Market Insight

Macro-Sensitive Portfolio Strategies

How We Define Macroeconomic Risk

Kurt Winkelmann, Ludger Hentschel, Raghu Suryanarayanan, and Katalin Varga

Kurt.Winkelmann@msci.com

Ludger.Hentschel@msci.com

Raghu.Suryanarayanan@msci.com

Katalin.Varga@msci.com

November 2012

Abstract:

Global economic conditions have seen a weak recovery since 2008, with major economies experiencing sub-par growth rates relative to long-term trend growth. As a result, investors are interested in designing portfolio strategies that explicitly recognize macroeconomic risk. The design of macro-sensitive portfolio strategies relies on how we define macroeconomic risk and measure the relationship between asset prices and macroeconomic risk. In this paper — the first in a series that addresses these issues — we argue that macroeconomic risk is the change in asset value due to persistent shocks to real economic growth. This definition underscores the role of long horizons in macroeconomic risk and the principal issue facing investors: how should asset allocation respond to large macroeconomic shocks, given that their consequences are likely to be resolved over long time periods? We advocate going back to the basics of asset pricing and analyzing the impact of macro shocks on both asset cash flows and discount factors.

Why This Matters:

- Practical design and risk management of macro-sensitive strategies rely on how we define macro risk and measure the link between asset prices and macro risk.
- *Current* valuations reflect investors' concerns about *long-term* economic and cash-flow growth.
- The impact of persistent shocks to economic growth is likely to be resolved over long horizons.

Introduction

In 2008, both the global real economy and global equity markets experienced sharp contractions. While global equity markets have recovered since 2008, the recovery in global macroeconomic conditions has been weak. As a result, investors are interested in designing portfolio strategies that explicitly recognize macroeconomic factors. In turn, macro-sensitive portfolio strategies depend on the measurement of the relationship between asset prices and macroeconomic factors.

Central to any discussion of measuring, managing, and potentially exploiting a source of risk is a definition of that risk; precisely defining the type of risk enhances the ability of investors to design strategies involving that risk. While investors are interested in many macroeconomic variables (e.g., inflation, commodity prices), we will argue in this paper that *macroeconomic risk is the change in asset values associated with persistent shocks (positive or negative) to the trend growth rate in real output*.

We have defined macroeconomic risk in this way because practitioners and academics alike have had limited success building models where asset risk and return can be accounted for by shocks to the real economy. Most existing valuation and risk models are geared towards short horizons with a very low contemporaneous correlation between real economic growth and asset returns. We intentionally focus on longer horizons.

Consequently, to construct macro-sensitive portfolio strategies, investors and analysts must confront these three issues:

- The relationship between trend growth and business cycle shocks.
- The role of horizon in describing macroeconomic risk.
- The asset pricing mechanism that reflects macroeconomic risk.

In this paper, we argue that investors should be less concerned about shocks to the business cycle, per se, and more concerned about the implications of those shocks for trend growth. In short, when confronted with large shocks to the real economy, investors should be asking the following questions:

- How persistent will the shocks be?
- What are the implications of the shocks for asset prices?
- What are the implications for asset allocation?
- How do we manage macroeconomic risk?

This paper is the first in a series that explores these questions. We will discuss macroeconomic risk by examining empirical evidence about the economy and equity returns. In future papers, we will argue that observed risk premiums can be interpreted in terms of asset exposures to macroeconomic shocks. We will also argue that an investor’s asset allocation depends on their willingness to tolerate exposure to macroeconomic shocks. We will use the framework developed through these papers to consider the treatment of other important macroeconomic variables, such as inflation and commodity prices.

Asset Returns are Not Correlated with the Business Cycle

A core belief for most investors is that shocks to the economy produce shocks to asset prices. The logic behind this belief is that asset returns are highly correlated with the real economy. The appeal of this story is that if we can predict the path of the economy, then we can predict the path of asset returns. And if we can make this linkage, then we can design portfolio strategies based on the path of the economy.

Over short horizons, this story is at odds with the empirical evidence. First, and as we will discuss in this section, there is at best a weak linkage between shocks to the economy and real asset returns. Secondly, there is extremely weak evidence that the economy is forecastable over short horizons. We will discuss this second point in the next section.

A simple way to see the weak linkage between real asset returns and the real economy over short horizons is to regress asset (excess) returns on real economic growth. The results of these regressions for five economies (the US, UK, Germany, Spain, and Japan) are in Table 1, which shows the estimated coefficients, the *t*-statistics, and the R^2 statistics for each economy. In none of the markets is the explanatory power especially strong. Although the coefficients are mostly positive (indicating a positive relationship between equity returns and the real economy), they are not statistically meaningful.[1]

Table 1: Weak Linkage between the Real Economy and Asset Returns.

	USA	UK	Germany	Spain	Japan
Regression Estimates					
Intercept	0.0	0.0	0.0	0.0	0.0
Real GDP Growth	1.3	-0.1	1.2	0.9	1.3
T-Stats					
Intercept	0.5	2.1	1.0	0.6	-0.5
Real GDP Growth	1.7	-0.1	1.4	0.6	1.4
R-Squared (%)	1.7	0.0	1.5	0.3	1.9

The Table shows results from regressing quarterly stock returns in excess of short-term government bill yields on quarterly growth in real GDP. We use the MSCI USA, MSCI UK, MSCI Germany, MSCI Spain, and MSCI Japan indices to measure stock returns including reinvested dividends. The regressions use quarterly data from 1970 Q1 to 2012 Q2 for US, from 1975 Q1 to 2012 Q2 for UK, and from 1988 Q1 to 2012 Q2 for Spain, Germany, and Japan

A second way to view the relationship between asset returns and the economy is to look at the behavior of both variables during extreme events.

Table 2 compares US equity returns and US real growth for two years: 1987 and 2008. The table shows quarter-by-quarter equity returns and GDP growth rates. In both years, equity returns were negative in the fourth quarter (and spectacularly so), with declines in excess of 20 percent. However, in 1987 the real economy grew during the fourth quarter compared to declines in equity return; in 2008, the real economy declined dramatically in the fourth quarter. Apparently, extreme events in the equity market cannot be explained systematically by extreme events in the real economy.

[1] The weak explanatory power is consistent with the variance bounds tests of Leroy and Porter (1981). Essentially, stock market volatility is quite high relative to output volatility.

Table 2: Extreme Market Events Cannot Be Explained by the Real Economy.

	1987		2008	
	Equity Returns (%)	Real Output Growth (%)	Equity Returns (%)	Real Output Growth (%)
Q1	20.51	0.70	-9.32	-0.48
Q2	4.82	1.07	-2.07	0.38
Q3	6.56	0.86	-8.98	-0.90
Q4	-22.80	1.74	-22.22	-2.33
Y	3.91	4.44	-37.14	-3.31

The Table shows quarterly US equity returns and contemporaneous quarterly growth in real US GDP during 1987 and 2008, two years of dramatic declines in US equity markets. The equity returns are based on the MSCI USA index and include reinvested dividends.

