Build Risk Parity Portfolios with Correlation Risk Attribution (x-σ-ρ)

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The concept of portfolio efficiency, where a rational institutional investor is expected to optimize his expected return for a given level of volatility, was established by Markowitz when he introduced the mean-variance framework (1952, and 1959). While the elegant theory is mathematically sound, it is difficult to implement in practice. The solution typically lacks diversification and the resulting optimal portfolios tend to be excessively concentrated in the lowest volatility assets. It is extremely sensitive to the input parameters especially when it comes to alpha (Merton (1980)) and this leads to inconsistency of portfolio composition and turnover. To move beyond these issues, practitioners and academics have investigated more heuristic and robust solutions which do not rely on alpha and are in fact computationally simpler. Two renowned portfolio construction strategies namely, *equal weighted portfolio* (1/N strategies) and *minimum-variance portfolio* (MV strategies) are widely implemented in practice.

In equal weighted portfolios, each constituent has equal weight in the portfolio, $w_i = \frac{1}{n}$, so that the portfolio construction process is independent from any estimated parameters. The portfolio is well diversified in terms of all portfolio constituents. The research of Windcliff and Boyle (2004) found that equal weighted portfolios were far from inefficient. Additionally, DeMiguel, Garlappi and Uppal (2009) stated, "*The intuition for the good performance of the* $\frac{1}{n}$ *strategy is that the loss from naïve rather than optimal diversification is smaller than the loss arising from the estimation error in the parameters needed to implement optimal asset allocation rules.*" On the other hand, the main drawback is that it has very limited risk diversification as it does not take into account individual risk or the correlation between known sources of risk in the portfolio.

Minimum variance portfolio optimization is a well-known in practice and Modern Portfolio Theory. Given the volatility, the portfolio is defined as ---

$$\sigma(w) = \sqrt{w' \Omega \, w}$$

where w, the vector of weights; Ω, the covariance matrix,

the minimum variance (MV) portfolio can be constructed by solving the following optimization problem:

$$\min \quad \frac{1}{2} w' \Omega \, w$$

$$\text{subject to } w' \cdot 1 = 1$$

It is the minimum variance portfolio that sits at the left most tip of the mean-variance frontier. Not only does it offer low volatility but also it produces good out-of-sample performance (Clarke, De Silva and Thorley (2006)). One main weakness of the minimum variance portfolios is that it diversifies the volatility instead of the weight; thus, it tends to create concentrated portfolios that consist of a few assets with low volatility.

More recent research has taken the above findings with further investigation and found that employing equal weighted risk contribution provides a promising alternative to construct portfolios. Equal weighted risk contribution portfolio, which is also referred to as *Risk Parity portfolio*, corresponds to the portfolio that contains all constituents where risk contributions are equal. The risk contribution of the m^{th}asset (or factor) is defined as,

$$\sigma(w) = \sum_{m=1}^{n} Risk\ Contribution_m = \sum_{m=1}^{n} w_m \frac{\partial \sigma(w)}{\partial w_m} = \sum_{m=1}^{n} w_m MCR_m$$

where MCR_m is the marginal contribution to risk.

An article written by Maillard, Roncalli and Teiletche (2011) provides more insights in the above equal weighted risk contribution portfolios. The paper indicates that the equal weighted risk contribution portfolio avoids the major drawbacks of the two mentioned strategies. *"Despite their robustness, both approaches have their own limitations, mainly being a lack of risk monitoring for* $\frac{1}{n}$ *portfolios and dramatic asset concentration for minimum variance portfolio*." It states that the volatility of equal weighted risk contribution portfolios is located between those of minimum variance and equal weighted portfolios and this has been proven both theoretically and empirically. The empirical studies that compare equal weighted risk contribution portfolios and other construction methodologies also show that the former is often associated with a high return and low drawdown. It could be said that equal weighted risk contribution portfolios offer a good trade-off between return, volatility and diversification because of their strong risk-adjusted performance.

