The Perils of Parity

Investigating the Role of Leverage in Multi-Asset Class Portfolios

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Introduction

In recent months, several large asset owners reported that they were considering the addition of leverage to their multi-asset class portfolios. These institutional investors noted that equity allocations introduce more volatility than their overall weighting in a typical multi-asset class portfolio. They argue that a levered fixed income allocation will reduce volatility by reducing equity exposures and increasing allocations to lower returning and lower risk diversifying assets, while still meeting expected actuarial returns.

The introduction of leverage to the fixed income allocation is linked to the concept of "risk parity" portfolios. Proponents of this concept argue that in a well diversified portfolio each asset class should provide an equal contribution to overall portfolio risk, reducing the potential loss impact from any individual asset class allocation.[1] Unlike the traditional approach, where capital allocation drives the risk of the portfolio, in risk parity portfolios the risk allocation drives the capital allocation. When a portfolio is constructed using this concept, it is dominated by low volatility/low returning assets. As noted by Qian (2005) and Allen (2010), leverage is necessary to achieve the expected return required by institutional investors. To balance the risk profile of a portfolio, leverage is added to the fixed income allocation.

In this paper, we derive the conditions necessary for a portfolio with a levered fixed income allocation to:

a) achieve lower volatility than an unlevered portfolio; and
b) achieve a better risk-return profile than an unlevered portfolio.

We show that these conditions define a threshold for correlations between the core unlevered portfolio and the extension levered allocation. For simple two asset class portfolios with equity and fixed income allocations, these conditions can be expressed in terms of correlations between equities and bonds. The potential benefits of adding leverage to reduce volatility depends on the correlations between bonds and equities, the relative volatility of bonds versus equities, and the weights of the two asset classes in the portfolio. If the goal is to improve the risk-return profile of the portfolio, the decision to add leverage also depends on the relative Sharpe-ratios of the core portfolio and the extension. When the portfolio includes only equities and fixed income, we can examine the problem in terms of relative Sharpe ratios of equities and bonds. We proceed to examine the empirical plausibility of the derived conditions by looking at the historical behavior of correlations, volatilities, and Sharpe ratios between equities and fixed income. Asset owners considering adding leverage to their fixed income allocation can examine these influences to decide whether negative correlations between bonds and equities, a low ratio of bond to equity volatility, and higher risk-adjusted returns of bonds relative to equities are likely to persist.

1. Can a Levered Fixed Income Allocation Reduce Portfolio Volatility?

In this section we examine whether adding leverage to the fixed income allocation can reduce overall portfolio risk. It is important to examine this independent of any return enhancement provided by a levered fixed income allocation, as risk reduction can be valuable even if the return of the levered extension is negative. Risk reduction is also frequently cited as the major reason for risk parity type strategies.

[1] For examples, see Qian (2005), Morris and Haeusler (2010).

Consider an unlevered (long only) reference portfolio with positive weights w_B in fixed income and $w_E = 1 - w_B$ in equities. As shown in the Appendix, adding leverage to the fixed income allocation lowers portfolio volatility when the correlation between bond and equity returns ρ satisfies:

$$\rho < -\frac{1}{2} \cdot (1+k) \cdot \frac{w_B}{w_E} \cdot \frac{\sigma_B}{\sigma_E}, \qquad (1)$$

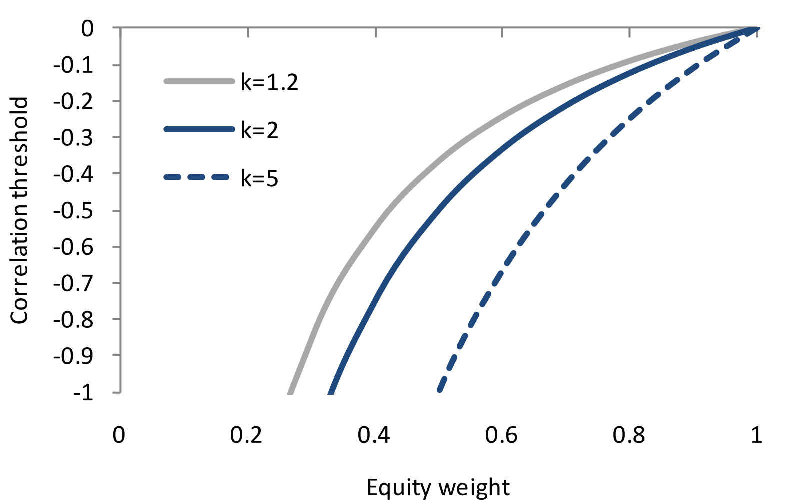
where k is the leverage coefficient applied to the fixed income allocation ($k>1$), σ_E is the volatility (standard deviation) of the equity allocation, and σ_B is the volatility of the fixed income allocation. We see that this correlation threshold becomes more negative as:

1. The leverage coefficient k rises;
2. The weight of fixed income in the portfolio w_B rises; and
3. The ratio of bond to equity volatility σ_B / σ_E rises.