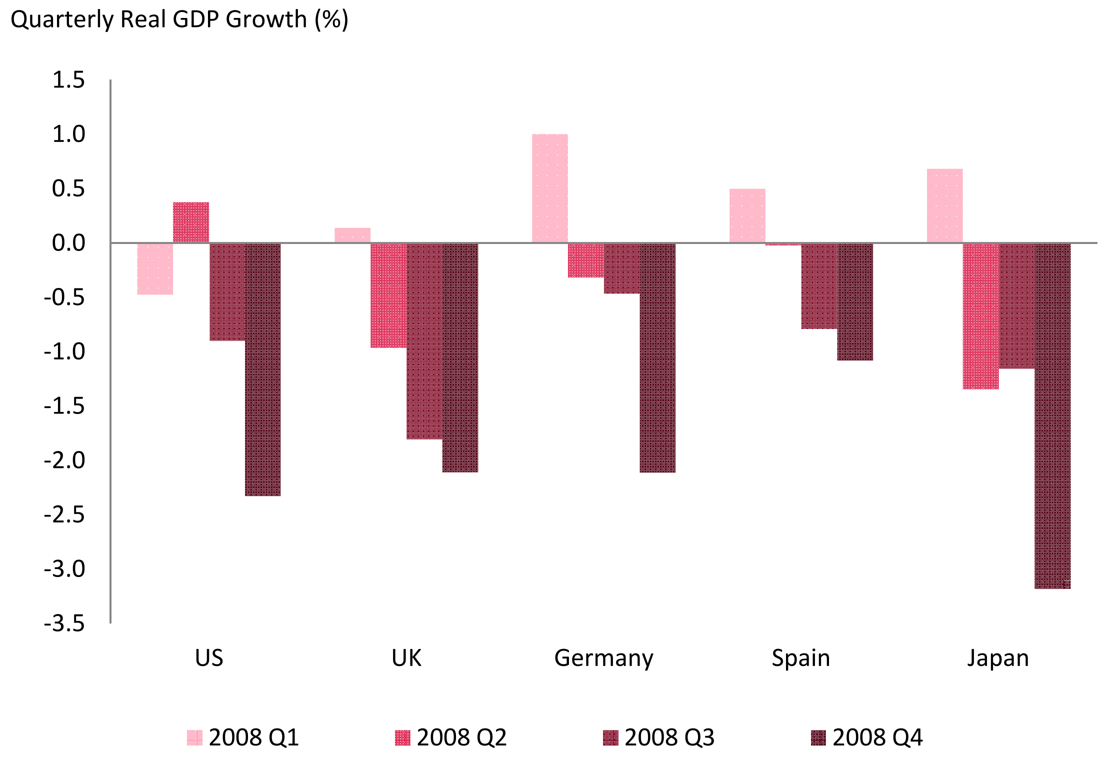
Two issues are reflected in the results shown in Tables 1 and 2. First, stock returns are observed at high frequency while economic activity is measured at low frequency with subsequent revisions. It could be that there actually is a causal link between contemporaneous macroeconomic events that we are not capturing because of these data issues.

Second, asset prices reflect both current market conditions and expectations about the future. Thus, they may be sensitive to macroeconomic shocks that provide a signal about future market conditions. This point gets to the core issue in defining macroeconomic risk: is this really a short horizon risk (e.g., phases of the business cycle) or does it reflect factors that emerge over longer time horizons? To more fully address this question, we need to assemble and interpret information about the macroeconomy.

Persistent Shocks to Trend Growth Do Occur -- Macro Risk is Regime Risk

In 2008, many major economies experienced a large negative shock to real output. The pattern and magnitude of this decline in real output is shown in Figure 1 for the US, UK, Japanese, German, and Spanish economies. The Figure makes clear that real output fell materially in each of these economies in the third and fourth quarters of 2008.

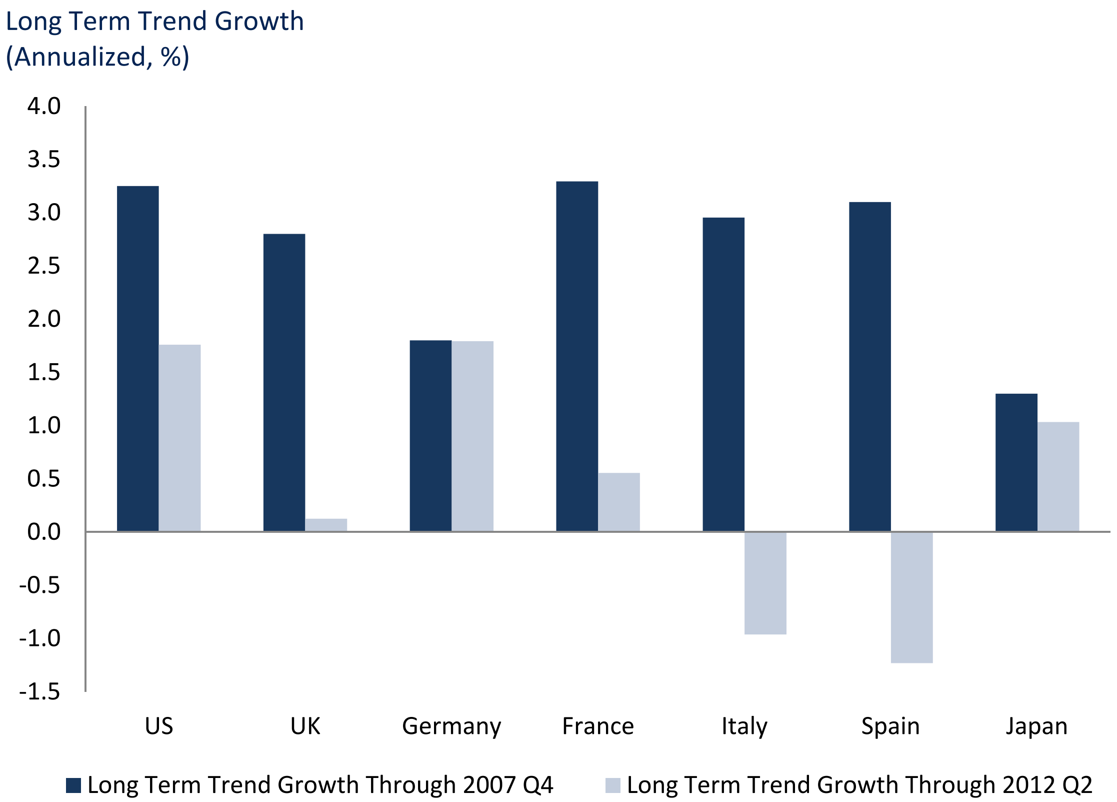
Figure 1: Significant Declines in Real GDP in 2008.



The Figure shows quarterly real GDP growth for the US, the UK, Germany, Spain, and Japan during 2008.

Since 2008, most of the major economies have experienced sub-par growth rates relative to long-term trend growth; this point is illustrated in Figure 2, which shows the long-term trend growth rate through 2007, and compares it to the long-term trend growth rate through the second quarter of 2012. The Figure makes clear a potential decline in long-term trend growth; the graphs in Appendix C that show the recent evolution of real output portray a similar outlook. For example, if current conditions persist, then trend growth in the US is likely to be 1.6 percent going forward, versus 3.3 percent through 2007. Put differently, for the US economy to revert to the GDP levels predicted by trend growth before 2007, real output would need to enter a prolonged period of growth in excess of 3.3 percent. For another example: in Spain, trend growth is projected to be negative; only in Germany is the current trend growth rate approximately the same as before 2007. It is understandable, based on Figure 2, why investors raise questions about a change in economic regimes.

Figure 2: Lower Trend Growth Going Forward?