Jose Menchero and Ben Davis's MSCI Research Insight paper on "Risk Contribution Is Exposure Times Volatility Times Correlation: Decomposing Risk Using the X-Sigma-Rho Formula," which was recently published in the Journal of Portfolio Management [2011] presents a flexible and more insightful approach to risk attribution that combines the strengths of stand-along volatility analysis with the virtues of the marginal contribution approach. The authors decompose the risk contribution into a product of exposure, volatility, and correlation. Based on this concept, we can construct equal weighted risk contribution portfolios where each constituent contributes the same amount of risk to the portfolio as the marginal contribution is given by the product of volatility and correlation.

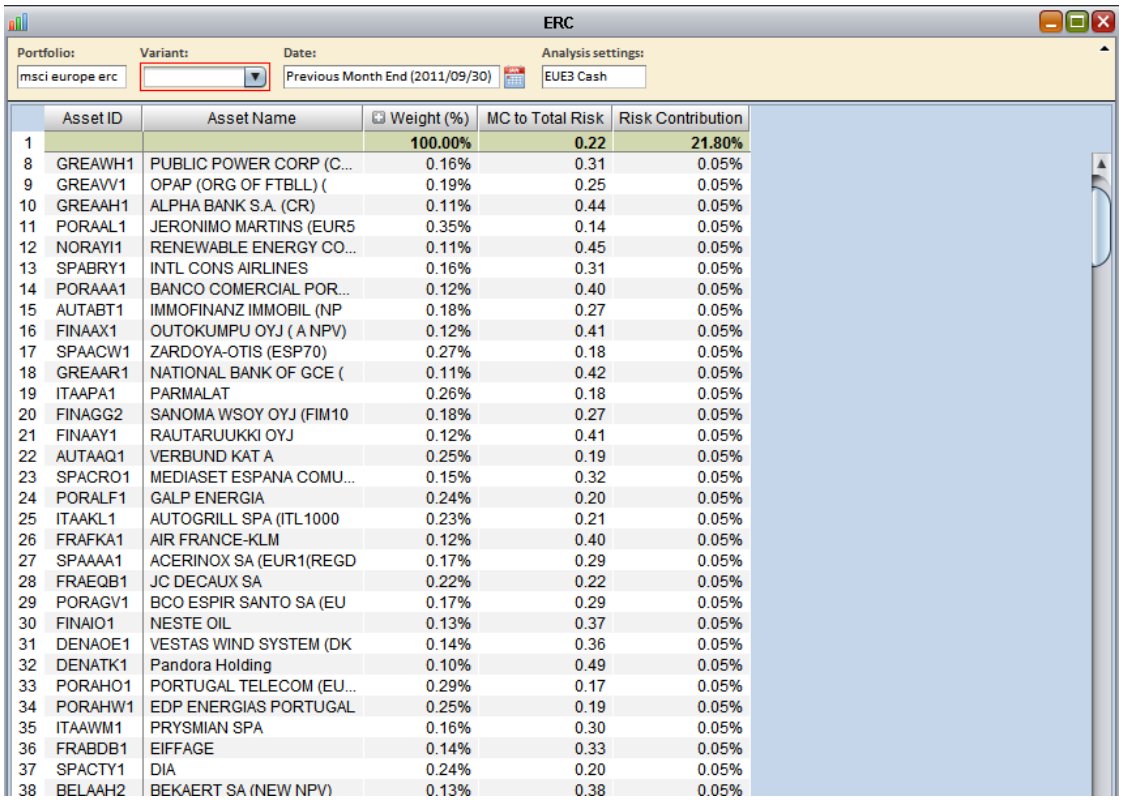
Risk Parity Portfolio: An Example

In this illustration, we attribute risk to fundamental factors using the **Barra European Equity Model** (EUE3), construct equal weighted risk contribution portfolio using MSCI Europe Index and utilize MSCI's **Barra Portfolio Manager (BPM)** as main platform for portfolio construction and analysis. Figure 1 provides a snapshot on the construction of equal weighted risk contribution portfolio with its constituents at the end of September 2011. Figure 2 compares the equal weighted portfolio and the equal weighted risk contribution portfolio in terms of weights and risk contributions. As reported in

