Stress Testing Market Report

Credit Risk: Default, Migration and Correlation Shocks

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Introduction

The 2008 Financial Crisis taught us several lessons about the dynamics of credit ratings and the probability of default. The focus of this report is to explain how shocks stemming from differing migration and default probabilities, as well as correlation changes, impact expected shortfall1 and expected loss2 in our sample scenarios.3

Our analysis will utilize the CreditMetrics framework using CreditManager.4 We start by generating a loss distribution by simulating thousands of profit and loss values for a portfolio. This loss distribution, which is typically measured at a one-year horizon, is largely a function of the obligor (or issuing entity), ratings migration, default probability, and correlation. The stresses we introduce in this paper affect this loss distribution on two fronts. First, we stress ratings migrations and defaults. Next, we stress correlation. From the loss distribution, expected shortfall at varying confidence intervals is calculated.

Our research suggested the following:

- Capital (measured by the expected shortfall as a percentage of value) doubles, and in a few cases nearly triples, on the back of our joint correlation and default stress.
- Assets held at face value have lower risk figures; however, they are relatively more sensitive to stress tests when compared to assets held at market value.
- The correlation stress increasingly ‘takes hold’ as ratings deteriorate, reflecting the exponentially increasing probability of default as ratings decline.

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1 Defined as the average PnL of the portfolio that exceeds VaR.
2 Defined as current value minus mean horizon value (or the expected value in 1-year).
3 The Basel Committee on Banking Supervision suggests the use of expected shortfall may offer advantages relative to VaR for the internal models-based approach. Reviewing the FRTB, Christopher Finger, September 2012. http://www.msci.com/resources/research/articles/2012/Market_Insight_Reviewing_the_FRTB_Sept%202012.pdf
4 Please see CreditMetrics Technical Document for further information.
Portfolio Overview

For our analysis, we focus on three portfolios which we describe in Table 1. Here, we will analyze one portfolio consisting of only government bonds (the ‘Government Bond’ portfolio), and two portfolios consisting of corporate bonds. The ‘Diversified Corporate’ portfolio consists of bonds with ratings predominantly in the AAA and AA range, while the ‘High Yield’ portfolio consists of bonds with a BBB and below rating.5

Table 1: Bond Portfolio Weighted Average Probability of Default and Composition by Rating.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Weighted Avg p(Def)</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Bond (Gov)</td>
<td>4bp</td>
<td>53%</td>
<td>36%</td>
<td>8%</td>
<td>4%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Diversified Corp (Corp)</td>
<td>8bp</td>
<td>46%</td>
<td>29%</td>
<td>14%</td>
<td>11%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>High Yield (HY)</td>
<td>400bp</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>89%</td>
<td>9%</td>
<td>2%</td>
<td>0%</td>
</tr>
</tbody>
</table>

As a summary statistic, each portfolio is assigned a weighted average probability of default.6 For example, if one were to hold a representative unit of the portfolio for the next year, this metric explains the expected probability of this unit defaulting (i.e., four basis points or the ‘Government Bond’ portfolio).

Credit Shocks

Turning to our analysis, we now define the credit shocks used to build our stress scenarios with the CreditManager platform.7 These shocks are described below and summarized in Table 2.

1) Ratings Migration and Default Shock: The slowdown in global growth and the persistence of the Eurozone crisis motivates the development of a shock linking future estimates of macro scenarios with ratings transitions. Here, a long-term transition matrix is employed coupled with a migrations stress to simulate ratings impairment due to crisis.

2) Asset Correlation Shock: Taking cues from increases in financial crisis asset correlations, as well as liquidity constraints that are a typical knock-on effect of crisis, this analysis also introduces a correlation shock. Beginning with a ‘through the cycle’ 10-year window, we increase the weight of the systemic component to effectively increase obligor correlations.

Our analysis will be based in both the ‘Market Mode,’ where changes in ratings and defaults drive P&L, and ‘Book Mode,’ where default is the primary driver of P&L since assets are held at face value. Note that ‘Book Mode’ assumes the investor holds bonds at book value, then default leads to recovery values.

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5 We loosely define HY to included BBB rated bonds in our analysis.
6 Weighted average probability of default can be thought of as an average loss spread over the entire portfolio. Generally speaking, if you own a AAA-rated bond, you have a 1bp chance of default in one year. For a BBB-rated bond this probability is higher. Here we are assessing the average chance of default changes.
7 A detailed methodology of transition matrix macro stress is included in the Appendix of this document.
Table 2: Description of Credit Shocks and Base Scenario.