The Figure shows the trend growth of real GDP in 2007 Q4 and 2012 Q2 for the US, UK, Germany, France, Italy, Spain, and Japan. We estimate the trend growth by applying a Hodrick-Prescott filter to quarterly real GDP data.

To frame the issues differently, the experience of 2008 can be compared with that of 1987. Table 2 shows that equity returns declined by about the same magnitude in the fourth quarter of each year; the economy declined in 2008 but grew in 1987. Table 3 reinterprets the real growth of Table 2 in terms of standard deviations away from trend. What the Table makes clear is that in the fourth quarter of 1987, the US economy *grew* by about three standard deviations above trend, while in the fourth quarter of 2008, the US economy *contracted* by about five standard deviations.[2] Judged in this context, the shocks in 2008 should have prompted the following question: was the economic shock in the fourth quarter of 2008 simply bad luck in the short term, or did it represent the beginning of a prolonged period of below-trend growth? Inherently, questions about long-term growth and its predictability must be resolved over longer time horizons.[3]

[2] The US real GDP trend growth rate was already decelerating at the start of 2008. The largest decline in realized growth, however, occurred during the fourth quarter of 2008.

[3] In the event, the actual experience since 2008 suggests that we have had a persistent shock to trend growth, not simply a bad draw.

Table 3: Is Macro Risk Really Regime Risk?

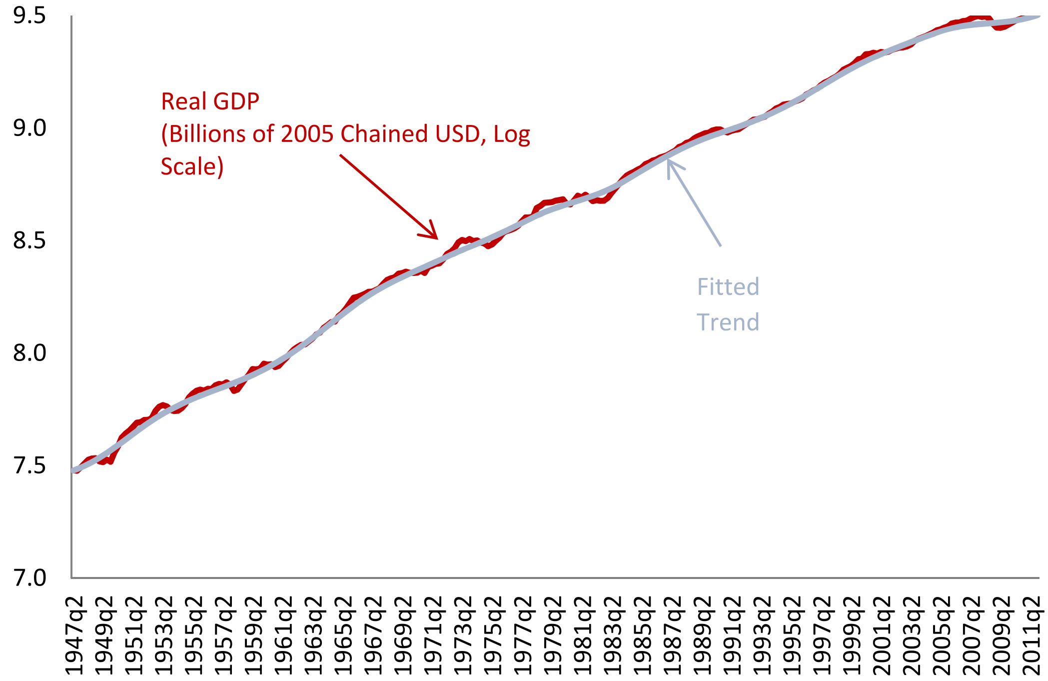
	1987	2008
Real GDP Trend Growth	3.69%	2.39%
Real GDP Growth/Drop	6.92%	-9.31%
Normalized GDP Growth/Drop	3.1 σ	(-4.9) σ

The Table contrasts three measures of real US GDP growth rates for the fourth quarters of 1987 and 2008. The first row contains the trend growth rates at the start of each year. The second row shows the actual change in real GDP during the fourth quarter of each year. The third row shows the change divided by the volatility of changes from 1950 to the third quarter of each year.

Is Real Output Growth Predictable?

A simple way to analyze the macroeconomy is to plot the historical evolution of real output. In Figure 3, quarterly real output for the US economy from 1947 through 2012 is plotted and compared to a trend line.[4] Appendix A has similar plots for the UK, Germany, Japan, and Spain.

Figure 3: It is Hard to Distinguish Real Output from its Trend.



The Figure shows quarterly observations of real US GDP in 2005 dollars (billions) and a smoothed trend line fitted with a Hodrick-Prescott filter. The vertical axis employs a logarithmic scale.

[4] The plot is of the log of real output. The trend line is fit using a Hodrick-Prescott filter. The plot does not show output on a per capita basis nor on a per working person basis.

The basic pattern in Figure 3 is that real output cycles tightly around the trend line, and deviations from the trend seem short-lived. At first sight, real output may be mean reverting around a deterministic time trend, and in a highly predictable fashion. In this view, output growth would also be highly predictable over short horizons. An alternative view is that real output follows a random walk with constant growth. In this case, the trending pattern would be driven by a series of unpredictable random shocks to output. Under this second interpretation, the best predictor of quarterly real output growth may be the constant trend growth rate.

A standard statistical test that attempts to disentangle these two competing views is the unit root test. We are interested in testing the hypothesis that real GDP follows a random walk with a trend versus a trend-stationary alternative.[5] Table 4 shows the results of unit root tests for real output in the US, the UK, Japan, Germany, and Spain. The Table includes the test statistics for each country, as well as the critical values of the test for each country. The test fails to reject the random walk hypothesis when the test statistic is above the critical value, as it is for the five countries in Table 4.

Table 4: Real GDP has Followed a Random Walk.

Dickey Fuller (DF) Test Statistics (95% Confidence Level)	US	UK	Germany	Spain	Japan
P-Value	0.79	0.83	0.56	0.97	1.00
Test Value	-1.60	-1.48	-2.06	-0.73	0.34
DF Critical Value	-3.43	-3.43	-3.45	-3.43	-3.43

The Table shows results from tests of the hypothesis that real GDP follows a random walk with trend against its trend stationary alternative. All test results in the table fail to reject the random walk hypothesis. We use data from 1950 for the US, 1960 for the UK, Spain and Japan, and 1980 for Germany. All data run through 2012 Q2.

These results present a bleak picture about predictability in real output growth because they focus attention on short-term movements in real output. The focus on the short horizon actually presents two important challenges.

