Factor allocation to asset allocation

The power of a factor-based framework

Chenlu Zhou, Andrea Amato

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Introduction

While asset allocation is an important determinant of multi-asset-class portfolio returns, allocation decisions have traditionally been made on an ad-hoc basis. Rules of thumb have evolved from a 60-40 domestic equity-fixed income split to global portfolios including alternative assets. More recently, institutional investors have adopted risk budgeting – rather than capital budgeting – across asset classes to recognize risk drivers and adapt to market regime changes.

However, asset-class buckets are not always clear-cut risk and return drivers. For example, both public and private equity are exposed to systematic equity risk. While private equity also has a "pure private" illiquidity premium, its returns historically had more in common with public equity than with other alternative assets such as hedge funds or real estate. Similarly, fixed income asset returns are distinctly driven by interest rate, inflation and credit factors: high grade corporate bonds have been highly correlated with government bonds, whereas high yield credit has behaved more like equity in volatile markets.

Rather than make allocation decisions among traditional asset classes, some large institutions have begun to look through asset classes to the underlying drivers of risk and returns: factors. On a high level, investors may consider three key questions when implementing factor-based asset allocation.

1. What are the key factors driving risk and returns?

The MSCI Multi-Asset Class Factor Model (MAC) (Shepard 2018) addresses the first question by introducing a unified suite of models, spanning a broad range of public and private asset classes and geographies. At the highest level, the Tier 1 factors identify the factors driving multi-asset class portfolios, supplemented at Tier 2 with tilts and the most common systematic strategies, both designed to support strategic asset allocation. Tiers 3 and 4, with a broader systematic strategy view and granular sector and country factors investors may use to examine tactical asset allocation and sector rotation strategies.

2. How can I determine an optimal factor allocation?

For investors who have adopted a factor-based approach to strategic allocation, the second question continues to be one of the biggest challenges. Factor allocations may reflect the investor's goals, preferences, constraints and views on expected returns and future macroeconomics scenarios. Idiosyncratic characteristics such as liabilities, horizon and liquidity needs could be modeled holistically. Investors can also translate their discretionary views to a tilt away from an otherwise "passive" allocation. This is an area of active research and is out of the scope of this note, where we take the optimal factor allocation as given.

3. **How can I implement a given factor allocation?**

In this note, we present a framework to answer the third question: how can investors construct a portfolio of assets that reflect a target factor allocation? Specifically, given the vast universe of investment vehicles, how can investors select assets that best represent the target factor allocation? If the investor's existing allocation deviates from the target allocation, how can they adjust the investment universe and asset weights to achieve a better result? We introduce a factor-to-asset methodology and present case studies of multi-asset-class portfolio construction with minimal tracking error against a strategic factor allocation and a tactical allocation with a style factor tilt. We also show the power of diagnostic tools to open the "black box" and provide intuition to the process.

1 Methodology

Recent literature (e.g., Greenberg 2016, Bass 2017, Bender 2018) has provided insights into a factor-based asset allocation framework and the potential benefit of diversification across factors, rather than asset classes.

Our main contribution is to address three practical challenges investors face when *implementing* a factor-based asset allocation: a multi-asset-class factor model that combines traditional beta and systematic strategy factors, a methodology that expresses asset-to-factor exposures and a transparent framework that translates target factor exposures to asset weights.

First, the MSCI MAC Model combines traditional beta factors (e.g., equity, rates, commodities) and systematic strategy factors (e.g., value, momentum, carry), and informs both strategic asset allocation and style factor tilts. See Shepard (2018) for more details on the factor model.

Second, we express asset-to-factor exposures based on fundamental characteristics of the assets. This may be contrasted to the approach taken in recent literature (e.g., Greenberg 2016, Bender 2018), which performs time-series regressions to derive asset exposures to the factors. The fundamental approach – which does not depend on the selection and definition of factors – is more intuitive and robust.

Finally, we present an interactive, guided asset selection framework for investors to translate factor allocation to asset allocation. Given a target factor allocation, the optimal asset allocation is defined as one that minimizes tracking error to the factor portfolio, subject to constraints. Our framework starts with a small set of core assets and uses diagnostics to identify optimal additions to the investment opportunity set

that conform to the target factor exposure and improve tracking error; it proceeds step-wise until an optimal portfolio is reached.

In summary, our factor allocation to asset allocation framework has these steps:

1. Choose a factor model and define a desired factor allocation.[1]
2. Define an investment universe of assets suitable for the chosen factor model subject to investor's constraints.
3. Find optimal asset allocation by minimizing tracking error against the target factor portfolio, with a core asset set as a starting point.
4. Generate diagnostics to identify optimal additional assets to the investment universe, and alternatives that generate comparable exposures and tracking error.
5. Update the investment universe and repeat the process of optimization and asset addition until tracking error against the target factor portfolio can no longer be meaningfully reduced.