For example, assume that a portfolio has an allocation of 50% equity and 50% fixed income, and the ratio of bond to equity volatility is $\sigma_B / \sigma_E = 1/3$ (equities are three times more volatile than bonds). If the fixed income component is levered 2 times, this portfolio will have lower volatility than the unlevered portfolio only if the correlation between fixed income and equity $\rho < -0.5$.

Figure 1 shows the maximum correlation for which a levered portfolio has lower volatility than a corresponding unlevered portfolio as leverage varies in the fixed income allocation. Here we assume that equities are three times as volatile as bonds and plot threshold correlations against the weight of equity in the portfolio. The chart shows, for example, that if the fixed income component is levered 5 times, it becomes theoretically impossible for a levered portfolio to have lower volatility relative to an unlevered portfolio if the equity weight is 50% or below. In Section 3, we examine the empirical plausibility of Formula (1) by looking at the historical behavior of correlation ρ between stocks and bonds and the ratio of bond to equity volatilities σ_B / σ_E.

Figure 1: Threshold bond-equity correlation that leads to a levered portfolio with lower volatility than a corresponding unlevered volatility (assumes equities are three times riskier than bonds)



2. What is the Effect of Leverage on Portfolio Risk-Return Profile?

Qian (2009) argues strongly in favor of building risk parity portfolios noting that "…backtest results show that in all cases without exception, the Risk Parity portfolios provide Sharpe ratios with higher long-term returns relative to traditional benchmarks with similar or lower risk". In the Appendix, we show that adding a levered extension to an unlevered core portfolio generally improves the portfolio's risk return profile (Sharpe ratio) when the gross amount of portfolio leverage *K* satisfies:

$$K < \frac{\sigma_C}{\sigma_X}\left(\frac{s_X}{s_C} - 2\rho_X\right),$$

where σ_X is the volatility of the portfolio extension, σ_C is the volatility of the core portfolio, s_X and s_C are the Sharpe ratios of the portfolio extension and the core portfolio respectively, and ρ_X is the correlation coefficient between the returns of the core portfolio and the extension portfolio.[2] We see that the threshold amount of leverage falls when:

1. The ratio of core of volatilities of the core portfolio and portfolio extension σ_C / σ_X falls
2. The ratio of Sharpe ratios of the extension and core portfolios s_X / s_C falls
3. The correlation between the core portfolio returns and portfolio extension returns ρ_X rises.

Qian (2005) notes that risk parity portfolios are mean-variance optimal if the underlying components have equal Sharpe ratios and their returns are uncorrelated. Equal Sharpe ratios have some theoretical appeal—they imply that expected return is proportional to risk for each asset class or that assets are priced by their risk. As we will show in Section 3 however, empirically *average* correlations between bonds and equities have been close to zero and slightly positive during the last 35 years. Assuming that the portfolio extension is one of the asset classes in the original portfolio (e.g. bonds), that equities are three times riskier than bonds, and equity returns are uncorrelated with fixed income returns, we can derive that for a 50% equity and 50% fixed income allocation the ratio $s_X / s_C \approx 0.8$ and the ratio $\sigma_X / \sigma_C \approx 0.63$. Therefore, to improve the Sharpe ratio, gross leverage *K* must be no larger than 0.25 or the leverage applied to the fixed income allocation should not be higher than 1.5.

