Technology shock, Japan growth shock).

Macroeconomic variables can be particularly suited to scenario analysis for several reasons. First, macroeconomic forecasts are readily available and include predictions made by central banks, government agencies, broker-dealers and other private sector organizations, especially for developed markets. Second, the distribution of macroeconomic forecasts often lends itself to constructing several well defined scenarios. Figure 2 looks at the distribution of US consumer price forecasts for 2010 made in February of that year. While the majority of forecasts cluster around the central consensus view of 2.2%, there are clearly significant outlier scenarios at both the right and left tails of the distribution.

Figure 2: Distribution of US consumer price inflation forecasts



Many economic variables influence the profitability of the corporate sector and therefore its ability to generate cashflows and pay dividends to shareholders; these variables include evolution of gross domestic product (GDP) growth, business investment, and consumer spending. Moreover, macroeconomic factors such as inflation and interest rates have a direct impact on discount rates used in asset pricing. In practice, the forecast for the joint development of macroeconomic variables tends to be linked to a scenario for the performance of the economy. Different macroeconomic scenarios would have different implications for portfolio performance.

For example, in the second half of 2009 a steady flow of macroeconomic news reassured institutional investors that the global economy had turned around. In light of record low policy rates, widespread unconventional monetary policy measures, and fiscal stimulus, concerns about inflationary risk started to surface. While recovery seemed the central scenario, there was uncertainty about its future shape and strength, especially as stimulus measures might be removed. These factors, coupled with concerns about sovereign finances, contributed to market uncertainty in early 2010.

Frequently the set of factors used to analyze portfolio risk and return does not explicitly include macroeconomic variables. For example, fundamental factor models examine portfolio risk and return in terms of transparent and financially meaningful attributes of individual securities, such as balance sheet or income statement items, market capitalization, analyst forecasts, industry or country membership, etc. Factor returns are then estimated by cross-sectional regression. Statistical factor models extract factor exposures and returns directly from the asset return data using a version of principal component analysis. A straightforward method of adding a macroeconomic layer to the factor model is discussed in Melas and Liu (2007). We will examine the topic of stress testing with macroeconomic variables in greater detail in a later research bulletin.

2. Selecting a Severe, but Plausible Scenario

2.1. Historical and Hypothetical Scenarios

The usual guidance for selecting scenarios for stress testing is that such scenarios must be “severe but plausible”. For example, the Financial Services Authority (FSA 2009) specifies that firms should consider “a severe downturn scenario based on forward-looking hypothetical events that are calibrated against the most adverse movements in individual risk drivers experienced over a long historical period.” Traditionally, scenarios were designed using two methods—historical and hypothetical. *Historical* scenarios are based on events in the past, for example the emerging market debt and currency crises of the late 1990s. They are fully articulated and involve little judgment in implementation. They can be useful when some aspect of a historical scenario is expected to reoccur and the scenario is of an appropriate magnitude. However, such scenarios are backward looking and may lose relevance through time. Correlations between assets and asset classes can be viewed as a function of the institutional investment process. Past events can induce changes in this process, making historical relationships unreliable. For example, historical stress tests would not have captured the risks in new products that have been at the center of the recent credit crisis. As the Bank for International Settlements (BIS 2009) notes "…the severity levels and duration of the stress indicated by previous episodes proved to be inadequate. The length of the stress period was viewed as unprecedented and so historically based stress tests underestimated the level of risk and interaction between risks."

Hypothetical scenarios consider plausible future developments. They allow a flexible formulation of an event and can use a mixture of elements—a shock from a previous historical event can be combined with other developments that never occurred. The advantage of such scenarios is that they can be tailored to be relevant to the risk profile of the portfolio. However, the building of a well articulated hypothetical scenario can be a labor intensive process, especially if the underlying model considers many factors, and it is important to understand the implicit assumptions made in scenario construction. Unlike historical scenarios, hypothetical scenarios can involve simulating shocks that reflect structural breaks that never occurred. For example, we could create a hypothetical scenario that examines the impact of a country exiting the eurozone by making a prediction of returns, volatilities, and correlations that would apply in this scenario.

However, it may be difficult to convince decision makers that truly innovative scenarios are plausible and so the construction of hypothetical scenarios can still be limited by historical events. As BIS (2009) notes: "Scenarios that were considered extreme or innovative were often regarded as implausible by the board and senior management."

2.2. Reverse Stress Testing

Traditional approaches to stress testing first define what qualifies as a significant deterioration in portfolio risk factors and then assess the impact of these changes on the portfolio. For example, we might examine the impact of the rise in US interest rates by 100 bps on an international government bond portfolio. *Reverse* stress testing, on the other hand, is used to assess the resilience of a portfolio to extreme events by identifying which particular events could lead to losses that exceed a given level. It starts from an outcome, such as a portfolio loss, and identifies the circumstances that would cause this outcome to occur. Thus, reverse stress testing provides insight into likely scenarios that are the most relevant to the loss profile of a portfolio.