In the following section, we present two case studies of our framework, and show how diagnostics may provide insights in an asset allocation process. The first case reflects a typical strategic allocation of a U.S. public pension fund. In the second case, we explore a tactical allocation to achieve a "value" style tilt.

2 Case study: Strategic allocation of public pension fund

2.1 Factor allocation

We express the desired factor allocation in terms of target factor risk contribution, defined as:

$$\text{factor risk contribution} = \text{factor exposure} \times \text{marginal factor risk}$$

Factor risk contribution adds up to total portfolio risk, allowing investors to assign risk budget to each factor. We choose this definition not only because it can be intuitively thought of as "risk budgeting," but also because of its desirable feature that, in the case of a mean-variance optimal risk allocation, the return contribution of each factor is proportional to its risk contribution.[2]

[1] In this note, we take target factor allocation as given. In practice, optimal factor allocation may reflect goals, constraints and preferences of the investors and their views on expected returns.

[2] Bruder (2012] provides a proof in the context of risk-budgeting across assets. The same proof applies to risk-budgeting across factors.

Let's consider a U.S. public pension fund whose main objectives are to minimize volatility of its funding ratio while improving its funding status. Key considerations in this liability-driven[3] investment problem include: 1) meeting payout obligations to the pensioners through income-generating assets/factors such as credit and real estate; 2) hedging against long-term, inflation-indexed liabilities through long-duration, inflation-linked assets; 3) hedging against a shortfall in home-economy growth by reducing home bias.

The MSCI MAC Model Tier 1 USA Home Variant can be used to analyze this strategic asset allocation problem. The model has a total of nine factors: the equity, real estate, pure private equity and commodity factors are defined globally, while the home bias, foreign currency, rates, credit and inflation factors are estimated relative to the U.S. markets.

Exhibit 1 shows a stylized factor allocation[4] through risk allocation to the nine factors, reflecting desired minimal risk contribution from home bias factor, and the shift of growth exposure from equity to credit and real estate factors in search of income returns. In practice, the optimal factor allocation is subject to an array of investor preferences and constraints and may be different from this stylized example.

[3] Here we illustrate some intuitive considerations for the optimal allocation given a typical pension fund's liability profile and the link between contribution to the fund and home economy. Providing quantitative guidance of such optimal portfolio is an area of active research and is out of the scope of this note.

[4] Note that the target allocation presented in this stylized example is intentionally chosen to be balanced across factors rather than have concentrated risk contribution from Equity. This choice leads to a lower volatility than that of a typical pension fund portfolio. Leverage could be used to reach a higher volatility level.

Exhibit 1: Target annualized risk contribution

Key factors	Target risk contribution (% of total)	Target risk contribution (annualized)	Factor exposure (at target)	Factor exposure unit
Equity	65%	2.74%	0.33	beta to public equity
USA Home Bias	0%	0.00%	0.00	overweight to the home market
Foreign Currency	0%	0.00%	0.00	foreign currency asset weight
USD Rates	2%	0.08%	3.51	nominal duration (Years)
USA Credit	15%	0.63%	0.05	spread duration × OAS (Years × %)
USD Inflation	3%	0.13%	-1.28	negative real duration (Years)
Real Estate	10%	0.42%	0.15	leverage-adjusted weight
Pure Private Equity	0%	0.00%	0.00	strategy beta
Commodities	5%	0.21%	0.04	leverage-adjusted weight
Total Factor Risk	100%	4.21%		

Target annualized risk contribution to Tier 1 USA Home Variant factors shown in terms of risk contribution and corresponding factor exposures. Risk is calculated as of June 1, 2018. Target risk contribution is expressed in absolute terms ("Target Risk Contribution (annualized)") and as a proportion to the total portfolio risk ("Target Risk Contribution (% of total)").

Exhibit 1 also presents a set of factor exposures corresponding to the target risk contribution, which could, on its own, be useful in understanding the characteristics of the portfolio.[5] For example, equity exposure (0.33) represents the total weight of public equity and beta-adjusted private equity in the portfolio, while rates exposure (3.51 years) reflects the effective duration of the fixed income assets in the portfolio. Note that portfolios with the same factor exposures may present very different factor risk contribution at different times, and vice versa, as factor covariances change due to regime shifts.

2.2 Asset exposure to factors

We establish the investment opportunity set in this example by selecting investable assets that provide exposures spanning the factor space. Furthermore, we define a simple set of "core assets" and add flexibility with a "candidate assets" menu.[6] In practice, index-tracking ETFs have been used as vehicles to replicate index returns. In this note, we perform analysis using indexes as proxies for investable assets and

[5] See Bruder [2012] for discussion on technical details, the existence and uniqueness of such solutions.