Figure 2 shows the maximum amount of leverage *k* that can be applied to the fixed income allocation for a portfolio with a levered portfolio to have a more desirable risk-return profile relative to an unlevered portfolio. Here we assume that equities are three times as volatile as bonds and plot threshold leverage coefficients *k* against the correlation between equities and fixed income ρ for different levels of the relative Sharpe ratio of equities to bonds. It is clear that the amount of leverage that will improve the risk-return profile of the levered portfolio relative to an unlevered portfolio, for a given level of correlation between bonds and equities, critically depends on the relative risk-adjusted performance of the two asset classes. If bonds have a Sharpe ratio of zero or below, then a levered fixed income allocation will improve the Sharpe ratio of the portfolio only if the correlations between bonds and equities are negative. A portfolio where the fixed income component is levered 2 times will have a better risk-return profile relative to an unlevered portfolio for a given level of bond-equity correlation only if the ratio of Sharpe ratios of bonds to equities is greater than 4. It is clear that adding a levered fixed income allocation to a corresponding unlevered portfolio is most advantageous from a risk-return standpoint when the correlations between bonds and equities are low and bond risk adjusted returns are higher than equity risk adjusted returns. Table 1 in the Appendix constructs several typical scenarios for portfolio

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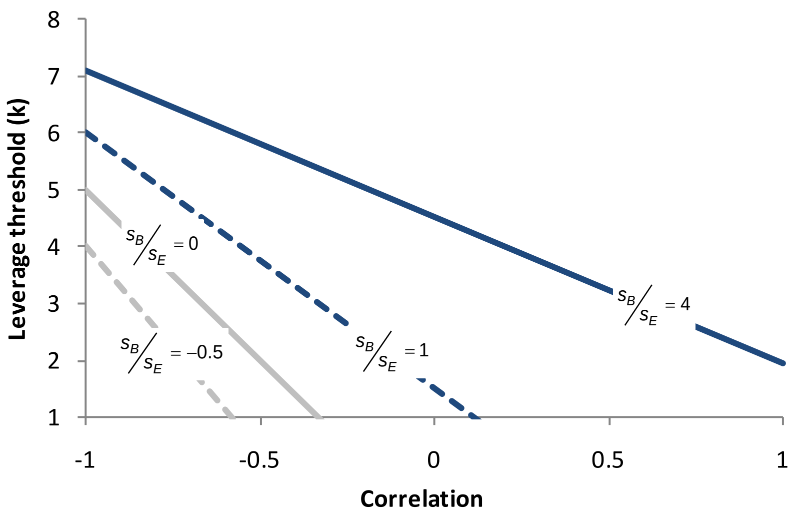
[2] In the simple two asset class portfolio with a levered fixed income allocation, note that $K = (k-1)w_B$ and $\rho_X = \frac{1}{\sigma_C}\left[w_E \sigma_E \rho + w_B \sigma_B\right]$, where *k* is the amount of leverage applied to the fixed income allocation

weights, asset class returns and volatilities, correlations between equities and fixed income, as well as the amount of leverage. It examines the impact of these variables on the risk and return of levered and unlevered portfolios.

Figure 2: Threshold leverage coefficients *k* that lead to a levered portfolio with a better risk-return profile than a corresponding unlevered portfolio (assumes a 50% equity, 50% fixed income allocation and that equities are three times riskier than bonds)



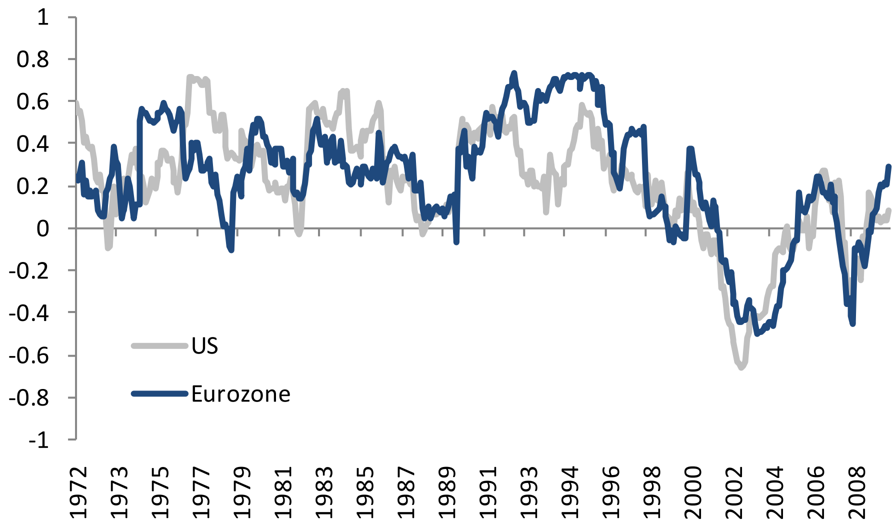
3. Historical Behavior of Bond-Equity Correlations and Volatilities

Figure 3 shows that the stock-bond return correlation displays substantial time variation. The average correlation over the last 40 years (1970-2010) was 19% in the US and 24% in Europe.[3] This is consistent with academic studies looking at longer horizons.[4] However, it rises as high as 70% and falls as low as -60%. Sustained negative realized correlations are confined to the last 10 years in both the US and Europe.