Let's consider some examples of reverse stress testing in a multifactor model. A reverse scenario can be built using the return and risk decomposition of a portfolio. This can be done in volatility space or in shortfall space. If we can decompose portfolio return into several components

$$r_P = \sum_k X_k r_k \ ,$$

then reverse stress testing may be thought of as a problem of determining the expected return of every factor, given that a portfolio sustains a loss of L, or, more formally

$$L_k = E\left[r_k \middle| r_P = L\right] = \beta_{k,P} L \ ,$$

where $\beta_{k,P}$ is the beta of factor k with respect to the portfolio.[3] The beta of the factor to the portfolio can be written as the marginal contribution to risk of the factor MCR_k divided by the total risk of the portfolio σ_P

$$L_k = E\left[r_k \middle| r_P = L\right] = \beta_{k,P} L = \frac{\operatorname{cov}(r_k, r_P)}{\sigma_P^2} L = \frac{MCR_k}{\sigma_P} L \qquad (1)$$

[3] Throughout the derivations in this section, for simplicity we assume that the mean portfolio return and mean factor returns are zero.

As an example, let us consider a simple equity portfolio that has exposures to only two factors: 0.5 exposure to value and 0.8 exposure to momentum. Let us assume that the volatilities of the two factors are 3% for value and 5% for momentum and the correlation between them is 0.2. Table 1 presents a range of analytics for this portfolio (the calculations for these analytics are given in the Appendix).

Table 1: Portfolio Characteristics

Factor	Exposure	Volatility	Marginal Contribution to Risk (MCR)	Beta to portfolio	Beta of portfolio to factor	Beta to value	Beta to momentum
Momentum	0.8	5%	0.0473	1.04	0.86	0.33	1
Value	0.5	3%	0.0152	0.33	0.77	1	0.12

Correlation (Value, Momentum) = 0.2
Portfolio volatility = 4.54%

Substituting $MCR_{momentum} = 0.0473$, $MCR_{value} = 0.0152$ and $\sigma_P = 4.54\%$ into equation (1), we see that a 10% loss in a portfolio can be caused by a momentum factor loss of 10.4% combined with a value factor loss of 3.3%, verified by noting that 0.5*3.3%+0.8*10.4%=10%. The probability of this scenario can be evaluated by comparing the size of the loss L with forecast portfolio volatility σ_P. In summary, this approach derives the expected returns for all factors given a certain loss in a portfolio.

An alternative approach looks at how specific factor shocks can impact the portfolio by considering individual factor shocks and employing the methodology of correlated stress tests. In correlated stress tests, the shock is driven by a certain factor and the returns of other factors are determined through the factor covariance matrix. For example, a portfolio manager might be interested in determining the size of a shock to momentum that would result in a 10% portfolio loss. In this case, we would look for the size of the shock driven by a single factor which, together with secondary effects given by the correlations of this factor with other factors, will give the specified portfolio loss *L*. We can back out the required return for the driving factor, given that we know the required portfolio loss and the factor covariance matrix, which tells us the expected returns of all the other factors given a return in the driving factor.

More formally, the return of a portfolio given a factor loss of L_k is given by

$$E\left(r_P \middle| r_k = L_k\right) = \beta_{P,k} L_k \,,$$

where $\beta_{P,k}$ is the beta of the portfolio with respect to factor *k*

$$\beta_{P,k} = \frac{\mathrm{cov}(r_P, r_k)}{\sigma_k^2} = \beta_{k,P}\frac{\sigma_P^2}{\sigma_k^2} = MCR_k\frac{\sigma_P}{\sigma_k^2}.$$

To derive the loss for the factor that drives the shock, we simply need to divide the required portfolio loss by the beta of the portfolio with respect to factor *k*

$$L_k = \frac{L}{\beta_{P,k}} = \frac{L\sigma_k^2}{MCR_k \sigma_P}\,. \tag{2}$$

For our simple example portfolio, we can calculate that a 10% loss in a portfolio can be driven by a 11.6% momentum factor loss. Because the correlation between the two factors is not zero, a return in the momentum factor also implies a certain return in the value factor. The expected loss in factor ν given a loss of L_μ in factor μ is given by

$$E(L_\nu | L_\mu) = \beta_{\nu,\mu} L_\mu. \tag{3}$$

where $\beta_{\nu,\mu}$ is the beta of factor ν to factor μ. These betas are shown in Table 1. Note that because the two factors are positively correlated, a loss of 11.6% in the momentum factor also implies a loss of 1.4% in the value factor. Similarly, using equation (2) and (3) we can calculate that a 10% loss in a portfolio can be driven by a 13% value factor loss, which implies a loss of 4.3% for the momentum factor, due to the positive correlation between value and momentum. To assess the relative probability of these two scenarios, we could compare the required factor loss with the volatility of the factors. The momentum-driven loss scenario looks the more likely of the two, since it is a 2.3 standard deviation event, compared to 4.3 standard deviations for value.

In this section, we looked at two methods for conducting reverse stress tests using a multifactor model. The first approach derives the expected returns to all factors given a portfolio loss. This takes the factor composition of the portfolio and derives how the portfolio loss is expected to be distributed among the different factor components. The second approach investigates single factor shocks, applied in a correlated fashion, that could lead to a certain portfolio loss. This approach is particularly useful when the portfolio or risk manager is concerned about the possibility of negative developments in specific factors.

Reverse stress testing provides a method of discovering scenarios that could lead to a specified portfolio loss. While it is mathematically straightforward to derive factor shocks that could result in a certain portfolio loss, some of these shocks could be assessed as unrealistic in light of what we know about the factor return distributions. Hence, as discussed in the examples above, we also need a mechanism to assess the probabilities of the derived shocks. A related technique, maximum loss, combines portfolio and market information to derive scenarios that are realistic and relevant. As described by Finger (2005), Studer (1999), and Studer and Luthi (1996), to define maximum loss, it is first necessary to define a set of risk factor scenarios referred to as the *trust region*. The second step is to find the worst portfolio loss over the scenarios in the trust region, which is referred to as the maximum loss. Like reverse stress testing, the maximum loss framework can also be used to generate stress scenarios that are realistic and most relevant to a particular portfolio.