[6] This methodology can also be applied to a scenario where investors want to adjust an existing portfolio to achieve a target factor allocation.

use index names in lieu of the investable assets that track the corresponding index returns.

We define core assets as the most liquid and accessible broad market investable assets that best track the factor returns of the selected factor model and provide a reasonable starting point for asset allocation. Aligned with the Tier 1 USA Home Variant factor model, we include in the core assets set World equity and U.S. equity, and three USD fixed income assets representing nominal rates, inflation and credit exposures.

Candidate assets – which we define as including equity sector or style tilts, fixed income duration preferences, and less liquid private asset classes – can be thought of as a "menu" to select from to achieve the factor allocation target.

Factor exposures of core assets and candidate assets for this exercise are presented in Exhibit 2, and investors can tailor both sets of assets based on their preferences and constraints. A closer look at the factor exposures shows the power of the factor lens. For example, pursuing a momentum strategy – which has a positive U.S. home bias driven by the outperformance of U.S. equities relative to the global market in recent years – could result in an unintended bet on the U.S. market; similarly, a material sector tilt could lead to a negative U.S. home bias. In the factor-based framework, these complex – and sometimes not obvious – relationships are an integral part of the optimization, and the investor was able to manage the portfolio exposure and risk holistically.

We include in the candidate set representative private real estate and private equity indexes – IPD Global Property Index and a synthetic U.S. Private Equity Index[7] – to reflect institutional investors' search for yield outside of traditional asset classes. Although these representative indexes are not directly investable, it is important to account for their correlation with the broader investment universe when making strategic allocation decisions. These strategic decisions can then serve as guidelines as investors research and determine investable vehicles that provide desired risk and returns.

[7] We assume that U.S. Private Equity Index has a beta of 1.26 to public equity.

Exhibit 2: Factor exposures of investment universe

Assets / Factors	Equity	USA Home Bias	Foreign Currency	USD Rates	USA Credit	USD Inflation	Real Estate	Pure Private Equity	Commodities
MSCI ACWI IMI	1		0.47						
MSCI USA IMI	1	0.45							
US Treasury				6.28					
US TIPS				8.22		-8.22			
US Corporate IG				7.04	0.08				
IPD Global Property			0.64				1		
US Private Equity	1.26							1	
Global Commodity									1
MSCI Emerging Market IMI	1	-0.45	1						
ACWI Consumer Discretionary	1	0.01	0.45						
ACWI Consumer Staples	1	-0.08	0.57						
ACWI Energy	1	-0.01	0.50						
ACWI Enhanced Value	1	0.01	0.47						
ACWI Financials	1	-0.07	0.56						
ACWI Healthcare	1	0.11	0.34						
ACWI High Dividend Yield	1	-0.09	0.57						
ACWI Industrials	1	-0.03	0.53						
ACWI Information Technology	1	0.16	0.30						
ACWI Materials	1	-0.20	0.70						
ACWI Min Vol	1	0.05	0.44						
ACWI Momentum	1	0.16	0.31						
ACWI Quality	1	0.09	0.37						
ACWI Real Estate	1	-0.04	0.50						
ACWI Telecom	1	-0.12	0.64						
ACWI Utilities	1	-0.03	0.48						
US Treasury 6M				0.47					
US Treasury 1-3Y				1.89					
US Treasury 3-5Y				3.79					
US Treasury 5-7Y				5.52					
US Treasury 7-10Y				7.55					
US Treasury 15Y+				16.92					
US Corporate HY				4.14	0.12				

Factor exposures of investment universe to Tier 1 USA Home Variant factors, calculated as of June 1, 2018. "Core Assets" are in marigold background, while a selection of "Candidate Assets" are in teal background.

Asset exposures to factors are defined intuitively and transparently based on asset characteristics such as equity fundamentals and bond durations, and are the same exposures used in cross-sectional regression to estimate factor returns. Compared to a time-series regression-based approach, this definition is independent of the construction of the factor model and factor returns, and, therefore more robust and interpretable.

2.3 Factors to assets

2.3.1 Constraints

In this study, we impose fully-invested long-only constraints on asset weights. We consider scenarios both with and without leverage: when leverage is allowed, investors can fund investments by borrowing USD cash. Foreign exchange (FX) assets are exempt from the constraints as we assume that FX hedging can be achieved through instruments like FX swaps that have relatively low nominal values.

Other common practical constraints could include lower and upper bounds for instruments or asset classes, liquidity and transaction costs. For simplicity, we show results based on long-only, no-leverage constraints only.