Figure 3 with the period from September 2010 until September 2011, equal weighted risk contribution portfolio yields an annualized return of -9.2% (with annualized volatility of 17.7%), compared to -13.5% (with annualized volatility of 20.0%) for equal weighted portfolio and -9.7% (with annualized volatility of 18.9%) for index portfolio, which is 4.3% and 0.5% higher accordingly. Higher information ratio is also achieved by equal weighted risk contribution portfolio.

In conclusion, a risk parity portfolio certainly provides an alternative for institutional investors to construct well diversified portfolio. As an investment strategy, this approach has attracted significant attention in the media and among professional investors in recent years. Further empirical work remains to be done and additional studies are needed to understand the best methods of implementation.

Figure 1



	Asset ID	Asset Name	Weight (%)	MC to Total Risk	Risk Contribution
1			100.00%	0.22	21.80%
8	GREAWH1	PUBLIC POWER CORP (C...	0.16%	0.31	0.05%
9	GREAVV1	OPAP (ORG OF FTBLL) (	0.19%	0.25	0.05%
10	GREAAH1	ALPHA BANK S.A. (CR)	0.11%	0.44	0.05%
11	PORAAL1	JERONIMO MARTINS (EUR5	0.35%	0.14	0.05%
12	NORAYI1	RENEWABLE ENERGY CO...	0.11%	0.45	0.05%
13	SPABRY1	INTL CONS AIRLINES	0.16%	0.31	0.05%
14	PORAAA1	BANCO COMERCIAL POR...	0.12%	0.40	0.05%
15	AUTABT1	IMMOFINANZ IMMOBIL (NP	0.18%	0.27	0.05%
16	FINAAX1	OUTOKUMPU OYJ (A NPV)	0.12%	0.41	0.05%
17	SPAACW1	ZARDOYA-OTIS (ESP70)	0.27%	0.18	0.05%
18	GREAAR1	NATIONAL BANK OF GCE (	0.11%	0.42	0.05%
19	ITAAPA1	PARMALAT	0.26%	0.18	0.05%
20	FINAGG2	SANOMA WSOY OYJ (FIM10	0.18%	0.27	0.05%
21	FINAAY1	RAUTARUUKKI OYJ	0.12%	0.41	0.05%
22	AUTAAQ1	VERBUND KAT A	0.25%	0.19	0.05%
23	SPACRO1	MEDIASET ESPANA COMU...	0.15%	0.32	0.05%
24	PORALF1	GALP ENERGIA	0.24%	0.20	0.05%
25	ITAAKL1	AUTOGRILL SPA (ITL1000	0.23%	0.21	0.05%
26	FRAFKA1	AIR FRANCE-KLM	0.12%	0.40	0.05%
27	SPAAAA1	ACERINOX SA (EUR1(REGD	0.17%	0.29	0.05%
28	FRAEQB1	JC DECAUX SA	0.22%	0.22	0.05%
29	PORAGV1	BCO ESPIR SANTO SA (EU	0.17%	0.29	0.05%
30	FINAIO1	NESTE OIL	0.13%	0.37	0.05%
31	DENAOE1	VESTAS WIND SYSTEM (DK	0.14%	0.36	0.05%
32	DENATK1	Pandora Holding	0.10%	0.49	0.05%
33	PORAHO1	PORTUGAL TELECOM (EU...	0.29%	0.17	0.05%
34	PORAHW1	EDP ENERGIAS PORTUGAL	0.25%	0.19	0.05%
35	ITAAWM1	PRYSMIAN SPA	0.16%	0.30	0.05%
36	FRABDB1	EIFFAGE	0.14%	0.33	0.05%
37	SPACTY1	DIA	0.24%	0.20	0.05%
38	BELAAH2	BEKAERT SA (NEW NPV)	0.13%	0.38	0.05%