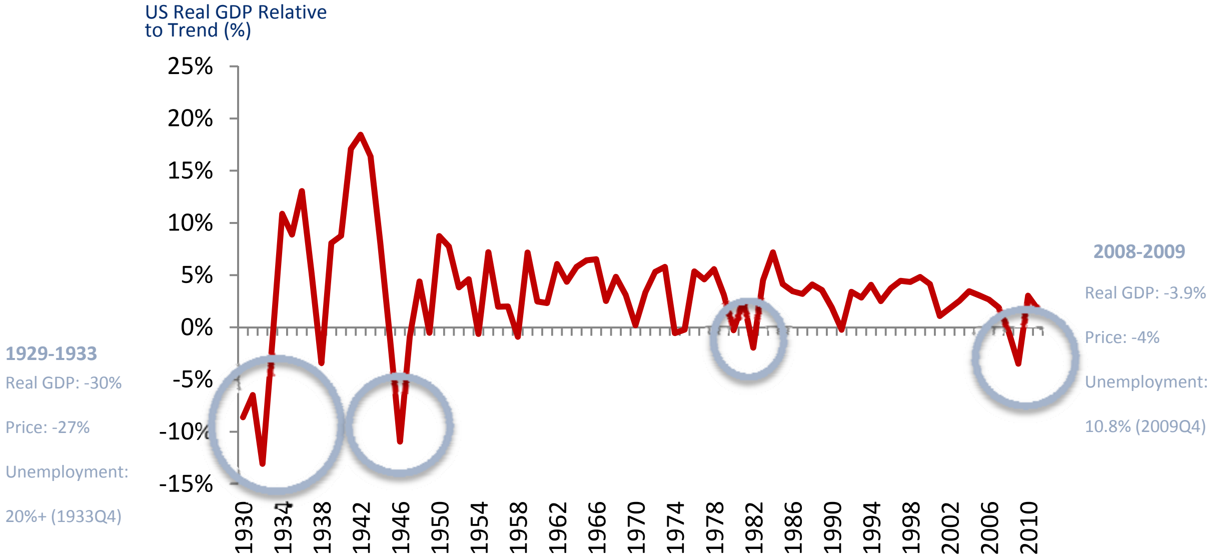
First, this type of focus means that we miss the potential impact that extreme, but rare, events may have on trend growth. Figure 4 examines the impact of large shocks to the economy by plotting de-trended real output for the US economy.[6] In other words, Figure 4 looks at the "cycle" around the trend. As anticipated, large shocks to real output do occur, but infrequently.

---

[5] In their simplest form, unit root tests check whether, in a regression of current real output on lagged real output and a constant, the coefficient on lagged real output is one. When the coefficient is less than one, then knowledge of current output (relative to trend) is useful in predicting next period's output growth. By contrast, when the coefficient equals one, knowledge of current real output has no predictive power for next period's growth.

[6] Barro (2006) followed by Barro and Urzua (2008) document the size, frequency and duration of disasters for 40 countries from World War I (and from 1870 for most OECD countries) to 2006. They identify 183 declines in GDP of more than 10 percent over one year or more, across all countries and over the entire sample history. The average size of the disasters is about 21 percent in their sample, and the average duration about three-and-a-half years.

Figure 4: Infrequent Disasters Do Occur.



The Figure plots quarterly deviations of US real GDP from its trend growth between 1929 and 2012.

Second, the limitations of data availability mean that statistical tests find it hard to distinguish between the random walk model with constant growth (no predictability of GDP growth) and one that includes a *near*-random walk (long-term predictability of GDP growth). The latter would imply that output growth could still be weakly related to a highly persistent factor. Although this persistent factor will only explain a tiny fraction of the variation in output growth over short horizons, its predictive power will grow at longer horizons.[7] Thus, the failure to detect statistically meaningful predictability over short horizons may still be compatible with longer-horizon predictability. In particular, shocks to the economy, especially large ones, may actually have important effects on output growth for a prolonged period of time. Both of these challenges suggest that investment decisions can be improved if investors focus on understanding the impact of persistent shocks to trend growth.

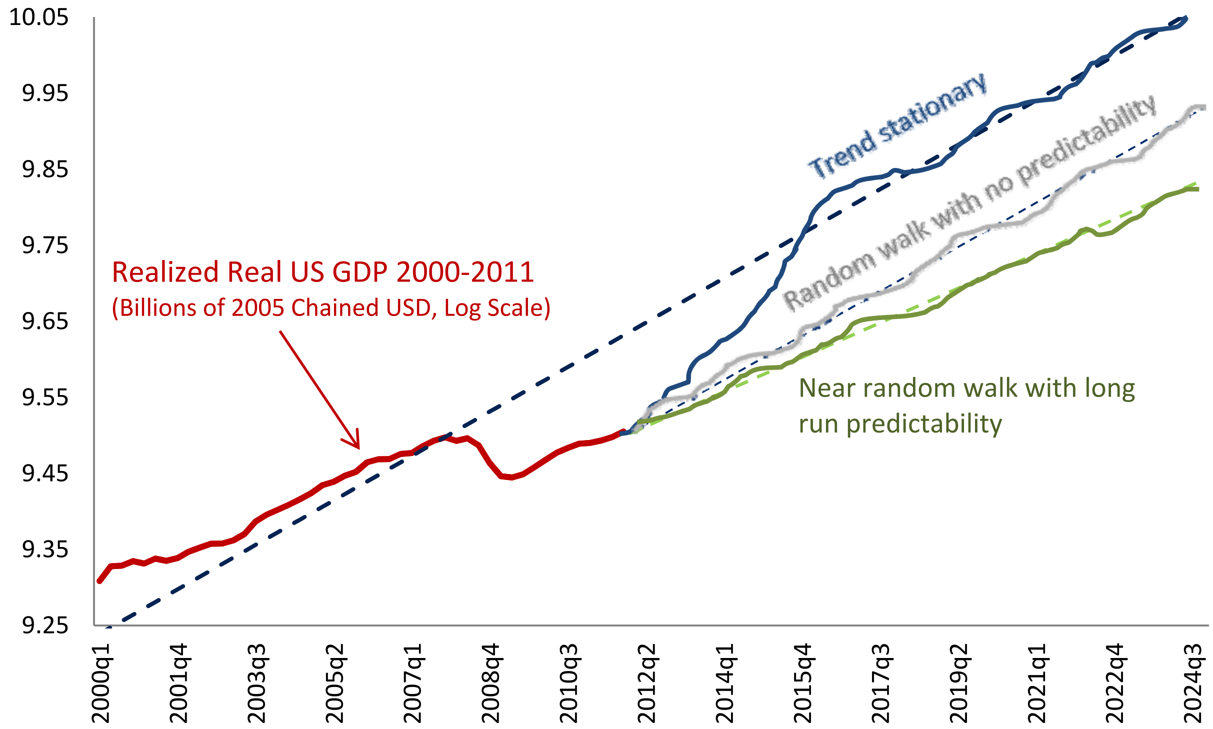
Figure 5 plots realized output through 2011 Q4, plus three projected paths for future output over the next 15 years:

- In the top path, real output is trend stationary and mean reverts to its pre-crisis long-term trend line. The shock to GDP in 2008 is simply bad luck in the short term that quickly corrects through above-normal growth rates in subsequent quarters.
- In the middle path, real output follows a random walk. The shock in 2008 permanently affects the *level* of real GDP, although *trend growth* remains around its long-term historical average of 3.3 percent. Again, the 2008 shock to output is bad luck. The shock has no implications for future GDP *growth*. However, in this scenario, the consequences for the level of GDP are permanent.

[7] See Appendix B for a more detailed description of the near random walk model following Bansal and Yaron (2004), and Hansen, Heaton and Li (2008).

- In the bottom path, a *large economic* shock can carry a persistent effect on output *growth*. Following the large negative shock, the economy shifts to a new, lower growth regime. This lower growth is indicated by the lower slope of the bottom dashed line.[8]

Figure 5: There is Uncertainty about the Long-Term Trend.