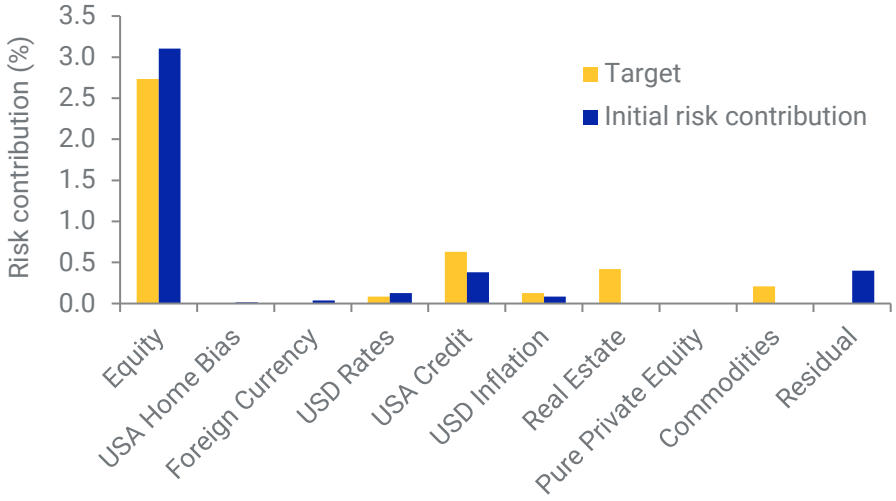
2.3.2 Initial weights

We start with an investment universe of just the core assets and find the optimal holdings that minimize the tracking error to the target factor portfolio under the constraints.[8] The results are shown in Exhibit 3. With five assets, we achieved a 70 basis point (bp) tracking error under long-only no-leverage constraints. The tracking error has two main contributors: 1) factor exposures mismatch, due to lack of real estate and commodity assets; 2) factor residual risk, because core asset returns do not replicate the Tier 1 factor returns exactly.[9]

[8] See Appendix for the mathematical representation of the optimization problem.

[9] Factor residuals capture the components of the detailed factors that the high-level factors miss, and they provide the warning light to indicate when the simplified high-level view is missing important details. For the total risk of a broadly diversified global portfolio, factor residuals often diversify away almost completely.

Exhibit 3: Initial risk contribution



Risk contribution of initial optimal portfolio with an investment universe only including core assets, compared with target risk contribution. Note the lack of contribution from real estate and commodities factors due to lack of relevant assets.

2.3.3 Diagnostics

A natural question follows: among the candidate assets, which additions may best bring us to the target?

As shown in the Appendix, the tracking error can be decomposed to two main components:

$$\text{Tracking Error} = \text{TE}(\text{factors}) + \text{TE}(\text{factor residuals})$$

With an objective of reducing the tracking error, we ask: given the optimal holdings of the current asset selection, what's the marginal effect – or "gradient" – on tracking error, given a unit of holding of *one* additional asset?

Intuitively, the sign of the marginal effect depends on the correlation of the additional asset and the residual return R_{TE}, and a negative gradient means that the additional asset is promising in terms of reducing tracking error. Similarly, such a gradient can be decomposed to gradient in $\text{TE}(\text{factors})$ and $\text{TE}(\text{factor residuals})$. A large negative gradient on factors often means that inclusion of the asset could bring factor exposures closer to the target; a large negative gradient on factor residuals usually

indicates that the asset could improve the tracking error by reducing the factor residual risk.

Although intuitive, one caveat of the gradient measures is that the marginal effect may not be linearly extrapolated when a large reallocation is required. Depending on which asset is added, the turnover rate of the existing portfolio varies; the full impact of adding a new asset needs to be measured by the total reduction of tracking error after re-optimization.

To evaluate the potential benefit of an additional asset, we assume only one of the candidate assets is added and calculate the gradient measures for each candidate, as well as the relative reduction of tracking error and turnover rate of the total portfolio as a result of re-optimization. The results are shown in Exhibit 4.

Exhibit 4: Effects of adding one asset to the mix

Candidate asset	Tracking error reduction	Gradient: Total	Gradient: Factors	Gradient: Factor residuals	Turnover
IPD Global Property	-19.2%	-3.0%	-2.9%	-0.1%	24.8%
Global Commodity	-17.9%	-6.8%	-7.6%	0.8%	15.8%
ACWI Energy	-8.1%	-5.4%	-0.4%	-5.0%	15.6%
ACWI Real Estate	-3.4%	-2.1%	-0.4%	-1.8%	10.3%
US Corporate HY	-3.4%	-0.7%	-0.5%	-0.3%	24.1%
ACWI Value	-2.0%	-0.5%	-0.4%	-0.1%	18.0%
ACWI Materials	-1.2%	-1.2%	-0.4%	-0.7%	6.9%
ACWI Utilities	-0.4%	-1.0%	-0.4%	-0.7%	3.1%
ACWI Telecom	-0.1%	-0.4%	-0.4%	0.1%	1.6%
…	…	…	…	…	…
ACWI Industrials	0.0%	0.1%	-0.4%	0.4%	0.0%

Diagnostics clarify the impact of adding one asset into the investment universe mix, ranked by tracking error reduction. Each row shows the result of the experiment of adding a single candidate asset to the investment universe.