Figure 2

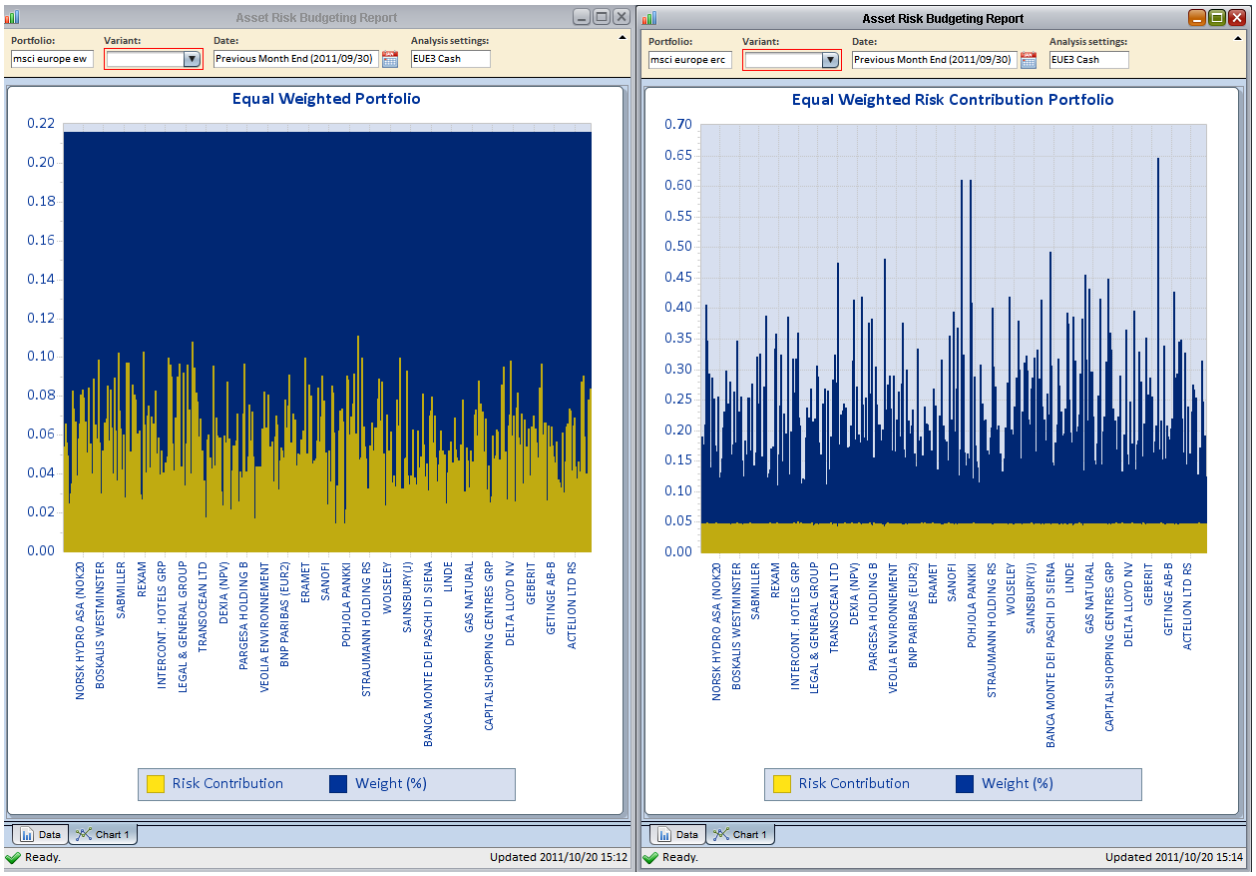
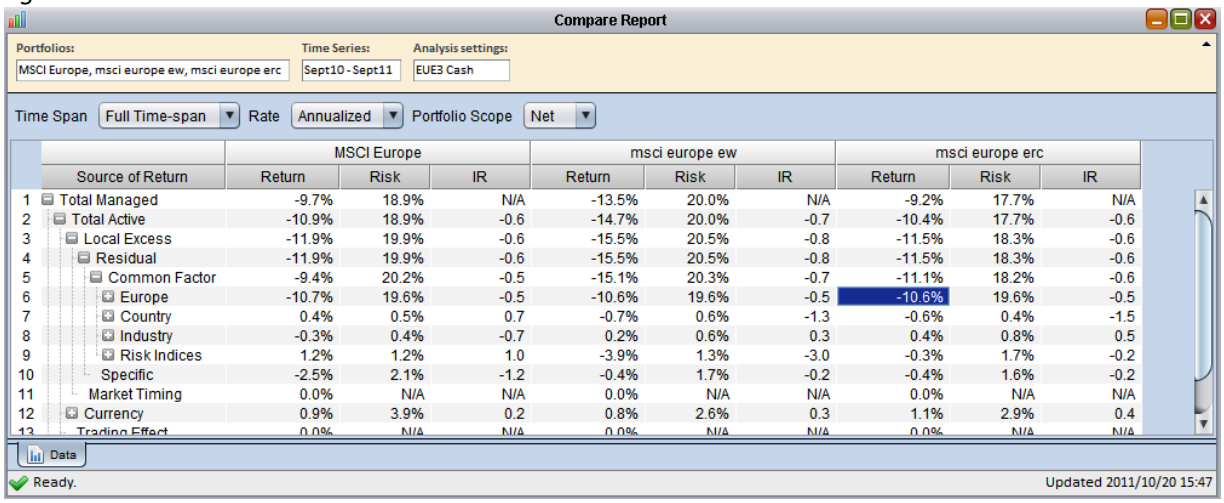