3. Transmitting the Shock to the Portfolio

Two typical types of stress tests are sensitivity analysis and scenario analysis. Sensitivity analysis estimates the impact of a change in a single factor, while scenario analysis studies the effect of a simultaneous move in a group of risk factors.

Sensitivity tests are the most basic level of stress test, where a single parameter is shocked, often without relating the shock to the wider context of an underlying event or real world outcome. An example of such a test might be assessing the impact of a 200 bps shift in interest rates on a portfolio. The main benefit of these tests is that they can provide a fast initial assessment of portfolio sensitivity to a given risk factor and identify certain risk concentrations. They are most appropriate in situations where fluctuations in portfolio value depend primarily on a single source of risk. The results of a sensitivity analysis are easy to communicate to senior decision makers and provide an intuitive link between changes in risk parameters and outcomes. While sensitivity analysis focuses on shocking a single factor, such tests can be performed in a correlated manner, accounting for the expected co-movements between different factors.

Scenario analysis examines a portfolio's response to a complete scenario. It is reasonable to start by postulating the state of the world that concerns the decision maker and then inferring the movements of market variables in that state. Scenarios can be designed to encompass both movements in the levels of market variables (prices) and changes in underlying relationships between different assets or markets (volatilities and correlations). Such testing provides a more complete assessment of portfolio risk. As already noted, scenarios can be historical and hypothetical, with the latter being more difficult to construct in a comprehensive way, but potentially more relevant to the current investment environment and the risk profile of a portfolio. Ideally, hypothetical scenarios should be run as correlated shocks of the key risk drivers with other relevant variables (see Kupiec, 1998; or Rubandhas, 2007 for examples). However, correlations derived from "normal" market conditions may no longer be valid in extreme situations implied by a stress scenario, an issue to which we return later in this section.

It is worth noting that both sensitivity analysis and scenario analysis can be applied to problems of different scope, not just shocks to return drivers in a portfolio. For example, Ruban and Melas (2010) use sensitivity analysis to examine the risk reduction and return enhancement properties of risk parity strategies, which apply different degrees of leverage to the fixed income allocation in a multi-asset class portfolio.

Traditionally, stress testing was focused on shocking market prices or *levels* of risk factors. While changes in prices are a defining feature of stress events, it is also important to consider other changes in the structure of the return generating process during times of stress, especially when considering applications of stress testing in portfolio construction. Specifically, the second moments of asset price return distributions tend to change, as stress events are likely to be characterized by increases in *volatilities* as well as breakdowns in *correlations*.

There are a number of established empirical facts about the behavior of financial market volatility. First, volatility exhibits persistence, or a tendency to cluster, as noted by Mandelbrot (1963) and others since then. While periods of low volatility are typically followed by other periods of low volatility, large returns of either sign tend to be followed by other large returns. A quantitative manifestation of this is that while returns themselves may have low autocorrelation, absolute returns, or their squares, display a positive, significant and slowly decaying autocorrelation function for intervals ranging from a few minutes to a few weeks (Cont, 2005). Second, the response of volatility can be asymmetrical depending on the sign of the latest return. Volatility tends to be higher following a negative return shock than following a positive shock of the same magnitude. Another way of putting this is that return and volatility are negatively correlated.[4] For equity returns, this asymmetry is often referred to as the leverage effect, as labeled by Black (1976). However, it can also be due to time-varying risk premia (e.g. Campbell and Hentschel, 1992). If volatility is priced, then an expected increase in volatility raises the required return on equity, leading to an immediate decline in prices of risky assets. Therefore, it is important to incorporate changes in volatilities that could result from the stress event when transmitting the shock to the portfolio.

It is also known that periods of greater volatility suffer from the problem of "correlations breakdown" (Loretan and English, 2000), meaning that measured correlations between asset returns in volatile periods can differ substantially from those seen in quieter markets. One explanation for the increase in correlations during volatile periods is a shift in the joint distribution of asset returns due to market contagion, the nature of the shocks or changes in market structures and practices. However, even if the behavior of asset returns is governed by an unchanged process, measured correlations are likely to rise in periods of high measured volatility.[5] As stress testing often implies simulating a large return in one or more risk drivers, it is important to account for any regime shifts or breakdowns in correlations. Moreover, as correlation

[4] This empirical regularity has led to the development of several volatility trading strategies, see Briere, Burgues, and Signori (2010)
[5] To borrow an example from Loretan and English, consider the case of two independently and identically distributed bivariate normal random variables with zero means and standard deviation 1. One could take a large number of draws of pairs of these variables and split them into a low volatility subsample and a high volatility subsample using a threshold of 1.96 for one of the variables. The effect of trimming the ends off the joint distribution in a low volatility subsample would be to reduce the sample correlation between the variables. By contrast, the correlation in the high volatility subsample would be enhanced because the support of its distribution is disjoined, picking up large negative values and large positive values.

is one of the most important parameters in financial models, manipulating correlations is a vital aspect of stress testing. Risk and portfolio managers may be interested in the impact on their portfolio if correlations between certain assets and asset classes change.