Perhaps unsurprisingly, the IPD Global Property Index and the Global Commodity Index ranked as the top two, echoing the lack of real estate and commodity exposure in the initial core asset set. Both indexes had the highest score in gradient on factors, driven by their direct exposure to relevant factors.

What's illuminating is that we see the MSCI ACWI Energy and MSCI ACWI Real Estate Indexes ranked in third and fourth place, respectively. The interpretation is that factor correlations reveal relevant investable proxies. For an investor without the capacity

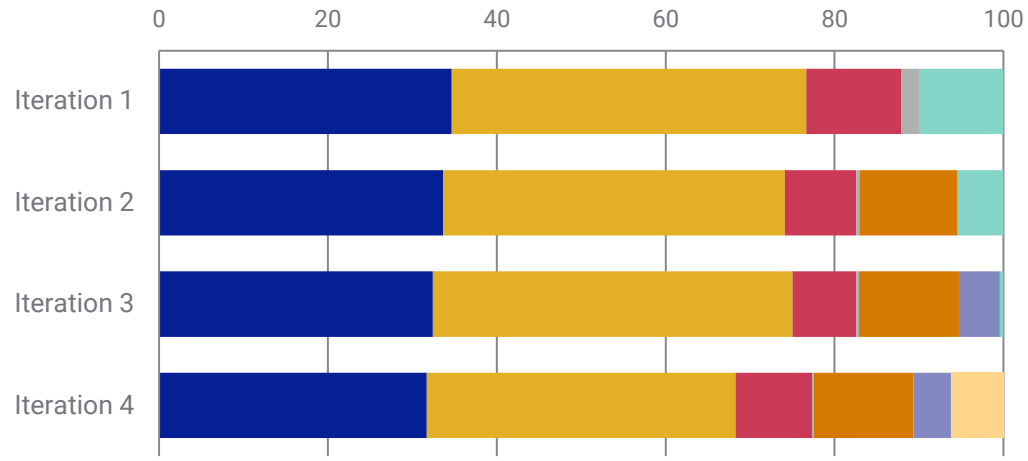
to invest in a less liquid private real estate asset, an equity portfolio with real estate exposure could have been a strong alternative. Similarly, energy stocks were shown to be good alternatives for commodity assets in our study. This highlights the power of a factor-based asset allocation framework.

2.3.4 Rebalance, rinse and repeat

As suggested by our diagnostics, we first add the IPD Global Property Index into the investment universe and then repeat the process of optimization, diagnostics and tweaking the investment universe. The capital allocation and the factor risk contribution of the optimal portfolios in the first four iterations of the asset allocation problem are reported in Exhibit 5 and Exhibit 6.

"Iteration 1" reflects the initial optimal portfolio. With the help of diagnostics, we update the investment universe by adding candidate assets one at a time and find the updated optimal portfolio with the updated universe. Allocation to real estate, commodity and high yield credit portfolios are funded by reduction in equity, investment grade and cash positions, and bring the relevant factor risk contribution closer to target.