Campbell and Ammer (1993) note three main offsetting fundamental effects underlying the correlation between stock and bond returns. Variations in real interest rates promote a positive correlation, since the prices of both stocks and bonds are negatively related to the discount rate. On the other hand, variation in expected inflation can promote a negative correlation, since high inflation has a negative effect on bond returns and an ambiguous effect on stock returns. Furthermore, common movements in future expected returns promote a positive correlation. As an example of these influences during a historical period, the large negative correlations observed around 2002-2003 are often attributed to a deflation scare, where bad economic prospects drove stock market values lower, while low inflation expectations drove up bond market values.

[3] Note that these figures were calculated with government bonds. Corporate bond returns are likely to have higher correlations with equities, as spreads tend to fall when equity prices rise.
[4] Campbell and Ammer (1993); Baele, Bekaert and Inghelbrecht (2009)

Figure 3: Historical realized correlation between government bonds and equities (24-month rolling correlations)



There are effects unrelated to long-run economic dynamics that have a significant influence on stock-bond correlations. For example, Connolly, Stivers, and Sun (2005) attribute some of the negative stock-bond return correlations observed since 1997 to a flight-to-safety phenomenon, finding that stock and bond return co-movements are negatively and significantly related to stock market uncertainty. Baele, Bekaert, and Inghelbrecht (2009) find that heteroskedasticity in macroeconomic shocks is a key driver of the time-variation in stock-bond return co-movements. They speculate that the lower variability of inflation and output growth observed during the so-called "Great Moderation" could lead to lower correlation between stock and bond returns. They also find that changes in stock and bond market liquidity significantly affect co-movements. There are two main reasons for this. First liquidity may affect the betas, as economic shocks may not be transmitted quickly to the observed returns in illiquid markets. Second, liquidity may be a priced factor—shocks that improve liquidity should increase returns and the impact on equity-bond correlations then depends on how liquidity shocks co-move across markets.

The relative volatility of bonds to stocks also exhibits a substantial time variation, as illustrated in Figure 4. In times of high inflation, bond volatility rises and hence the ratio of bond to equity volatility also rises (for example early 1980s in the US). The ratio of bond to equity volatility also tends to rise in periods when stock market risk falls (for example the mid 1990s or the mid 2000s). The average value of σ_B / σ_E since 1970 is 0.45 in the US and 0.28 in the Eurozone.

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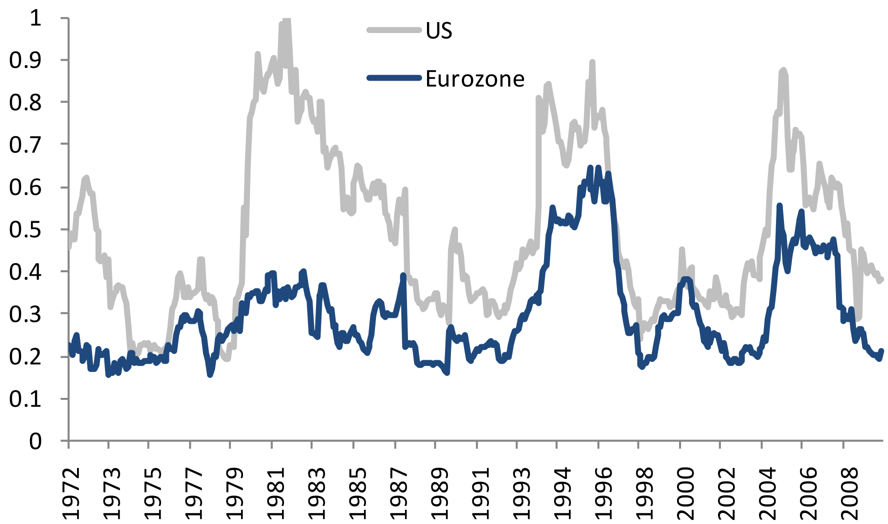
Figure 4: Ratio of bond to equity risk (24 month rolling standard deviations)



The relative Sharpe ratios of fixed income and equities have also seen significant variation through the last 35 years, as illustrated for the US in Figure 5. While sustained negative fixed income Sharpe ratios have not been seen since the fast rise of interest rates in the late 1970s, periods of rising interest rates often coincide with bond Sharpe ratios close to zero, as seen around 1994 and between 2004-2006.