Changing model correlations can be a challenging problem: the correlation matrix cannot be modified arbitrarily since modifications do not guarantee that it retains the positive semi-definite property. In other words, ad-hoc changes in correlation coefficients may make the matrix invalid. Moreover, modified correlations should ideally be consistent with some model of a return process, to avoid illogical changes. In Appendix 2 we illustrate a simple version of the latent factor approach to demonstrate a mechanism that can cause seemingly uncorrelated risk drivers to become highly correlated in times of stress (Lo, 2005). We believe this is a promising method for manipulating the correlation matrix for stress testing and other functions (see Bender, Lee and Stefek, 2010).

4. Stress Adjusted Portfolio Construction

In this section we show how to incorporate scenarios into portfolio construction. Our method is illustrated using an international government bond portfolio and two macroeconomic scenarios frequently mentioned by institutional investors.[6]

Under a benign macroeconomic scenario, governments reduce budget deficits through tighter fiscal policy, causing public debt to fall in future years. Interest rates and inflation remain low, while the yield curve remains steep. In a benign scenario, government bonds in southern Europe would rally as risk premia decline, while safe haven central European bond markets would remain unchanged.

In an adverse macroeconomic scenario, governments fail to reduce deficits and debt remains high for a prolonged period, leading to inflationary pressure. Interest rates and inflation rise, while sovereign bond performance suffers. In this scenario, bonds of highly indebted European countries would continue to trade at current high yield levels.

Typically, portfolio construction uses as inputs expected returns, which correspond to a single scenario for performance, and a covariance matrix, which accounts for the dispersion of returns around that mean scenario. However, a portfolio that is optimal for one set of expected returns and a covariance matrix will almost certainly not be optimal for another set of expected returns and a different covariance matrix. For example, a portfolio that is constructed for the benign scenario in which debt is under control and the yields and volatilities of Southern European sovereigns decline, is likely to be overweight the bonds of these countries. Such a portfolio could have negative returns and substantial risk in an adverse scenario of persisting deficits, resulting in yields and volatilities of Southern European bonds remaining at their current high levels. Conversely, a portfolio created for an adverse scenario is likely to be overweight bonds of relatively safe haven European countries and the US. Such a portfolio may miss out on return opportunities if a benign scenario occurs.

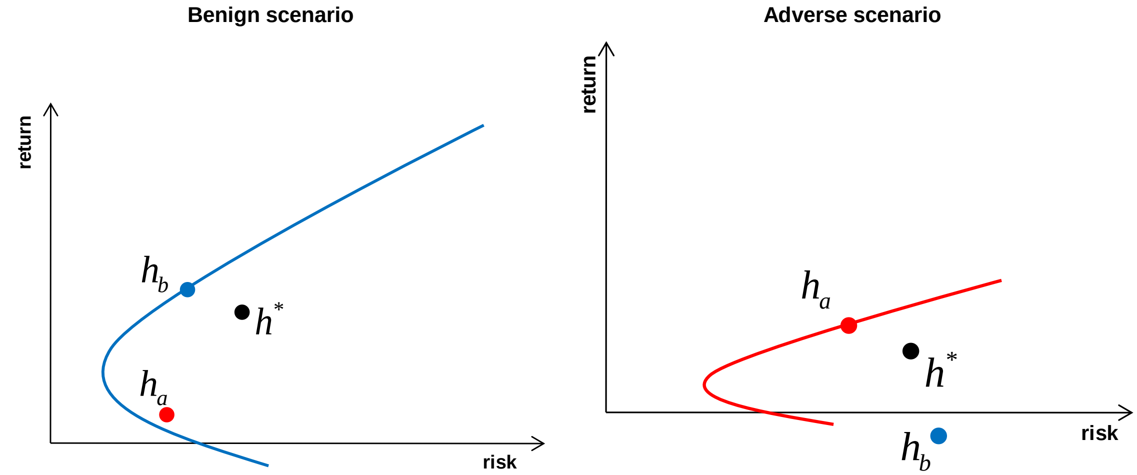
We illustrate this argument in Figure 3. A portfolio with weights h_b is optimal in the benign macro scenario and lies on the efficient frontier given the expected returns and the covariance matrix in the benign state of the world. However, this portfolio has high weights in assets that are expected to underperform if an adverse scenario occurs. Therefore, this portfolio will lie far below the efficient frontier in the adverse state of the world. Similarly, a portfolio with weights h_a that is optimal and lies on the efficient frontier in the adverse state of the world will lie below the efficient frontier in a benign scenario. As this portfolio was optimized for the adverse state of the world, it is not tilted towards securities expected to outperform in a benign state of the world. Hence,

[6] Please see the Financial Times article *Eurozone: State of the Union* on May 31 2010 for further scenarios related to the sovereign risk turmoil in the euro area.

constructing a portfolio with one specific scenario in mind may lead to sub-optimal performance if that scenario does not materialize.

Given that we cannot predict which scenario will occur, we could aim to design a portfolio that performs reasonably well in all scenarios, while not being optimal in any one of them. This is the portfolio with weights h^* in Figure 3, which lies below the efficient frontier in both the benign and adverse state of the world, but offers acceptable performance in both. We call this portfolio "event safe" since it hedges against a significant loss or high volatility of returns whichever state of the world occurs.