The Figure shows realized US real GDP in 2005 dollars (billions) in red and a fitted trend line using blue dashes based on average growth between 1947 and 2007. The lines from 2012 onward show stylized predictions from different GDP models. The trend stationary model predicts a fairly rapid return to the long-term GDP trend, in blue. The random walk model predicts constant growth from the most recent GDP observation, in gray. The near random walk model with long-term predictability predicts growth from the most recent GDP observation, but at a growth rate that differs from the historical average. The vertical axis employs a logarithmic scale.

Although the Figure shows a permanent decline in growth for the third scenario, this is not inevitable. A long-term decline in growth followed by a gradual reversion to the prior trend line is compatible with this scenario. In this sense, the third scenario can bridge the gap between the first and second models.

The distinction among these three paths highlights the challenges facing investors after the large shock to real growth in 2008. The economy could either revert to its pre-crisis average growth rate after a prolonged period of sub-par growth, or trend growth could remain permanently fixed at the lower level. Unfortunately, the uncertainty around the ultimate trend level will only be resolved in the very distant future.

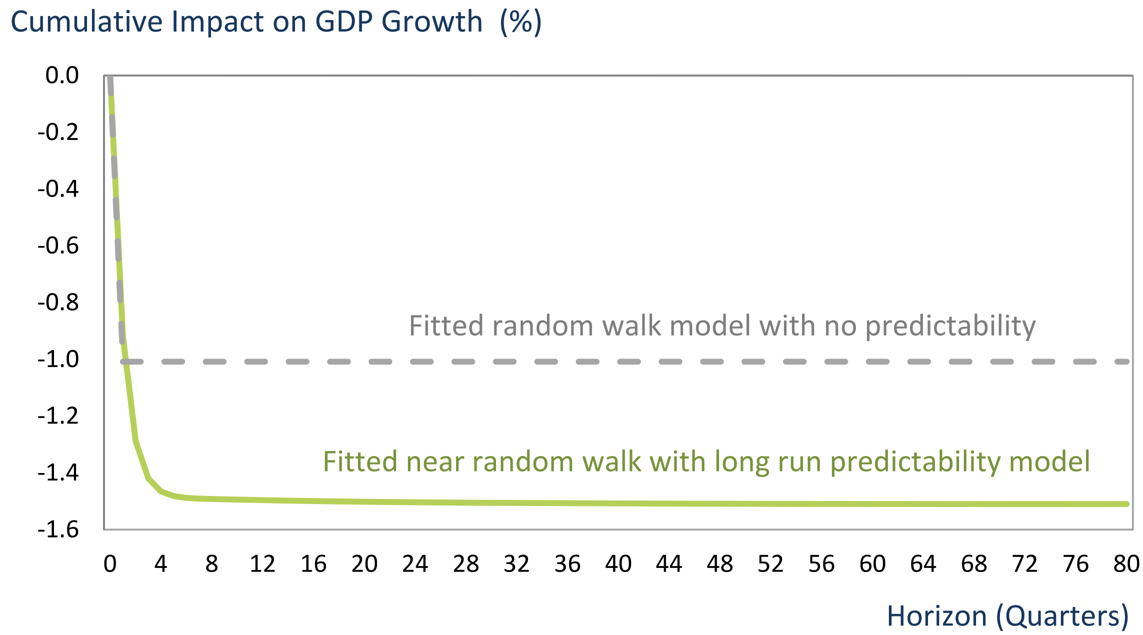
[8] Appendix C has similar figures showing realized and long-term trend GDP growth for the UK, Germany, Spain, and Japan.

Although the first scenario still plays an important role in popular conversation about the economy, it is now widely discredited in academic and financial circles. The random walk model of the second scenario is the current mainstay of academic work on business cycles.[9]

The third scenario is a relatively new interpretation in academic work that attempts to relate business cycles to asset prices. Since the difference between the second and third model only manifests itself over long horizons, data limitations make it hard to choose among these models conclusively. Nonetheless, observed GDP growth is consistent with this newer model.

Figure 6 depicts the cumulative impact on US GDP over 20 years following a 1.01 percent (one quarterly standard deviation) negative shock to US GDP for the random walk model and a near random walk model.[10] We fitted both models to quarterly US GDP data from 1947 to 2011.[11] The random walk model, shown with a dashed line, restricts the impact of the initial one percent negative shock to a permanent one percent decline in GDP. In contrast, the estimates of the near random walk model, shown in green (solid line), imply a one percent decline in GDP followed by an expected gradual decline of an additional 0.5 percent. When we remove the restrictions of the random walk model, the data show predictable variations in GDP growth. The difference in the projected long-term decline in GDP implied by the two models is material and must not be ignored by investors.

Figure 6: There is Evidence for Long Term Persistence in GDP Growth.



The Figure shows the cumulative impact on GDP growth over 20 years following a negative (one quarterly standard deviation or 1.01 percent) shock to US GDP growth, for the random walk with constant growth model (grey dashed line) and a near random walk model (green solid line). We fitted both models to quarterly US GDP data from 1947 to 2011, available from the Bureau of Economic Analysis.

[9] Nelson and Plosser (1982) and many subsequent papers find evidence for the random walk model with constant growth.

[10] The near random walk model is adapted from the one described in Hansen, Heaton and Li (2008). We used real GDP instead of aggregate real consumption, and ran a vector autoregression (VAR) of log real GDP growth and log corporate profits to GDP ratio, with two lags. The latter variable helps to identify a persistent component in GDP growth.

[11] The plots in figure 5 are robust to the time period used to fit the models, even if we exclude post 2008 data.

An important feature of this new description of GDP growth is that it helps to explain the behavior of financial market participants and asset prices. We will explore these implications in the next paper of this series.

These insights prompt the following questions:

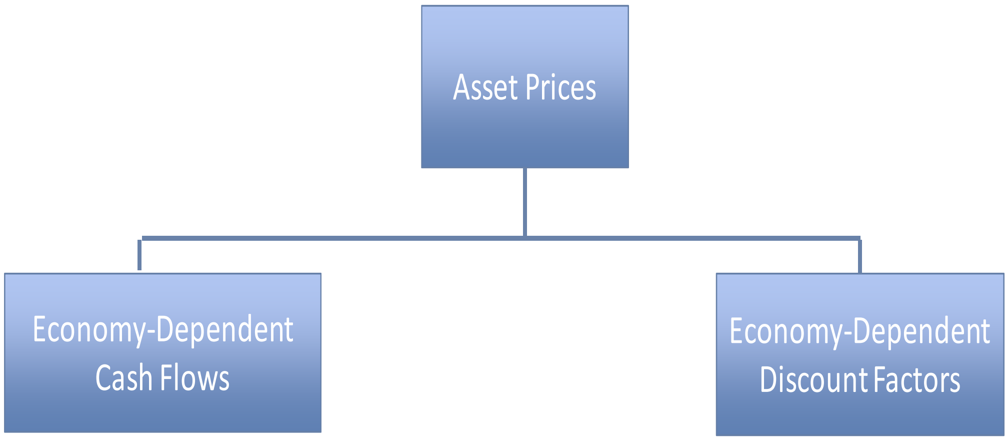
1. What is the impact on asset prices of persistent shocks to the economy, given that their impact will only be revealed over long horizons?
2. Are there additional signals that investors can examine to determine whether a persistent shock to the economy has occurred?
3. How do we design portfolio strategies conditioned on the potential for persistent, and adverse, shocks to trend growth?