Figure 5: US equity and fixed income Sharpe ratios (24 month rolling)

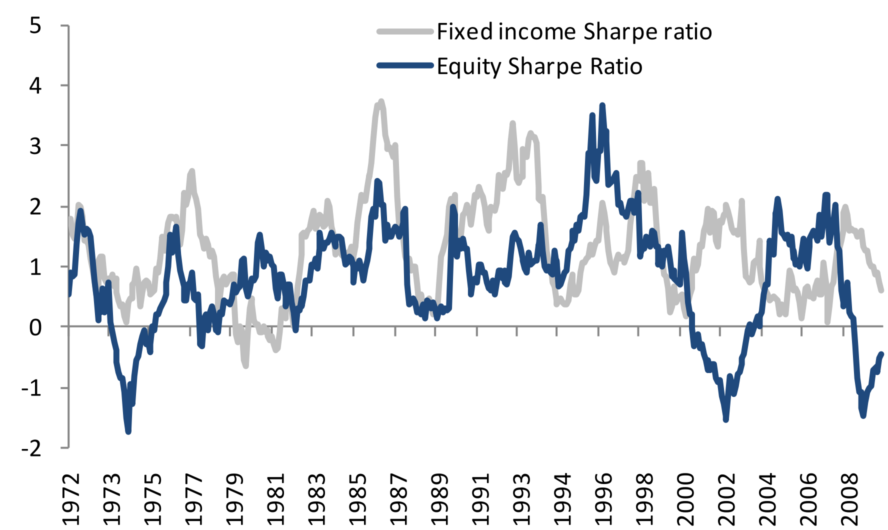
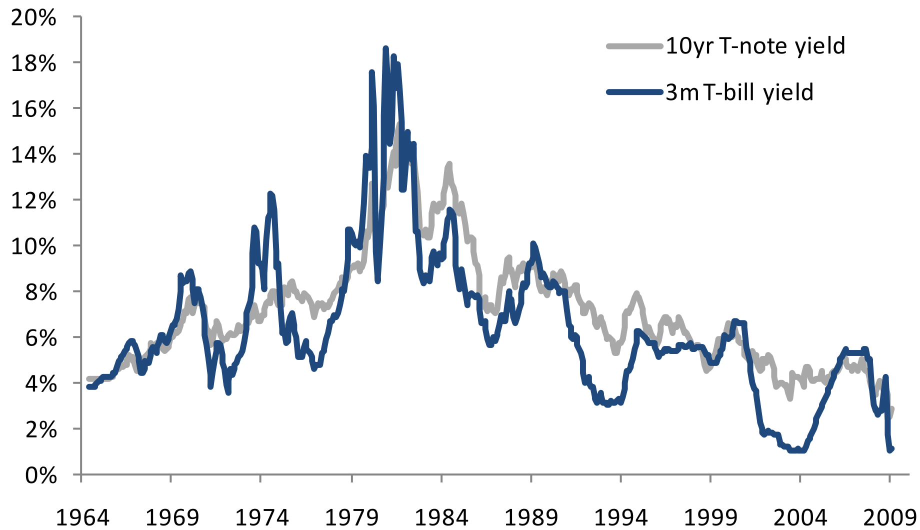


Figure 6 illustrates that the current US interest rates, both for short and long maturities, are at historical lows. We see that the inflationary periods of the 1970s and 1980s were marked by significant spikes in the short term yield. Recently, the US term structure has experienced a sharp steepening, with the 2-10 year spread widening to a multi-decade high of 291bp in February 2010 (for more details see Vannerem and Iyer, 2010). A benchmark US sovereign bond portfolio, like

Merrill Lynch US Treasuries Master, returned -3.7%, in 2009. Therefore, large interest rate rises and negative bond returns are not beyond the realm of possibilities

Figure 6: US short and long term government bond yields



4. Aspects of Leverage Risk not Reflected in Volatility

Our focus on volatility should not imply that volatility is a complete measure of risk. Traditional risk measures like volatility can underestimate the true risk associated with leverage. Risks due to negative skewness in fixed income returns are not captured by symmetric risk measures such as volatility (Inker, 2010). There are also additional risks due to liquidity and cash flow associated with leveraged portfolios. For example, in extreme conditions where wholesale money markets cease to function, liquidity mismatches between the short and long side of the portfolio may make it difficult to roll over the short term loans used to lever the fixed income allocation. In this situation, an institutional investor with a levered fixed income allocation would be forced to reduce leverage aggressively by liquidating the fixed income portfolio. The recent credit crisis showed that while developed government bond markets typically remained liquid, rapid sales of corporate bonds and other credit instruments could be extremely difficult. In a hypothetical but plausible sovereign risk crisis, fire sales of government bonds may also prove challenging. Note that we ignore the cost of leverage in our analysis above, which is likely to rise as the amount of leverage increases and also during times of financial stress.