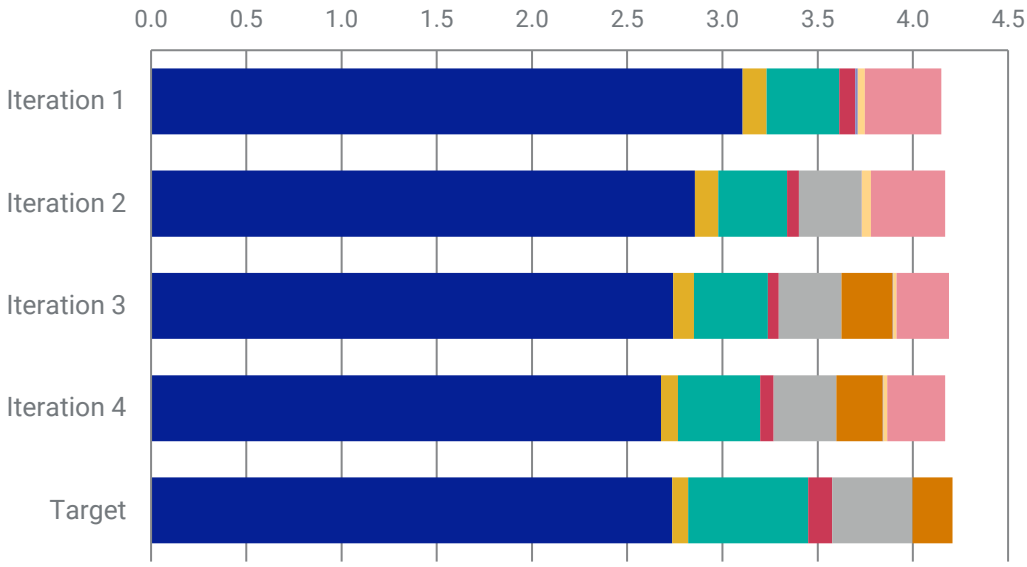
Exhibit 5: Capital allocation



	Iteration 1	Iteration 2	Iteration 3	Iteration 4
ACWI IMI	34.65	33.64	32.41	31.69
US Corporate IG	42.01	40.48	42.60	36.59
US Treasury	0.00	0.00	0.00	0.00
US TIPS	11.21	8.46	7.54	9.04
USA IMI	2.14	0.37	0.35	0.23
IPD Global Property	0.00	11.63	11.82	11.77
Global Commodity	0.00	0.00	4.87	4.45
US Corporate HY	0.00	0.00	0.00	6.22
USD Cash	9.99	5.43	0.42	0.00

Capital allocation of optimal portfolios in iteration 1 through 4. Values represent the percentage weight of each asset as a proportion of the total portfolio.

Exhibit 6: Annualized risk contribution (%)



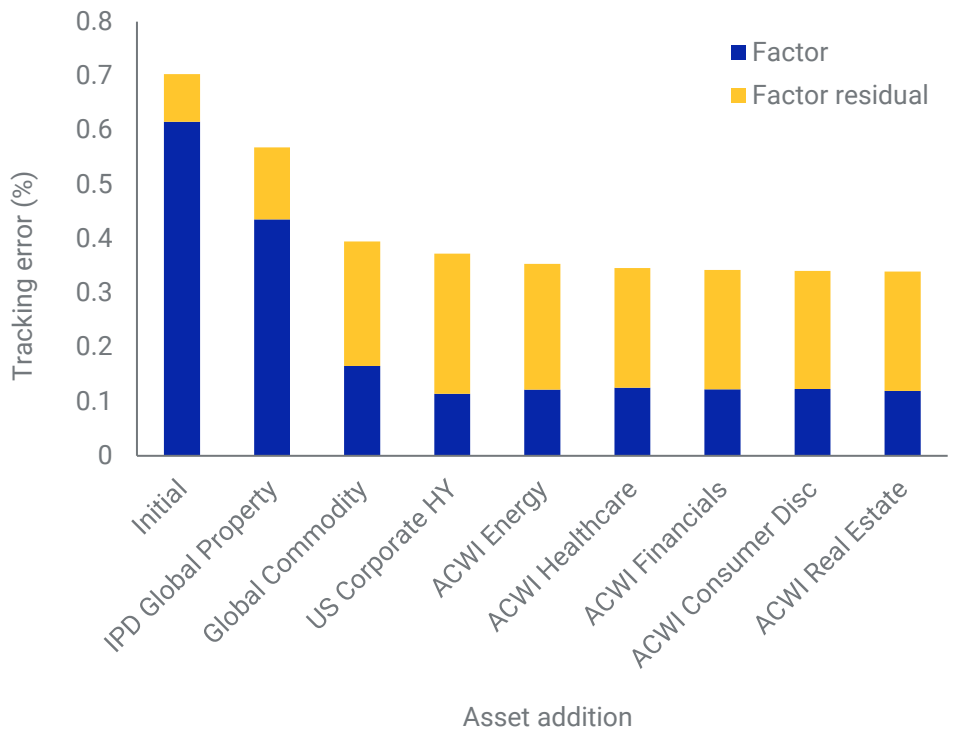
	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Target
Equity	3.11	2.86	2.74	2.68	2.74
USD Rates	0.13	0.12	0.11	0.09	0.08
USA Credit	0.38	0.36	0.39	0.43	0.63
USD Inflation	0.08	0.06	0.06	0.07	0.13
Real Estate	0.00	0.33	0.33	0.33	0.42
Commodities	0.00	0.00	0.27	0.25	0.21
USA Home Bias	0.01	0.00	0.00	0.00	0.00
Foreign Currency	0.04	0.05	0.02	0.02	0.00
Pure Private Equity	0.00	0.00	0.00	0.00	0.00
Residual	0.40	0.39	0.28	0.30	0.00

Factor risk contribution of optimal portfolios in iteration 1 through 4, compared to target factor risk contribution.

We repeat this process until satisfying results are reached. However, it is important to know the limit: Can factor residual shown in Exhibit 6 be further reduced? When do additional assets stop providing meaningful enhancements?