<table>
<thead>
<tr>
<th></th>
<th>Correlation Structure</th>
<th>Migration Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Case</strong></td>
<td>Long Term</td>
<td>Long Term</td>
</tr>
<tr>
<td><strong>Migration &amp; Default Only Stress</strong></td>
<td>Long Term</td>
<td>Stress</td>
</tr>
<tr>
<td><strong>Correlation Only Stress</strong></td>
<td>Stress</td>
<td>Long Term</td>
</tr>
<tr>
<td><strong>Correlation + Migration &amp; Default Stress</strong></td>
<td>Stress</td>
<td>Stress</td>
</tr>
</tbody>
</table>

Migration and Default Stress

We begin our stress scenario with a migration and default analysis. For this, we utilize a long-run, ‘though the cycle’ transition matrix found in CreditManager. We then transform the corresponding average probabilities of migration and default into a set of thresholds and then apply a shock to these thresholds. These shocks are illustrated in Figure 1 below.8

Investment Grade (IG) shocks target the AAA to BBB sections of the ratings matrix, while Speculative Grade (SG) shocks target the BB to CCC sections. The shocks are created via a minimization function that effectively transfers the desirable properties of a quarterly version of our long term transition matrix, to those from quarterly empirical transition matrices. This minimization produces the shocks shown in Figure 1; each pair of bars in Figure 1 corresponds to one empirical transition matrix. A negative shock implies a decrease in threshold; within CreditManager’s Merton model framework, this suggests a smaller distance for a negative asset return to travel in order for a downgrade or default to occur.

Following the application of the shocks, we transform the thresholds back to transition probabilities, to arrive at a stressed transition matrix.9 To capture and import this stress in CreditManager, we focus specifically on the cumulative probability of downgrade at each rating state of the stressed matrix. We compare these probabilities for each rating state to the respective values in the base matrix.

The increase in cumulative downgrade probabilities due to the stress are shown in Figure 2. Portfolio-level average probabilities of default are summarized in Table 3. We incorporate the increased probability of downgrade into CreditManager via a scenario and transition event.

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8 When we look at all the transitions together we look at an average shock across Investment Grade or Speculative Grade sectors as a filtering mechanism. An exact match to company-level data is less desirable in the context of this analysis.

9 Further detail of the mechanics of the stressed transition matrix can be found in the Appendix.
Asset Correlations

Having considered migration and default as the first half of our stress scenario, we now turn to asset correlations. Within the context of portfolio credit risk, correlation plays a crucial role, especially in the tail of portfolio profit and loss distributions. During periods of crises, tails lengthen significantly, with severe implications for statistics such as Expected Shortfall.

Obligors in CreditManager are mapped to a common set of MSCI equity indices, known as factors. The correlation between the equity factors filters back to the obligors via an $R^2$ statistic, which acts as a valve. $R^2$ values are bound between zero and one. Therefore, if $R^2$ is equal to zero, no correlation among the factors filters back to the obligors. Conversely, if $R^2$ is equal to one, all the correlation between the factors filters back to the obligor.

Table 3 summarizes both the migration and default stress and correlation stress scenarios. Here, our average value for equity factor correlations in this study is about 59 percent. Average $R^2$ values in our
portfolio are also about 60 percent. This suggests an average pair-wise obligor correlation of 36 percent.\(^{10}\)

We now consider our correlation stress. Specifically, we refer to a period overlapping with our default stress and consider pairwise obligor correlation. In his 2011 study,\(^{11}\) Zazzara found that pairwise obligor correlations increased approximately 50 percent during the period of 2007 to 2011 when compared to 2005 to 2007. To reflect this increase on, our three portfolios would require setting \(R^2\) to about 90 percent; we decide to move to 100 percent for a more potent stress and increased intuition. That is, at 100 percent, the \(R^2\) valve is opened entirely, allowing correlation to be explained fully by the equity index factor mappings and driven only by systematic factors. The resulting average obligor correlations are 60-70 percent higher than our base scenario levels.