Figure 3: Constructing Event Safe Portfolios



Formally, one way to construct the portfolio h^* is to add constraints to the standard portfolio optimization problem. In the simplest case, we use the baseline scenario for optimization, but look for a portfolio that also has an acceptable risk-return profile should the other scenarios occur. In theexample above, if the portfolio manager believes a benign scenario is most likely to occur, he could optimize the portfolio using the expected returns and covariance matrix corresponding to the benign scenario, adding constraints that ensure an acceptable risk-return profile in the adverse scenario. Denoting the vectors of expected returns and the covariance matrices in the benign and adverse scenarios by r_b, r_a, V_b and V_a respectively (where the subscript *b* corresponds to the benign scenario and the subscript *a* to the adverse scenario), we could construct the portfolio h^* as the solution to the following optimization problem

Maximize	$h' r_b - \lambda h' V_b h$	
subject to:	$h' r_a > c_r$	minimum return in the adverse scenario
	$h' V_a h < c_\sigma$	maximum risk in the adverse scenario.

Additional constraints, such as a constraint on portfolio shortfall, can be added and constraints based on multiple scenarios can be incorporated.[7] A key advantage to this simple framework is that it doesn't require assigning probabilities to the different scenarios. It can also be used with portfolio construction processes other than mean variance optimization — for example, we can determine asset weights based on constraints in adverse scenarios. The difference between the expected return for the event safe portfolio h^* and a portfolio that lies on the efficient frontier in the benign scenario and has the same level of risk can be viewed as an insurance premium that pays for better performance in the adverse scenario.

[7] Note that as the number of scenarios grows, it is likely to become harder to satisfy all constraints.

For a simple numerical illustration of this concept, consider an investor who wants to create a portfolio of 10-year government bonds from the following 5 countries: US, UK, Japan, Greece, and Germany. Assume that this portfolio manager wants to consider two scenarios. In a benign scenario, there is a gradual recovery in the debt of Southern European countries, represented by Greece. The other scenario is an adverse scenario with current volatility in sovereign debt markets continuing and investors increasingly concerned about the fiscal outlooks in other developed markets, represented here by UK and Japan. The returns, volatilities, and correlations corresponding to these scenarios are presented in Table 2.[8]

Table 2: Returns, Volatilities and Correlations of 10 year Government Benchmarks in different scenarios

Benign scenario

	Greece	Germany	US	UK	Japan
Return	0.06	0.03	0.04	0.04	0.01
Risk	0.06	0.06	0.08	0.07	0.05
Correlations					
	Greece	Germany	US	UK	Japan
Greece	1.00	0.80	0.67	0.75	0.18
Germany	0.80	1.00	0.82	0.91	0.22
US	0.67	0.82	1.00	0.77	0.20
UK	0.75	0.91	0.77	1.00	0.27
Japan	0.18	0.22	0.20	0.27	1.00

Adverse scenario

	Greece	Germany	US	UK	Japan
Return	0	0.03	0.01	0	-0.02
Risk	0.09	0.06	0.08	0.08	0.08
Correlations					
	Greece	Germany	US	UK	Japan
Greece	1.00	0.07	0.08	0.69	0.60
Germany	0.07	1.00	0.72	0.40	0.59
US	0.08	0.72	1.00	0.50	0.49
UK	0.69	0.40	0.50	1.00	0.56
Japan	0.60	0.59	0.49	0.56	1.00

Note that in the benign scenario, the risk for the Greek benchmark declines from the current high levels and converges to that of the German benchmark. The Greek benchmark also exhibits a moderate rally. The historical correlations typical in the calm market conditions preceding the 2007-2010 period re-establish themselves, with the Greek benchmark becoming increasingly correlated to the German benchmark. In the adverse scenario, the risk for the Greek benchmark also decreases from the current elevated levels, but remains higher than the German benchmark. There is no rally in returns for the Greek bonds, which continue to trade at their current distressed levels. UK and Japanese bonds also experience weaker performance and slightly higher volatilities than those seen in the benign scenario. The correlations between the Greek benchmark and the German benchmark remain close to zero, while the correlations of Greek bonds with UK and Japanese bonds rise, reflecting rising investor concerns about similarities in fiscal outlooks.[9]

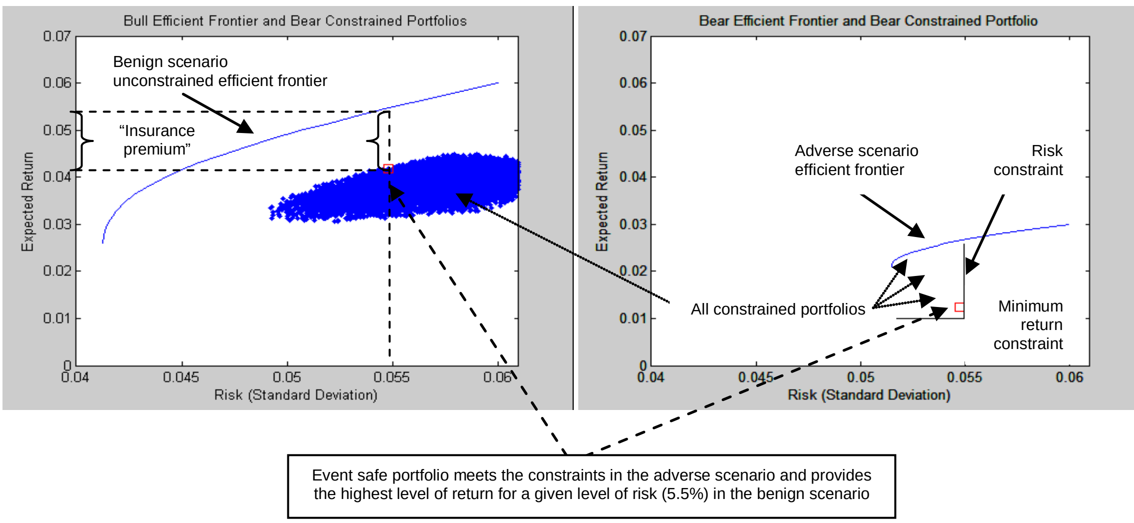
Figures 4 and 5 illustrate the process of constructing the "event safe" portfolios. First, we find the set of portfolios that meet two constraints in the adverse scenario: the portfolio must have a return of at least 1% and risk of no more than 5.5% in the adverse scenario. From this set, we then select a portfolio that offers the best return in the benign scenario for a given level of risk, which we set to 5.5%. Figure 4 shows the risk-return combinations attainable in the benign scenario, while Figure 5 shows the same for the adverse scenario. If we compare the unconstrained efficient frontiers available in the two scenarios, we notice that the adverse scenario frontier lies lower and to the right of the benign scenario frontier. In other words, you would attain a lower level of return for any given level of risk in the adverse scenario, while the minimum level of risk attainable using only the five assets we consider is higher in the adverse scenario. The set of portfolios that meets the minimum return and maximum risk constraints in the adverse scenario lies below the efficient frontier and above and to the left of the two lines that specify the constraints in Figure 5. The risk and return of all portfolios in this set in the benign scenario is given by the cloud of points that lies below and to the right of the efficient frontier in Figure 4. To find the "event safe" portfolio for any given level of risk, we chose the highest return portfolio for the level of risk from the "cloud" of all portfolios that meet our constraints. The risk