Figure 3



	Source of Return	MSCI Europe Return	MSCI Europe Risk	MSCI Europe IR	msci europe ew Return	msci europe ew Risk	msci europe ew IR	msci europe erc Return	msci europe erc Risk	msci europe erc IR
1	Total Managed	-9.7%	18.9%	N/A	-13.5%	20.0%	N/A	-9.2%	17.7%	N/A
2	Total Active	-10.9%	18.9%	-0.6	-14.7%	20.0%	-0.7	-10.4%	17.7%	-0.6
3	Local Excess	-11.9%	19.9%	-0.6	-15.5%	20.5%	-0.8	-11.5%	18.3%	-0.6
4	Residual	-11.9%	19.9%	-0.6	-15.5%	20.5%	-0.8	-11.5%	18.3%	-0.6
5	Common Factor	-9.4%	20.2%	-0.5	-15.1%	20.3%	-0.7	-11.1%	18.2%	-0.6
6	Europe	-10.7%	19.6%	-0.5	-10.6%	19.6%	-0.5	-10.6%	19.6%	-0.5
7	Country	0.4%	0.5%	0.7	-0.7%	0.6%	-1.3	-0.6%	0.4%	-1.5
8	Industry	-0.3%	0.4%	-0.7	0.2%	0.6%	0.3	0.4%	0.8%	0.5
9	Risk Indices	1.2%	1.2%	1.0	-3.9%	1.3%	-3.0	-0.3%	1.7%	-0.2
10	Specific	-2.5%	2.1%	-1.2	-0.4%	1.7%	-0.2	-0.4%	1.6%	-0.2
11	Market Timing	0.0%	N/A	N/A	0.0%	N/A	N/A	0.0%	N/A	N/A
12	Currency	0.9%	3.9%	0.2	0.8%	2.6%	0.3	1.1%	2.9%	0.4
13	Trading Effect	0.0%	N/A	N/A	0.0%	N/A	N/A	0.0%	N/A	N/A

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