**Table 3: Summary of Portfolio-level Base and Stress Average Probability of Default and Correlation Stress.**

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Migration and Default Stress (DEF)</th>
<th>Correlation Stress (COR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
<td>Stress</td>
</tr>
<tr>
<td>GOV</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>CORP</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>HY</td>
<td>400</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>Avg p(Def)</td>
<td>Avg p(Def)</td>
</tr>
</tbody>
</table>

**Summary of Results**

In the section that follows, we consider the effect of our stress scenarios on the three test portfolios. The metrics we use will be expected loss and expected shortfall at 99 percent and 99.5 percent confidence levels.

**Expected Loss**

Expected loss is a closed form metric driven by credit rating changes and/or default and the passage of time. Hence, we will see that our migration and default stress (‘DEF’ in the figures below) will impact our portfolios, unlike correlation stress (‘COR’ in the figures below). Our results in Figure 3 and Figure 4 suggest these conclusions:

- Migration mode produces a larger value for expected loss across all portfolios, since this setting allows for downgrades to be reflected in the valuation at the horizon.
- Expected loss is less variable across portfolios under the migration mode.
- The sensitivity to the transition matrix stress is significantly greater under the book mode; this is related to the relative stability of migration mode and highlights an important difference in the two settings. Allowing for downgrades to be revalued essentially builds risk into the migration mode statistic. In book mode, an increase default probability is a binary shock in the sense that before the shock there was no ‘mass’ contributing to reduced valuation due to downgrade, and after the shock the mass was given a 100 percent weight in the default bucket. Comparing this

\(^{10}\) We reference the following formula: \(\sqrt{60\%} \times \sqrt{60\%} \times 59\% = 36\%\).

to migration mode, we see that risk is reshuffled around the ratings structure, not in a binary framework. For example, the risk contributing to a move to a CCC-rating in migration mode before the shock might be transferred to default after the shock. This is a relatively smoother ramp compared to book mode, and explains the difference in sensitivities.

Expected Shortfall, 99.0 Percent and 99.5 Percent Confidence
For each scenario, we ran our three portfolios through 100,000 simulations to produce a Profit and Loss (P&L) distribution. From this distribution, expected shortfall at 99 percent and 99.5 percent was calculated. Results are summarized in Figures 5-7. Our results suggest the following:

**Government Portfolio**
- Within book mode, highly rated portfolios can be relatively insensitive to our correlation stress. Increasing the pair-wise likelihood of joint movements is not enough to overcome the very low probability of default.
- Under migration mode, the effects of correlation are realized, as downgrades in addition to default allow for the correlation to ‘take hold’ and increase capital measures by about 75 bps and 100 bps at 99.0 percent and 99.5 percent confidence, respectively.
- Tail risk is heightened significantly due to our downgrade and default stress. This is seen in a nearly tripling of capital at 99.0 percent confidence.

**Corporate Portfolio**
- Correlation by itself produces significant increases in capital of 1.5 percent and over 2 percent at 99.0 percent and 99.5 percent respectively in migration mode. This reflects the relatively lower ratings compared to the Government portfolio, and the effects of correlation taking hold as ratings decline.
- The default stress is again a significant driver of risk in both modes, with the addition of correlation at 99.5 percent in migration mode providing a relatively large increase in capital.
High Yield

- Overall, this was the most sensitive portfolio to default, migration, and correlation shifts.
- Notably at 99.5 percent confidence in migration mode, the correlation shock nearly matches the shock produced by the default and migration stress. This suggests the portfolio is sitting on a relatively slippery slope in terms of increases in write-downs, thanks to the presence of heightened correlation in a relatively low rating environment.

Figure 5: Expected Shortfall, 99 percent confidence, Book Mode.

Figure 6: Expected Shortfall, 99 percent confidence, Migration Mode.

Figure 7: Expected Shortfall, 99.5 percent confidence, Book Mode.

Figure 8: Expected Shortfall, 99.5 percent confidence, Migration Mode.
Conclusion

Given the uncertain trajectory of global growth and the persistence of the Eurozone crisis, investors can analyze the effects of stressing parameters that affect credit rating migration, defaults, and correlation. CreditManager can be a useful tool in showing the stability, or lack thereof, of the loss profiles of credit portfolios.

In a buy and hold context, our analysis revealed that book mode generally produced lower risk figures that are more sensitive on a relative basis to ratings changes and correlation moves. Conversely, in a mark to market context, migration mode results produced larger risk figures which were relatively less sensitive to these aforementioned changes.