[8] The covariance matrices were ensured to be positive semi-definite, however the inputs are hypothetical.
[9] For a detailed analysis of issues underlying the current turmoil in sovereign debt markets see Iyer, Ruban and Vannerem (2010).

and return of this portfolio in the benign and adverse scenarios is highlighted by the square in both Figures 4 and 5. The reduction in the return in the benign scenario relative to a portfolio with the same level of risk that lies on the unconstrained efficient frontier is the “insurance premium” for the minimum performance requirements in the adverse scenario.

Figure 4: Benign scenario efficient frontier and constrained portfolios

Figure 5: Adverse scenario efficient frontier and constrained portfolios



A more sophisticated approach to constructing “event safe” portfolios is to assign probabilities to different scenarios and perform a probability weighted unconstrained optimization. While a detailed discussion about assigning probabilities to scenarios is beyond the scope of this paper, it is worth mentioning a few possible techniques. A historical method would examine the distributions of relevant variables, possibly re-scaled to make them consistent with current volatility levels, to imply the likelihood of shocks of different sizes. Alternatively, a fundamental valuation model would give an indication of the deviation from “fair value” (see Muellbauer and Murphy, 1997, for an example) and relate the size of a deviation to the likelihood of a correction. A variation of this approach would be to use a time-series (or technical) trend model, which can range from simple linear trends to univariate and multivariate filters. These tools can also help identify significant deviations of variables from their long-run levels. Finally, it may be possible to use prices of financial instruments to help imply a market assessment of the probabilities of extreme price movements. Derivative prices can derive risk-neutral probabilities of large movements in prices,[10] while tools such as structural models of default can relate information in one market (equities) to the probability of extreme price movements in another (corporate debt).

5. Conclusions

In this paper, we presented a framework for conducting effective stress tests and incorporating insights from stress tests in portfolio construction. Stress testing can be a useful complement to risk model outputs, such as volatility, VaR, and expected shortfall. The key advantage of stress tests is that the loss is linked to a specific event, which can be more meaningful to portfolio managers than a summary statistic of a loss distribution. Stress tests can be valuable at all stages of the investment process, including portfolio construction, limit setting, and hedging.

[10] Note that physical probabilities of large negative price movements are likely to be smaller than risk-neutral (or price implied) probabilities, if investors are risk averse.

Prior research on stress testing has concentrated on ways to develop realistic and relevant shocks. The framework presented here attempts to expand on this, by illustrating that stress testing is a broader process addressing a wide range of investment problems and is useful in all stages of investment decisions.

We started by examining the foundations of stress testing. We looked at the type of investment problem to address with a stress test—the scope of the test. The scope of the stress test determines how general or specific the shock should be. The methodology of stress testing can be applied to a wide range of possible shocks, from systemic shocks that have a direct effect across all markets, to targeted shocks that directly impact only a small subset of the investment universe.

Next, we examined decisions made while running the stress test. We first outlined the different methods for constructing the scenario. Traditionally, stress tests have been constructed either by relying on historical developments or considering hypothetical scenarios. Recently, reverse stress testing—starting from the makeup of the portfolio and determining the events necessary to result in a severe loss—has gained in popularity. We illustrated two complementary techniques of reverse stress testing using a factor model of risk.

We then discussed how to select the type of stress test. Sensitivity analyses are quick and easy to run and provide the initial view of the shock impact, especially for portfolios with one dominant source of risk. Scenario analyses provide a more comprehensive assessment of portfolio risk, but with more complex implementation.

This step also established a way of transmitting the shock to the portfolio. Shifts in volatilities and correlation breakdowns are important features of stress events. Correlations can change significantly under extreme conditions. Therefore, stressing the correlations between risk drivers is potentially as important as stressing the levels of risk drivers. Changing model correlations is a challenging problem: the correlation matrix cannot be modified arbitrarily because such modifications do not guarantee that the matrix retains the positive semi-definite property. In other words, ad-hoc changes in correlation coefficients may make the matrix invalid. In Appendix 2, we illustrate an approach that can mitigate this problem.