5. Further Discussion on Achieving Diversification

Proponents of risk parity portfolios argue that the approach offers a new way to engineer better risk/return trade-offs, better manage downside risk, and an opportunity to take advantage of traditional asset classes in a non-traditional way (Morris and Haeusler, 2010). However, as Allen (2010) notes, despite the intuitive appeal of the risk parity approach, it does not typically result in a risk minimizing portfolio. He finds that the risk parity portfolio lies below the efficient frontier, meaning that there are other portfolios on the frontier which, combined with leverage, could achieve the same expected return as the levered risk parity portfolio at an even lower level of risk.

It is interesting to contrast the risk parity approach with a minimum volatility portfolio. While risk parity emphasizes equal contribution to risk from each asset class, minimum volatility focuses on risk reduction on a total portfolio level, using the multi-asset class covariance matrix to achieve this result. Although the minimum volatility portfolio might be more concentrated than the risk parity portfolio, it also takes better advantage of the correlation structure by attempting to fully exploit diversification opportunities within each asset class.

Finally, it is worth noting that higher allocations to fixed income make the levered risk parity portfolio more sensitive to interest rates than a corresponding unlevered portfolio. This sensitivity is magnified by the use of leverage.

6. Conclusion

In this paper we examine the recent trend of adding leverage to fixed income allocations of multi-asset class portfolios of large asset owners. Whether it is optimal to add leverage from a volatility-reduction perspective depends on the correlations between bonds and equities, the relative volatility of bonds versus equities, and the weights of the two asset classes in the portfolio. If correlations between bonds and equities are negative, adding leverage could reduce the volatility of a portfolio, especially if the weight in fixed income assets is low, leverage is moderate, and bonds have a low risk relative to equities. Negative correlations also increase the likelihood that adding leverage will improve the risk-return profile of the portfolio.

In light of this relationship, we have examined the historical behavior of stock-bond correlations and relative volatilities of fixed income and equities, highlighting some explanations for the variations in these measures. In particular, while bond-equity correlations have been low over the past decade, over a longer period bonds and equities had a small positive correlation of approximately 0.2. Explanations for the recent low correlations include a flight to safety in the early 2000s, a deflationary environment later during the decade, and a perceived moderation in the size of macroeconomic shocks. The ratio of stock to bond volatility tends to increase when inflation rises (as bond risk rises) and also when equity market volatility falls. Asset owners looking to add leverage to their fixed income allocation may wish to examine these influences to decide whether negative correlations between bonds and equities, as well as a low ratio of bond to equity volatility, are likely to persist. They may also wish to assess the likelihood of bond outperformance, given the current interest rate and economic environment.

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Appendix

1. Deriving a condition that implies lower volatility in a levered portfolio

Let us denote:

σ_L : volatility of the levered portfolio

σ_U : volatility of the unlevered portfolio

σ_B : bond volatility

σ_E : equity volatility

w_B : bond allocation

w_E : equity allocation

ρ : correlation between equities and bonds

k : leverage applied to bonds (strictly greater than one)

We would like to derive a condition, under which

$$\sigma_L^2 - \sigma_U^2 < 0.$$

Using (1), we can write

$$\left(w_E^2\sigma_E^2 + k^2 w_B^2\sigma_B^2 + 2k w_B w_E \sigma_B \sigma_E \rho\right) - \left(w_E^2\sigma_E^2 + w_B^2\sigma_B^2 + 2 w_B w_E \sigma_B \sigma_E \rho\right) < 0.$$

Rearranging, and simplifying, we obtain

$$(k^2 - 1) w_B^2 \sigma_B^2 + 2(k-1) w_B w_E \sigma_B \sigma_E \rho < 0,$$

$$(1+k) w_B \sigma_B + 2 w_E \sigma_E \rho < 0.$$

Which leads to

$$\rho < -\frac{1}{2}\cdot(1+k)\cdot\frac{w_B}{w_E}\cdot\frac{\sigma_B}{\sigma_E}.$$

2. Deriving a condition that implies a better risk-return profile in a levered portfolio

Quadratic utility approach

Let us define :

r_L : the return of the leveraged portfolio

r_C : the return of the core portfolio

r_X : the return of the extension

K : gross leverage coefficient

We can then write

$r_L = r_C + Kr_x$

$\sigma_L^2 = \sigma_C^2 + K^2\sigma_X^2 + 2K\sigma_C\sigma_X\rho_X = \sigma_C^2 + K\sigma_X\left(K\sigma_X + 2\sigma_C\rho_X\right),$

where $\rho_X = corr(r_C, r_X)$.