In both cases, the stresses highlighted in this paper demonstrate significant increases in capital, in the range of two to three times, when we simulate a crisis environment. Our methodology captures the effect of correlation ‘taking hold’ of portfolio loss figures as credit quality diminishes.
Appendix

CreditManager Transition Matrix for Macro Stress Methodology

We have generated quarterly stress factors by empirically investigating the observed quarterly Moody’s transition probabilities from 1970 to 2011. Our methodology is based on the papers by Otani (2009) and Wei (2003).12

We do not directly use the transition probabilities as stressed scenarios, as these short term transition observations include idiosyncratic events, such as missing rating migration events from one rating to another rating state, or increased rating migrations to a certain rating because of coincidental rating changes on a few companies.

Our approach is to figure out factors that will shift the long-term average transition probabilities such that we get a view of the credit risk environment that is close to the observed transition ratings. Thus the final shifted transition matrix will not be affected greatly by peculiar rating changes in the observed transitions, but capture the overall changes dominating the transitions in that period.

In order to simplify the notation, we assign each alphabetic rating to a numeric value with the order from highest grade to default. For example eight (8) grade rating levels will use the following assignments; AAA=1, AA=2, A=3, BBB=4, BB=5, B=6, CCC=7 and Default=8.

The average probabilities in the transition matrix, is transformed into a set of thresholds \(Z_{i,j}\) for transition from rating \(i\) to rating \(j\) by the inverse of the cumulative standard normal distribution function:

\[
Z_{i,j} = \Phi^{-1}\left(\sum_{k=1}^{j} \tilde{p}_{i,k}\right)
\]

where \(\tilde{p}_{i,k}\) denotes the average transition probabilities from \(i^{th}\) to \(k^{th}\) rating state. Since we have decided to use quarterly macro series data, we need to transform the annual average transition matrix that is already available in CreditManager.13 Average transition probabilities for quarterly periods are calculated by transforming the annual matrix to a generator matrix, and diagonally adjusting to guarantee positive probabilities; see Inamura (2006)14 for more information.

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13 CreditManager stores annual average transition matrices from S&P and Moody’s. For the rest of the analysis, we used 1970-2011 average transition matrix from Moody’s. Our approach tries to obtain the best transition probability forecasts matching Moody’s data. CreditManager also allows the clients to input their own transition matrices. Our estimated macro factor regression coefficients can be used with such matrices, as long as the dynamics of the client’s transition matrix is similar to global or USA issuer transitions. Also we would like to point out that global diversified transition matrices are heavily weighted by the USA issuers in Moody’s transition data, because of the large amount of rated issuers in USA.

We assume that the thresholds will change through time by linear shocks.

$$Z_{i,j} = \bar{Z}_{i,j} + x_{s(i)}(t) + \epsilon_{i,j}(t)$$

where $x_{s(i)}(t)$ is the shock added to all the transition thresholds from the $i^{th}$ rating state for the quarterly period ending at time $t$. $\epsilon$ is the noise that is trying to capture the random changes in transitions. $s(i)$ is clustering function that is used to apply consistent shocks depending on the initial rating state of the transition. We have tested models where no clustering has been applied ($s(i) = i$), and investment grade (IG) and speculative grade (SG) ratings are separated ($s(i \in IG) = IG$ or $s(i \in SG) = SG$). The clustering by IG and SG grouping resulted in reducing the effects of individual company specific events. A positive shock suggests the thresholds will increase, thus the probability of transitions to lower rating levels and default will decrease.

Once we have a time series of shocks, the shocked transition probabilities are estimated as:

$$\hat{p}_{i,k} = \Phi(\bar{Z}_{i,j} + x_{s(i)}(t)) - \Phi(\bar{Z}_{i,j-1} + x_{s(i)}(t))$$

Transition to highest rating state is given by $\hat{p}_{i,1} = \Phi(\bar{Z}_{i,1} + x_{s(1)}(t))$. The time series of shocks are found by minimizing the difference between the observed rating transition probabilities and the shocked ones. We also use a weighted minimization target by the number of issuers in each rating cohort at any given quarter ($N_i(t)$). This allows the bias of the estimated shocks to be projected towards the rating bucket that has the most issuers in a rating cluster.

$$\min_{x_{s(t)}} \sum_{i \in s} N_i(t) \sum_{j} (p_{i,j}(t) - \hat{p}_{i,j}(t))^2$$
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1As of June 30, 2011, based on eVestment, Lipper and Bloomberg data.