Finally, we examined how to incorporate the results of stress tests in portfolio construction, by examining methodologies to build “event safe” portfolios. These portfolios provide protection against event risk by ensuring that the portfolio meets certain performance criteria in all scenarios.

This paper provided an outline for the use of stress testing in the investment process. In the future, we will examine other related topics, such as constructing macroeconomic scenarios and translating macroeconomic shocks to asset prices, as well as using the latent factor approach to conduct stress tests with “extreme” correlations.

Bibliography

Bender, J., J.-H. Lee and D. Stefek, 2010. “Manipulating Correlations through Latent Drivers.” MSCI Research Insight.

Bank for International Settlements, 2009. “Principles for Sound Stress Testing Practices and Supervision.” Consultative Document.

Black, F. 1976. “Studies of Stock Market Volatility Changes.” *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, 177-181.

Brière, M., A. Burgues and O. Signori, 2010. “Volatility Exposure for Strategic Asset Allocation.” *The Journal of Portfolio Management*, 36(3): 105-116.

Cont, R. 2005. “Volatility Clustering in Financial Markets: Empirical Facts and Agent—Based Models.” *in A. Kirman and G. Teyssiere (eds.) Long Memory in Economics*. Springer.

Financial Services Authority. 2009. “Stress and Scenario Testing.” Policy Statement

Finger, C. C. 2005. “What’s the Worst that Could Happen?” RiskMetrics Group Research Monthly.

Iyer, A. S., O. Ruban and P. Vannerem. 2010. “The Curse of Olympian Spending with International Borrowing”. MSCI Research Insight.

Jorion, P. 2007. *Value at Risk.* McGraw Hill.

Kupiec, P. 1998. “Stress Testing in a Value at Risk Framework.” *The Journal of Derivatives*, 6: 2-24.

Laubsch, A. J. 1999. *Risk Management: A Practical Guide.* Risk Metrics Group

Lo, A. 2005. *The Dynamics of the Hedge Fund Industry.* The Research Foundation of CFA Institute.

Loretan, M. and W. B. English. 2000. “Evaluating “Correlation Breakdowns” during Periods of Market Volatility.” Board of Governors of the Federal Reserve System International Finance Discussion Papers, Number 658.

Mandelbrot, B. 1963. “The Variation of Certain Speculative Prices.” *The Journal of Business*, 36: 394-419.

Melas, D. and Y. Liu. 2007. “Macroeconomic Factors in a Fundamental World.” MSCI Research Insight.

Muellbauer, J. and A. Murphy. 1997. “Booms and Busts in the UK Housing Market.” *Economic Journal*, 107(445): 1701-27.

Ruban, O. and D. Melas. 2010. “The Perils of Parity.” MSCI Barra Insight.

Rubandhas, S. 2007. “Stress Testing in a Multi-Factor Framework.” *MSCI Horizon Newsletter*, Q3-Q4 2007.

Studer, G. 1999. “Market Risk Computation for Nonlinear Portfolios.” *Journal of Risk,* 1(4): 33-55.

Studer, G. and Luthi, H.-J. 1996. “Quadratic Maximum Loss for Risk Measurement of Portfolios.” RiskLab Technical Report.

Appendix 1: Reverse Stress Testing Example Calculations

We can define the beta of a factor to a portfolio in the standard way as

$$\beta_{k,P} = \frac{\mathrm{cov}(r_k, r_P)}{\sigma_P^2}.$$

As shown by Goldberg, Hayes, Menchero and Mitra (2009), the marginal contribution to risk of a factor can be written as

$$MCR_k = \beta_{k,P}\sigma_P = \frac{\mathrm{cov}(r_k, r_P)}{\sigma_P}$$

or

$$\beta_{k,P} = \frac{MCR_k}{\sigma_P}.$$

As the marginal contribution to risk is a standard output of Barra analytics, it is helpful to define other relevant parameters.

The beta of a portfolio to a factor, $\beta_{P,k}$, can be expressed as

$$\beta_{P,k} = \frac{\mathrm{cov}(r_k, r_P)}{\sigma_k^2} = MCR_k \frac{\sigma_P}{\sigma_k^2}.$$

Similarly, the beta of factor ν to factor μ is

$$\beta_{\nu,\mu} = \frac{\mathrm{cov}(r_\mu, r_\nu)}{\sigma_\mu^2} = \frac{\sigma_\nu}{\sigma_\mu}\rho_{\nu,\mu} \tag{4}$$

For the simple portfolio considered in Table 1, we can calculate

$$\mathrm{cov}(r_{value}, r_P) = \mathrm{cov}(r_{value}, X_{value}r_{value} + X_{momentum}r_{momentum}) = X_{value}\sigma_{value}^2 + X_{momentum}\sigma_{momentum}\sigma_{value}\rho =$$
$$= 0.5 * 0.03^2 + 0.8 * 0.05 * 0.03 * 0.2 = 0.00069$$

Similarly

$$\mathrm{cov}(r_{momentum}, r_P) = 0.00215.$$

The volatility of the portfolio is given by

$$\sigma_P = \sqrt{(X_{momentum}\sigma_{momentum})^2 + (X_{value}\sigma_{value})^2 + 2X_{momentum}X_{value}\sigma_{momentum\sigma}\sigma_{value}\rho} =$$
$$= \sqrt{(0.8 * 0.05)^2 + (0.5 * 0.03)^2 + 2 * 0.8 * 0.5 * 0.05 * 0.03 * 0.2} = 0.0454$$