Note that in the case of a simple bond-equity portfolio, we can write

$K = (k-1)w_B$, $r_X = r_B$ and $\sigma_X = \sigma_B$.

Also, note that

$$\rho_X = \frac{\operatorname{cov}\{(w_E r_E + w_B r_B), r_B\}}{\sigma_C\sigma_B} = \frac{w_E \operatorname{cov}\{r_E, r_B\}}{\sigma_C\sigma_B} + \frac{w_B\sigma_B}{\sigma_C} = \frac{1}{\sigma_C}\left[w_E\sigma_E\rho + w_B\sigma_B\right].$$

Define $U = r - \lambda\sigma^2$.

Then, we can write

$U_L - U_U = \left(r_L - r_U\right) - \lambda(\sigma_L^2 - \sigma_U^2) = K\left[r_X - \lambda\sigma_X\left(K\sigma_X + 2\sigma_C\rho_X\right)\right].$

We require

$U_L - U_U > 0$, or

$r_X > \lambda\sigma_X\left(K\sigma_X + 2\sigma_C\rho_X\right).$

Re-arranging, we obtain

$\frac{s_X}{\lambda} > K\sigma_X + 2\sigma_C\rho_X$ or $K < \frac{1}{\sigma_X}\left(\frac{s_X}{\lambda} - 2\rho_X\sigma_C\right)$

If $\lambda = r_C / \sigma_C^2$, we can write $K < \frac{\sigma_C}{\sigma_X}\left(\frac{s_X}{s_C} - 2\rho_X\right).$

This particular choice of λ normalises the utility of the core unlevered portfolio to zero. Note that it requires the return of the core portfolio to be positive for *K* to be defined. For other ways of determining λ see Liu and Xu (2010).

Return-Variance Ratio approach

Let us define $\hat{s}_C = r_C / \sigma_C^2$, $\hat{s}_X = r_X / \sigma_X^2$ and $\hat{s}_L = r_L / \sigma_L^2$

We can write

$$\hat{s}_L = \frac{r_C + Kr_x}{\sigma_C^2 + K^2\sigma_X^2 + 2K\sigma_C\sigma_X\rho_X} = \frac{\hat{s}_C + K\left(r_x / \sigma_C^2\right)}{1 + K^2\left(\sigma_X^2 / \sigma_C^2\right) + 2K\left(\sigma_X / \sigma_C\right)\rho_X} = \frac{\hat{s}_C + K\theta^2\hat{s}_X}{1 + K^2\theta^2 + 2K\theta\rho_X},$$

where $\theta = \sigma_X / \sigma_C$. Therefore

$$\frac{\hat{s}_L}{\hat{s}_C} = \frac{1 + K\theta^2\left(\hat{s}_X / \hat{s}_C\right)}{1 + K^2\theta^2 + 2K\theta\rho_X}.$$

Note that the denominator is always positive, as when $\rho_X = -1$ we have

$1 + K^2\theta^2 - 2K\theta = \left(1 - K\theta\right)^2.$

Let's assume that the returns to the core and leveraged portfolios are both positive. In this case we require

$$\frac{\hat{s}_L}{\hat{s}_C} > 1.$$

This implies

$$1 + K\theta^2 \frac{\hat{s}_X}{\hat{s}_C} > 1 + K^2\theta^2 + 2K\theta\rho_X$$

$$\theta \frac{\hat{s}_X}{\hat{s}_C} > K\theta + 2\rho_X .$$

Note that $\theta \frac{\hat{s}_X}{\hat{s}_C} = \dfrac{r_X / \sigma_X}{r_C / \sigma_C} = \frac{s_X}{s_C}$, the ratio of Sharpe ratios of extension and core portfolios.

Therefore $K < \frac{\sigma_C}{\sigma_X}\left(\frac{s_X}{s_C} - 2\rho_X\right)$.