In Exhibit 7 we decompose the tracking error to contributions by factor exposure mismatch versus factor residuals. We observe that the first two asset additions substantially reduced a factor's contribution to tracking error (and, as a trade-off, somewhat increased the factor residual's contribution), but the effect became minimal after that, and the total tracking error flattened out at around 34 bps. Saturation appeared. This was because even though high-level factors did a good job capturing systematic risk of broad market indexes, they could not fully explain the total risk, because no indexes are perfectly diversified. To further reduce tracking error, especially the factor residuals, we need a more granular factor model that explains risk in more details.

Exhibit 7: Tracking error contribution



Decomposition of tracking error throughout the iterations. The first two additions of assets (IPD Global Property and Global Commodity) substantially reduced factors' contribution to tracking error, but, after that, total tracking error flattened out, indicating that it could no longer be meaningfully reduced without more granular factors and assets.

3 Case study: Tactical allocation with style tilt

In this section, we explore an example implementing a tactical allocation with a style-factor tilt.

Consider an institutional investor who wants to enhance her strategic equity allocation by introducing a value tilt, which overweights stocks with relatively low valuation metrics and underweights stocks with high valuation metrics.

The MSCI MAC Tier 2 USA Home Variant provides tools to evaluate risk and returns of such a tactical tilt. Expanding on the factors of Tier 1, Tier 2 introduces additional factors to reflect common tilts away from market-weighted portfolios. As listed in Exhibit 8, these include both traditional tilts, such as cyclical vs. defensive sectors and systematic strategy factors.

Exhibit 8: The MSCI MAC Tier 2 factors

Factor	Type	Factor	Type
Equity	Equity Market	**Commodities**	Real Asset
Home Bias	Equity Tilt	**Real Estate**	Real Asset
Emerging Market Equity	Equity Tilt	Pure Hedge Fund Strategy	Pure Alternative
Cyclical vs. Defensive	Equity Tilt	**Pure Private Equity**	Pure Alternative
DM Foreign Currency	Currency	Equity Value	Systematic Strategy
EM Foreign Currency	Currency	Equity Size	Systematic Strategy
DM Nominal Rates	Fixed Income	Equity Volatility	Systematic Strategy
DM Credit	Fixed Income	Equity Momentum	Systematic Strategy
EM Rates	Fixed Income	Commodity Momentum	Systematic Strategy
EM Hard Currency Credit	Fixed Income	Credit Carry	Systematic Strategy
Inflation	Fixed Income	Equity Implied Volatility	Systematic Strategy
Steepener	Fixed Income		
EM Nominal Rates	Fixed Income		

*Factors in **bold** are carried over from Tier 1. Factors in italics are market specific, while the other factors are defined globally.*

For simplicity, we focus on allocation within equity and use indexes as proxies to the index-tracking investable assets in the following analysis.

One way to obtain a value tilt is to combine assets tracking the MSCI ACWI IMI and the MSCI ACWI Enhanced Value Index. We first compare factor exposure and risk contribution of two portfolios, shown in Exhibit 9. “Portfolio A” is 100% invested in ACWI IMI and “Portfolio B” is 50% invested in ACWI IMI and 50% in the MSCI ACWI Enhanced Value Index. As expected, Portfolio B had a significant exposure (0.52) to

the equity value factor and a slightly higher total risk (9.8%) compared to Portfolio A's 9.4%.

Interestingly, Portfolio B also came with non-trivial exposures to the home bias, low vol, momentum and size factors. This is due to correlations among factors: value has historically been positively correlated with the low vol factor and negatively correlated with momentum. As of this analysis date, U.S. equities had a period of outperformance compared to world equities – therefore relatively overvalued – causing a global value strategy to have a negative exposure to U.S. home bias.

If these additional exposures are undesirable, a hypothetical investor could include additional country and factor indexes to find the optimized portfolio that has the same equity market and equity value factor exposures and zero exposures to all other factors. The result of this optimization, allowing short-selling, is shown in Exhibit 9 as Portfolio C. Portfolio C may not be realistic given its significant leverage, but it sheds light on how a pure value tilt could be achieved and could be useful in informing discretionary decisions aiming to cancel out undesirable exposures.