Hence, we can calculate

$$MCR_{momentum} = \frac{0.00215}{0.0454} = 0.0473$$

$$MCR_{value} = \frac{0.00069}{0.0454} = 0.0152$$

$$\beta_{momentum,P} = \frac{0.0473}{0.0454} = 1.04$$

$$\beta_{value,P} = \frac{0.0152}{0.0454} = 0.33$$

$$\beta_{P,momentum} = 0.0473 \times \frac{0.0454}{0.05^2} = 0.86$$

$$\beta_{P,value} = 0.0152 \times \frac{0.0454}{0.03^2} = 0.77$$

$$\beta_{momentum,value} = \frac{0.05}{0.03} \times (0.2) = 0.33$$

$$\beta_{value,momentum} = \frac{0.03}{0.05} \times (0.2) = 0.12$$

For the first approach to reverse stress testing, given a portfolio loss L=10%, we obtain, using equation (1)

$$L_{momentum} = \frac{0.0473}{0.0454} \times 0.1 = 10.4\%$$

$$L_{value} = \frac{0.0152}{0.0454} \times 0.1 = 3.34\%$$

For the second approach to reverse stress testing, in the momentum scenario we have, using equations (2) and (3)

$$L_{momentum} = \frac{0.1 \times (0.05)^2}{0.0473 \times 0.0454} = 11.6\%$$

$$L_{value|momentum} = 0.12 * 0.116 = 4.35\%$$

Similarly, for the value scenario

$$L_{value} = \frac{0.1 \times (0.03)^2}{0.0152 \times 0.0454} = 13.04\%$$

$$L_{momentum|value} = 0.33 * 0.1304 = 4.35\%$$

Appendix 2: Phase locking behavior and latent factors

As noted by Lo (2005), phase locking behavior describes sudden changes from low to high correlations in natural sciences. Actions that are uncorrelated the majority of the time may suddenly become correlated in the presence of latent factors. As an example, we can consider two risk factors with return generating processes given by

$$\begin{cases} f_1 = \bar{f}_1 + S_t \cdot Z_t \\ f_2 = \bar{f}_2 + S_t \cdot Z_t \end{cases}.$$

S_t is a phase locking indicator, such that it equals to 0 with probability 99% and equals to 1 with probability 1% and Z_t is some unobservable common driver or latent common factor. Let us further assume that $\bar{f}_1$ and $\bar{f}_2$ have an equal variance of σ_f^2 and that they are uncorrelated, that is $\text{cov}(\bar{f}_1, \bar{f}_2) = 0$. Further let's assume that Z_t has a considerably higher variance than $\bar{f}_i$, that is $\sigma_Z^2 = 10\sigma_f^2$. The returns of f_1 and f_2 are the sum of two components, their returns in "normal" times, which are uncorrelated and a highly volatile phase-locking component, which is identical for the two factors, but equals zero with a 99% probability. Most of the time the returns of the factors are determined by $\bar{f}_1$ and $\bar{f}_2$ and hence uncorrelated. However, with 1% probability the returns of both factors are dominated by Z_t. More formally, we can show that while unconditional correlation between the factors is approximately 0.1, the correlation conditional on $S_t = 1$ is 0.9 (and, of course, the correlation conditional on $S_t = 0$ is zero). Measured correlations between f_1 and f_2 will reflect the unconditional correlation and a standard risk assessment will not be able to detect the impact of Z_t.

Contact Information

clientservice@msci.com

Americas	
Americas	1.888.588.4567 (toll free)
Atlanta	+ 1.404.551.3212
Boston	+ 1.617.532.0920
Chicago	+ 1.312.675.0545
Montreal	+ 1.514.847.7506
Monterrey	+ 52.81.1253.4020
New York	+ 1.212.804.3901
San Francisco	+ 1.415.836.8800
Sao Paulo	+ 55.11.3706.1360
Stamford	+1.203.325.5630
Toronto	+ 1.416.628.1007

Europe, Middle East & Africa	
Amsterdam	+ 31.20.462.1382
Cape Town	+ 27.21.673.0100
Frankfurt	+ 49.69.133.859.00
Geneva	+ 41.22.817.9777
London	+ 44.20.7618.2222
Madrid	+ 34.91.700.7275
Milan	+ 39.02.5849.0415
Paris	0800.91.59.17 (toll free)
Zurich	+ 41.44.220.9300

Asia Pacific	
China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+ 852.2844.9333
Seoul	+827.0768.88984
Singapore	800.852.3749 (toll free)
Sydney	+ 61.2.9033.9333
Tokyo	+ 81.3.5226.8222

www.mscibarra.com | www.riskmetrics.com

Notice and Disclaimer

About MSCI

MSCI Inc. is a leading provider of investment decision support tools to investors globally, including asset managers, banks, hedge funds and pension funds. MSCI products and services include indices, portfolio risk and performance analytics, and governance tools.

The company’s flagship product offerings are: the MSCI indices which include over 120,000 daily indices covering more than 70 countries; Barra portfolio risk and performance analytics covering global equity and fixed income markets; RiskMetrics market and credit risk analytics; ISS governance research and outsourced proxy voting and reporting services; FEA valuation models and risk management software for the energy and commodities markets; and CFRA forensic accounting risk research, legal/regulatory risk assessment, and due-diligence. MSCI is headquartered in New York, with research and commercial offices around the world.