Sensitivity analysis

Table 1 below presents a sensitivity analysis of portfolio volatility and return to variance ratios to changes in the variables we identify above. Specifically, we vary the weights of equities and fixed income in the portfolio, their returns and volatilities, the correlation between equities and fixed income, as well as the amount of leverage applied to the fixed income allocation. The line in bold represents the scenario where the volatility of the unlevered portfolio is higher than the volatility of the corresponding leveraged portfolio The highlighted lines represent those scenarios where a leveraged portfolio has a worse risk-return profile (a higher return to variance ratio) than the corresponding unlevered portfolio. As an example, below we present a calculation using the inputs from the first line of the table.

Unlevered portfolio return:

$$r_U = w_E r_E + w_B r_B = 0.4 * 0.07 + 0.6 * 0.04 = 5.2\%$$

Levered portfolio return:

$$r_L = w_E r_E + k w_B r_B = 0.4 * 0.07 + 1.5 * 0.6 * 0.04 = 6.4\%$$

Unlevered portfolio volatility:

$$\sigma_U = \sqrt{w_E^2\sigma_E^2 + w_B^2\sigma_B^2 + 2w_E w_B \sigma_E \sigma_B \rho} =$$
$$= \sqrt{(0.4 * 0.15)^2 + (0.6 * 0.05)^2 + 2 * 0.4 * 0.6 * 0.15 * 0.05 * 0.2} = 7.2\%$$

Levered portfolio volatility:

$$\sigma_L = \sqrt{w_E^2\sigma_E^2 + k^2 w_B^2\sigma_B^2 + 2k w_E w_B \sigma_E \sigma_B \rho} =$$
$$= \sqrt{(0.4 * 0.15)^2 + (1.5 * 0.6 * 0.05)^2 + 2 * 1.5 * 0.4 * 0.6 * 0.15 * 0.05 * 0.2} = 8.2\%$$

Unlevered portfolio return to variance ratio:

$$\hat{s}_U = r_U / \sigma_U^2 = 0.052 / (0.072)^2 = 9.96$$

Levered portfolio return to variance ratio:

$$\hat{s}_L = {r_L}/{\sigma_L^2} = 0.064/(0.082)^2 = 9.55$$

Table 1: Sensitivity Analysis

Equity weight	Equity return	Equity volatility	Equity Sharpe ratio	Bond weight	Bond return	Bond volatility	Bond Sharpe Ratio	Bond-equity correlation	Leverage	Unlevered portfolio volatility	Levered portfolio volatility	Unlevered portfolio return-variance ratio	Levered portfolio return-variance ratio
40%	7%	15%	0.47	60%	4%	5%	0.80	0.2	1.5	7.2%	8.2%	9.96	9.55
60%	7%	15%	0.47	40%	4%	5%	0.80	0.2	1.5	9.6%	10.0%	6.29	6.55
80%	7%	15%	0.47	20%	4%	5%	0.80	0.2	1.5	12.2%	12.4%	4.27	4.43
60%	1%	15%	0.07	40%	4%	5%	0.80	0.2	1.5	9.6%	10.0%	2.39	2.98
60%	20%	15%	1.33	40%	4%	5%	0.80	0.2	1.5	9.6%	10.0%	14.75	14.29
60%	30%	15%	2.00	40%	4%	5%	0.80	0.2	1.5	9.6%	10.0%	21.26	20.24
60%	7%	15%	0.47	40%	4%	8%	0.53	0.2	1.5	10.0%	10.8%	5.75	5.62
60%	7%	15%	0.47	40%	4%	10%	0.40	0.2	1.5	10.6%	11.8%	5.21	4.76
60%	7%	15%	0.47	40%	4%	15%	0.27	0.2	1.5	11.8%	13.9%	4.18	3.40
60%	7%	15%	0.47	40%	4%	5%	0.80	0.6	1.5	10.3%	11.1%	5.44	5.39
60%	7%	15%	0.47	40%	4%	5%	0.80	-0.2	1.5	8.8%	8.9%	7.46	8.33
60%	**7%**	**15%**	**0.47**	**40%**	**4%**	**5%**	**0.80**	**-0.4**	**1.5**	**8.4%**	**8.3%**	**8.22**	**9.65**
60%	7%	15%	0.47	40%	4%	5%	0.80	0.2	1.5	9.6%	10.0%	6.29	6.55
60%	7%	15%	0.47	40%	4%	5%	0.80	0.2	2.0	9.6%	10.6%	6.29	6.64
60%	7%	15%	0.47	40%	4%	5%	0.80	0.2	5.0	9.6%	14.7%	6.29	5.62

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