Exhibit 9: Three stylized sample portfolios

Panel A: Asset weight of stylized portfolios

Asset	Portfolio A	Portfolio B	Portfolio C
MSCI ACWI IMI	100%	50%	86%
MSCI ACWI Enhanced Value	0%	50%	53%
MSCI USA IMI	0%	0%	16%
MSCI ACWI Momentum	0%	0%	3%
MSCI ACWI Quality	0%	0%	-37%
MSCI ACWI High Dividend Yield	0%	0%	-30%
MSCI ACWI Equal Weighted	0%	0%	-20%
MSCI ACWI Min Vol	0%	0%	29%

Panel B: Factor exposure and risk contribution of stylized portfolios

	Exposure			Risk contribution		
Factor	Portfolio A	Portfolio B	Portfolio C	Portfolio A	Portfolio B	Portfolio C
Equity	1.00	1.00	1.00	9.3%	9.2%	9.2%
Equity Value	0.04	0.52	0.52	0.0%	0.2%	0.2%
Equity Momentum	-0.01	-0.07	0.00	0.0%	0.0%	0.0%
Equity Size	0.00	0.19	0.00	0.0%	0.0%	0.0%
Equity Low Volatility	-0.02	0.09	0.00	-0.1%	0.2%	0.0%
Cyclical-Defensive Tilt	-0.01	0.01	0.00	0.0%	0.0%	0.0%
USA Home Bias	0.00	-0.13	0.00	0.0%	0.0%	0.0%
Emerging Market Tilt	0.00	0.07	0.00	0.0%	-0.1%	0.0%
Factor Residual				0.1%	0.2%	0.3%
Total Risk				9.4%	9.8%	9.8%

Asset weight, factor exposure and risk contribution of three stylized sample portfolios. Portfolio A has a 100% allocation to MSCI ACWI IMI-tracking asset; Portfolio B has a 50%/50% mix between ACWI IMI and ACWI Enhanced Value Index; Portfolio C aims to achieve the same equity market and equity value factor exposure as Portfolio B, while zeroing out all other factor exposures. Risk and exposure are calculated as of June 1, 2018.

4 Summary

We have demonstrated two examples of a robust and transparent approach to implementing strategic and tactical factor allocation using Tier 1 and Tier 2 of the MSCI Multi-Asset Class Factor Model. We use diagnostics based on tracking error decomposition to provide insights, enabling a more holistic asset selection and optimization process.

Although we treat the target factor allocation as a given in this note, we recognize that determining the optimal factor allocation remains one of the biggest challenges to institutional investors. Every investor is unique in their preferences, constraints and views on expected returns and future macroeconomics scenarios. In future research, we aim to consider these attributes holistically and better quantify the process of optimal factor allocation.

5 Appendix: Optimization and tracking error decomposition

For a portfolio comprised of N assets with returns R and $N \times L$ local factor exposure matrix X, and $N \times 1$ holdings vector h, the residual return R_{TE} can be written as:

$$R_{TE} \equiv h'R - Y'c = h'\left(\hat{Y}c + X\varphi\right) - Y'c,$$

where Y is a $K \times 1$ vector of target factor exposures, and c is the vector of factor returns; $\hat{Y}$ is a $N \times K$ factor exposure matrix of the assets, and φ is the factor residual satisfying $R = \hat{Y}c + X\varphi$. The optimal holding h^* is one that minimizes the volatility of R_{TE}.

In the absence of constraints, the optimal holding is:

$$h^* = cov(R,R)^{-1} \cdot cov(R,c) \cdot Y.$$

With constraints, the optimal holding can be found numerically using standard constrained optimization techniques such as quadratic programming.

Letting $X^\pi = h'X$ and $\Delta Y = \left(h'\hat{Y} - Y\right)$ we can decompose the tracking error TE into two components – risk due to mismatch of factor exposures and factor residual risk:

$$TE = var(R_{TE}) = \underbrace{\sum_{k=1}^{K} \Delta y_k \ \sigma_{g_k} \ \rho(c_k, R_{TE})}_{TE\ (factors)} + \underbrace{\sum_{l=1}^{L} x_l^\pi \ \sigma_{\varphi_l} \ \rho(\varphi_l, \ R_{TE})}_{TE(factor\ residuals)},$$

where lower case Δy_k and x_l^π are the elements of ΔY and X^π, respectively.

References

Bass, R., Gladstone, S., and Ang, A. 2017. "Total Portfolio Factor, Not Just Asset Allocation." *Journal of Portfolio Management* 43 (5).

Bender, J., Le Sun, J., and Thomas, R. 2018. "Asset Allocation vs. Factor Allocation—Can We Build a Unified Method?" *Journal of Portfolio Management* 45 (2).

Bruder, B. and Roncalli, T. 2012. "Managing risk exposures using the risk budgeting approach." SSRN.

Greenberg, D., Babu, A., and Ang, A. 2016. "Factors to Assets: Mapping Factor Exposures to Asset Allocations." *Journal of Portfolio Management* 42 (5).

Shepard, P., DeMond, A., Xiao, L., Zhou, C., and Ahlport, J. 2018. "The MSCI Multi-Asset Class Factor Model." MSCI Research Insight.

Contact us

clientservice@msci.com

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