We explore the first of these issues in the next section; we will address the second and third issues in future papers.

The Impact of Macro Shocks is Best Assessed by Returning to First Principles

To understand the impact of macroeconomic shocks on equilibrium asset prices, we need to go back to first principles. The most basic principle of modern asset pricing is that the competitive equilibrium value of an asset equals the expected discounted value of current and future asset cash flows. This principle is illustrated in Figure 7.

Figure 7: Cash Flows and Discount Factors Link Asset Prices to the Economy.



It is worth emphasizing what we mean by discount factors. Asset prices reflect the fundamental trade-off investors make by forgoing current consumption in order to invest in assets that pay off in an uncertain future. In order to arrive at the present value of uncertain future payoffs, investors apply discount factors that reflect investors' sensitivity to future consumption growth. Thus, discount factors do depend on the economy. [12]

[12] In usual valuation models, the discount factor is expressed as an interest rate and a risk premium. More modern asset pricing models show that both the risk-free interest rate and the risk premium in these valuation models depend on the economy and investor preferences.

The application of the fundamental principle of asset valuation leads to the conclusion that macro risk has an impact on valuation and risk via two channels:

- Cash flows
- Discount factors

As previously discussed, macroeconomic risk is the risk of persistent shocks to the real GDP trend growth. Given a large negative shock, investors (and other economic agents) face uncertainty as to precisely what the shock represents: is it bad luck for the current period likely to be offset by unusually high future growth, bad luck for the current period with no implications for future growth, or does it represent a transition to a new and lower trend growth rate? A complete analysis of investors' questions should also look at the impact of shocks on asset cash flows and discount factors. Since the resolution of this question is likely to occur over a long time period, analyzing the impact of macro shocks on cash flows and discount factors should also allow for horizon effects. These themes will be treated in more detail in subsequent papers.

Conclusion

Understanding the impact of macroeconomic events on asset prices and portfolio risk is an important component of asset allocation decision making. The importance of these issues has been accentuated by events in the global economy and global financial markets since 2008. Historically, linking portfolio strategy and risk management to the economy has been difficult. That difficulty has arisen in part because of the lack of a working definition of macroeconomic risk, and in part because of observed empirical regularities.

This paper is the first in a series that addresses themes related to macroeconomic risk. We have focused on providing a structure around the definition of macroeconomic risk, which we see as the risk of a *persistent* shock to real economic *growth*. This definition is consistent with observed regularities in the data: economic shocks seem to carry limited information about short-term future real economic growth; occasional, large negative shocks to the economy, however, do occur and have been followed by prolonged periods of sub-par growth.

The latter point underscores the principal issue facing investors: how should asset allocation respond to large macroeconomic shocks, given that their consequences are likely to be resolved over long time periods? We advocate going back to the basics of asset pricing and analyzing the impact of macro shocks on both asset cash flows and discount factors. Subsequent papers in this series will explore both of these themes, as well as the implications for asset allocation, investment strategy, and risk management.

References

Barro, Robert J., 2006, "Rare Disasters and Asset Markets in the Twentieth Century." Quarterly Journal of Economics, 121(3), 823–866.

Barro, Robert J. and José F. Ursúa, 2008, "Macroeconomic Crises since 1870." Brookings Papers on Economic Activity, 255–350.

Nelson, Charles R. and Charles I. Plosser, 1982, "Trends and Random Walks in Macroeconomic Time Series." Journal of Monetary Economics 10, 139-162.

Bansal, Ravi. and Amir Yaron, 2004, "Risks for the Long-Run: A Potential Resolution of Asset Pricing Puzzles." Journal of Finance 59, 1481-1509.

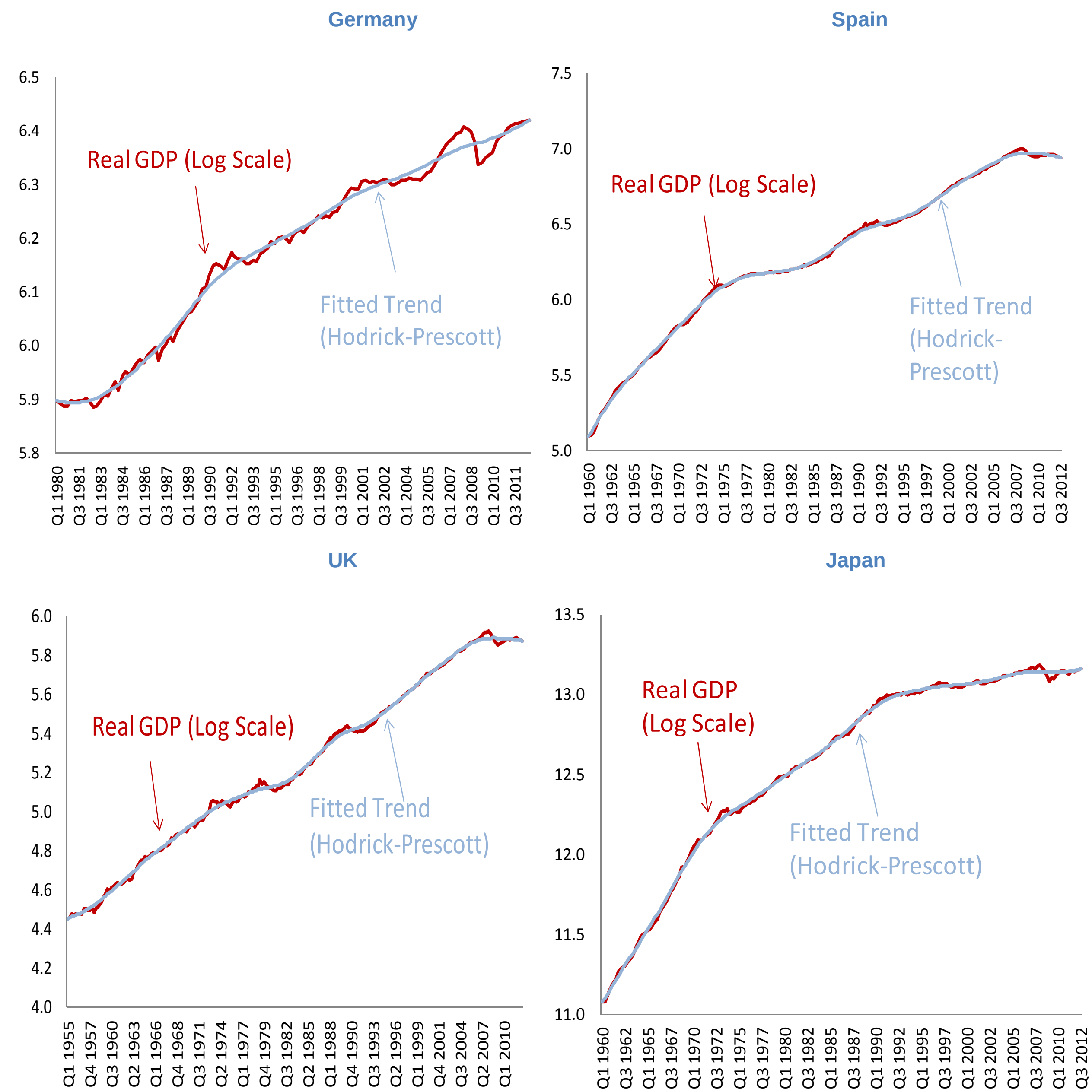
Hansen, Lars P., John Heaton and Nan Li, 2008, "Consumption Strikes Back? Measuring Long Run Risk." Journal of Political Economy 116 (2), 260-302

LeRoy, Stephen F., and Richard D. Porter, 1981, "The Present-Value Relation: Tests Based on Implied Variance Bounds." Econometrica 49 (May 1981), 555-574.

Appendix

A. It is Hard to Distinguish the Real Economy from its Trend

Long Term Evolution of Real Output for Germany, Spain, UK and Japan



B. Unit Roots in Macroeconomic Time Series

What Are Unit Roots?

Macroeconomic and financial data come in the form of time series. These are conveniently modeled as sequences of random variables over time or stochastic processes that follow a parsimoniously parameterized law of motion.

Available historical data suggest that macroeconomic *growth* series such as real output growth, inflation, change in commodity prices are *stationary.*[13] For example, as a first approximation, real GDP growth (Δy_t), expressed in logarithm, can be described as:

$$\Delta y_t = m + e_t \tag{1}$$

where e_t is independently and identically distributed over time, and m is the mean trend growth.

In contrast, macroeconomic "*level*" series such as real output, aggregate price level, or commodity prices trend over time. The common interpretation is that these series are generated by a non-stationary *unit root* process. Formally, a process is unit root if its first difference is stationary. For example, according to equation (2), log real GDP (y_t) has a unit root and follows a random walk with trend:

$$y_t = m + y_{t-1} + e_t \tag{2}$$

More general forms of unit root processes would allow the stationary residual term e_t to exhibit serial correlation, allowing for predictability in output growth.

There could be, in theory, another competing interpretation for the trending behavior of level series. When confronted with small samples –typical macroeconomic time series lengths are about 60 years or 240 quarters at best– it can be difficult to visually distinguish the trending pattern produced by a unit root process from the one generated by a "trend-stationary" process that is stationary around a *deterministic* time trend. In the latter case, real output would be better described as:

$$y_t = mt + ay_{t-1} + e_t \tag{3}$$

for some positive parameter a strictly less than one (instead of being equal to one as in equation (2)). In fact, many practitioners and academics alike used equation (3) to model real log GDP at least until the late 1970s, until they realized that past information added little value in predicting future GDP growth over short horizons.

Nelson and Plosser (1982), among others, showed that it is difficult to reject the unit root hypothesis. Their results suggest that the random walk model with constant growth as described in equation (2) may be a better approximation for real output in the short term.

[13] There are two popular notions of stationarity – strong and weak – among many refinements. A process (y_t) is strongly stationary if the joint distribution of any sequence of the form ($y_{t-s}, y_{t-s+1}, \dots, y_{t+s}$) does not depend on *t* nor *s*. A process (y_t) is weakly or covariance stationary if mean and second moment Ey_t and Ey_t^2 are finite, and if the covariance between y_t and any of its lags y_{t-s} only depends on the lag length *s*.

However, the recent academic literature attempting to relate asset prices to economic growth risk favors a third alternative. It argues that, in small samples, the standard statistical tests will find it hard to disentangle a random walk model from a near random walk alternative, which adds a small but very persistent component to output growth. An example of a near random walk model for output is the following:

$$\begin{aligned} y_t &= m + y_{t-1} + f_{t-1} + e_t \\ f_t &= \rho f_{t-1} + u_t \end{aligned} \tag{4}$$

Here the new variable f_t represents the predictable component of output growth.[14] The parameter ρ is close to one (about 0.98). This allows current shocks to output to carry persistent effects to output growth over time, with a slow reversion to the constant trend growth (*m*). However, over short horizons, the predictable component f_t may only explain a very small fraction of the volatility of output growth (say 1%), and it will be hard to distinguish model (4) from model (2) based on observed GDP. However, the two models have very different implications for asset prices. Assuming that investors fear the larger uncertainty around the long-term trend growth implied by a near random walk model similar to (4) helps to capture the observed link between asset prices and GDP.

Testing for Unit Roots

Suppose we want to test the hypothesis that real output follows a unit root process described by the random walk model in (2) against the trend stationary alternative described in (3). The common approach suggested by Dickey and Fuller is to run an Ordinary Least Squares (OLS) regression of current output on lagged output and test for the hypothesis that the parameter a is equal to one using the t-statistic:

$$t_a = \frac{\hat{a} - 1}{\hat{\sigma}_a}, \tag{5}$$

where $\hat{a}$ is the OLS estimate of a and $\hat{\sigma}_a$ is its OLS standard error

However, under the hypothesis that real output is truly unit root, the standard OLS test using the t-statistic is flawed. Indeed Dickey and Fuller (1979) showed that the t-statistic in this case tends to be biased downwards and does not converge towards a standard normal distribution in large samples[15]. In particular, the OLS test would tend to reject the unit root hypothesis more often than it should.

Dickey and Fuller (1979) tabulated the distribution of the t-statistic[16] and calculated the appropriate critical values to conduct a test of the unit root hypothesis at a given confidence interval.

---

[14] This is a version of the Bansal and Yaron (2004) model used to describe the evolution of US aggregate consumption. They assume that the predictable and persistent component to output growth (f_t) is unobserved. Hansen, Heaton, and Li (2008) identify the aggregate GDP to earnings ratio as a possible driver of this predictable component.

[15] The intuition for this result is that the OLS standard error $\hat{\sigma}_a$ of the estimated coefficient $\hat{a}$ is inversely proportional to the sample average of y_{t-1}^2, which grows to infinity with the sample size when y_t is a random walk. The t-statistics typically take large negative values.

16 Under the random walk hypothesis, the t-statistic is shown to converge in large samples to the following "degenerate" distribution (see Phillips (1987)):
$t_{\hat{a}} \rightarrow \int_0^1 B_s dB_s \Big/ \left(\int_0^1 B_s^2 dB_s \right)^{1/2}$ where (B_s) is a standard Brownian motion.

Multiple refinements of the Dickey Fuller test have been developed. A commonly used test is the Augmented Dickey Fuller test (ADF) to handle more general forms of unit root processes with serially correlated error terms. In this case, the unit root hypothesis takes the form

$$y_t = m + y_{t-1} + b_1 \Delta y_{t-1} + b_2 \Delta y_{t-2} + ... + b_p \Delta y_{t-p} + e_t$$

for some number of lags p. The trend stationary alternative is:

$$y_t = c + mt + a y_{t-1} + b_1 \Delta y_{t-1} + b_2 \Delta y_{t-2} + ... + b_p \Delta y_{t-p} + e_t$$

where $a<1$

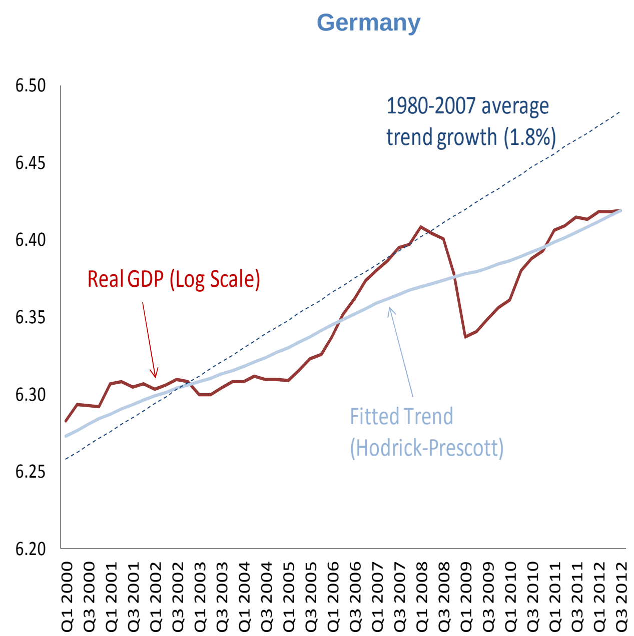
Said and Dickey (1984) tabulated the distribution and critical values for a test of the unit root hypothesis for the following test statistic:

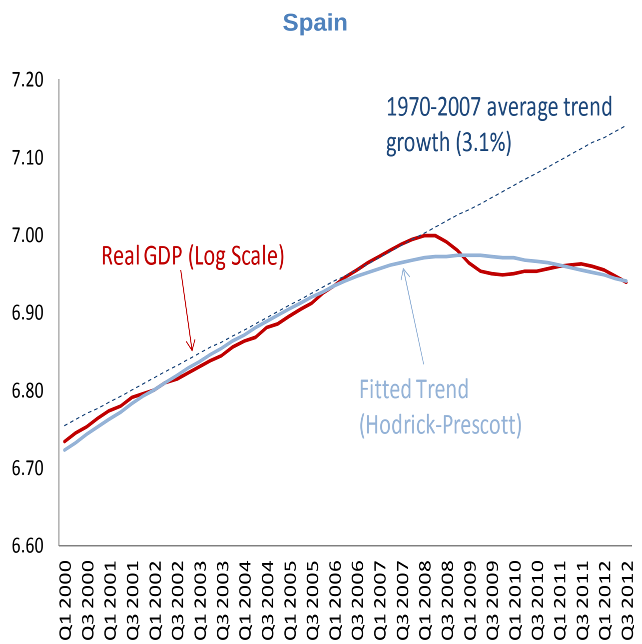
$$t_{ADF} = T(\hat{a} - 1) / (1 - \hat{b}_1 - ... - \hat{b}_p) \quad \text{where } T \text{ is the number of observations}$$

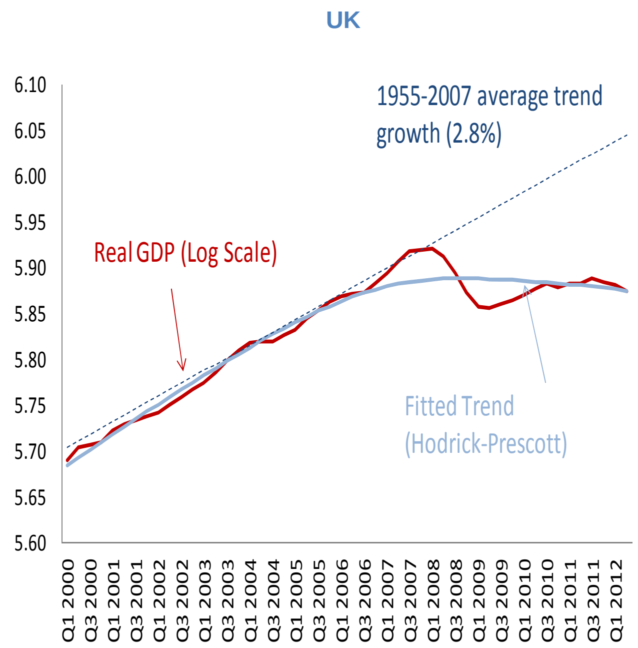
The Dickey and Fuller types of tests are heavily dependent on the number of observations. In particular, they are usually not well suited to distinguish unit root processes from near unit root alternatives in small samples. In such case, the Elliot, Rotemberg and Stock (2001) test is commonly preferred. For most macroeconomic level time series and typical post World War II samples, it is difficult to reject the unit root hypothesis, regardless of the type of test used.

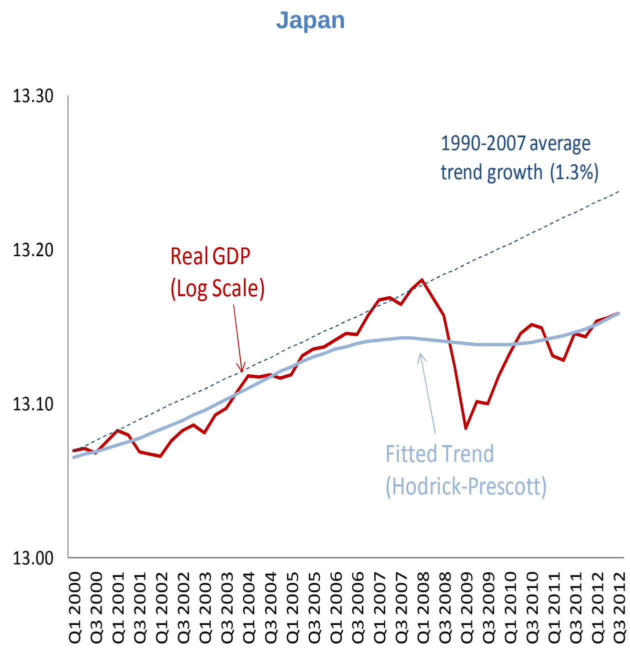
C. There is Uncertainty Around Trend Predictability

Evolution of Real GDP since 2000 and recent trend gap for Germany, Spain, UK, and Japan









Client Service Information is Available 24 Hours a Day

clientservice@msci.com

Americas

Americas	1.888.588.4567 (toll free)
Atlanta	+ 1.404.551.3212
Boston	+ 1.617.532.0920
Chicago	+ 1.312.675.0545
Montreal	+ 1.514.847.7506
Monterrey	+ 52.81.1253.4020
New York	+ 1.212.804.3901
San Francisco	+ 1.415.836.8800
Sao Paulo	+ 55.11.3706.1360
Stamford	+1.203.325.5630
Toronto	+ 1.416.628.1007

Europe, Middle East & Africa

Cape Town	+ 27.21.673.0100
Frankfurt	+ 49.69.133.859.00
Geneva	+ 41.22.817.9777
London	+ 44.20.7618.2222
Milan	+ 39.02.5849.0415
Paris	0800.91.59.17 (toll free)

Asia Pacific

China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+ 852.2844.9333
Seoul	798.8521.3392 (toll free)
Singapore	800.852.3749 (toll free)
Sydney	+ 61.2.9033.9333
Tokyo	+ 81.3.5226.8222

About MSCI

MSCI Inc. is a leading provider of investment decision support tools to investors globally, including asset managers, banks, hedge funds and pension funds. MSCI products and services include indices, portfolio risk and performance analytics, and governance tools.

The company's flagship product offerings are: the MSCI indices with approximately USD 7 trillion estimated to be benchmarked to them on a worldwide basis[1]; Barra multi-asset class factor models, portfolio risk and performance analytics; RiskMetrics multi-asset class market and credit risk analytics; MSCI ESG (environmental, social and governance) Research screening, analysis and ratings; ISS governance research and outsourced proxy voting and reporting services; FEA valuation models and risk management software for the energy and commodities markets; and CFRA forensic accounting risk research, legal/regulatory risk assessment, and due-diligence. MSCI is headquartered in New York, with research and commercial offices around the world.

[1]*As of June 30, 2011, based on eVestment, Lipper and Bloomberg data.*