Research Insight

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The MSCI Private Equity Factor Model

Research Notes

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Introduction

The rise of the Yale Model of asset allocation sparked many pension funds and endowments to steadily increase their allocations to private assets. The rule of thumb of asset allocation shifted to "40:40:20" from "60:40," with the 20% in private assets projected[1] to rise even further.

Private equity in particular has been an attractive asset class for institutional investors, drawn by extraordinary historical performance,[2] a possible liquidity premium and the opportunity to take on large levels of active risk to exploit manager skill (Shepard (2024)).

Increasingly, investors are looking at private equity beyond the United States. Asset owners have shown a strong U.S. bias in private equity. But investors outside the United States are increasingly allocating to private equity, and U.S. investors are exploring a broader opportunity set abroad. The shift requires investors to understand assets on the frontiers of investing.

There have been competing schools of thought on private equity's role in a multi-asset-class portfolio. Some observers have interpreted the low volatility and low correlations of quarterly private equity valuations as evidence that private equity is a high-return, low-risk "free lunch."[3] Others are more cautious, noting the subjective valuations of private equity are highly smoothed, masking what is actually a high degree of systematic risk in the long run. And some[4] have been even more conservative, arguing that private equity has a beta greater than two. This view would attribute the asset class's outperformance to a simple risk premium, rather than a liquidity premium or the skill of private equity managers.

This spectrum of views leads to four central questions for understanding private equity in the context of total plan risk and asset allocation:

- How large is the diversification effect of private equity?
- Is private equity's performance due to a liquidity premium and manager skill, or to high beta?
- How much opportunity for active risk is there in private versus public equity?
- Do we get different answers when we examine global versus domestic private equity?

Unfortunately, two common approaches to understanding private equity — one based on private equity valuations, the other proxies of publicly traded securities — can both lead to the wrong answers. Private equity valuations are subjective, and not market-based, resulting in smooth returns that can exaggerate private equity's benefits. Meanwhile, proxies in the public market ignore the liquidity premium, diversification benefits and active risk that draw investors to private equity in the first place.

[1] See Gilfedder (2014) and references therein.

[2] See the chart "Private equity has outperformed public equity."

[3] See Pedersen (2014) for a critique of the "free lunch" perspective of private assets

[4] See Sorensen (2013).

To tackle these challenges, MSCI Private Equity Factor Model consists of a suite of models and features:

- An innovative Bayesian desmoothing methodology provides robust risk estimates using "small data" techniques to bring together limited datasets.
- The incorporation of private equity data from Burgiss gauges the strong "pure private" component of private equity returns.[5]
- Use of the Barra public equity and fixed income models captures fundamental characteristics of private equity, and provides a consistent view of risk across public and private assets.
- Asset class coverage is expanded from Ventures and Buyouts to Mezzanine and Distressed Debt.
- Geographic coverage spanning 46 countries globally.
- Total coverage spans 17 private equity segments.

The MSCI Private Equity Factor Model represents a major advance in understanding the drivers of investments in global private equity. It reveals a high degree of systematic risk in private equity, but also large opportunities for global diversification and active risk compared with public equity.

Incorporated in the Barra Integrated Model, the MSCI Private Equity Factor Model provides a unique, like-to-like view of private equity investments among all asset classes — global stocks, bonds, commodities, currencies, volatility futures, hedge funds and private real estate — and enables consistent asset allocation and risk management decisions spanning the full portfolio.

[5] MSCI acquired The Burgiss Group, LLC, in October 2023.

Methodology overview

Typical approaches to modeling private equity have often fallen into one of two extremes: One extreme focuses on the "private" while the other focuses on the "equity" in "private equity." To understand private equity requires gauging the truth in each perspective.

From the "private" point of view, smooth private equity valuations can give the impression that private equity has little in common with public equity. This interpretation suggests that private equity is a source of little systematic risk, and the low beta implies a low return contribution from risk premia, making private equity's long-run performance even more remarkable.

The "equity" point of view recognizes that fund valuations smooth over much of the risk in private equity. Instead, this approach uses public proxies to represent investments in private equity. This approach has the benefit of using timely, market-based information, and can capture some of the systematic risk in private equity. But using a public proxy still requires answering the key question, "What is beta?" In addition, the proxy is blind to many of the key features that draw investors to private equity, such as the illiquidity premium.

The reality is surely somewhere in the middle of these extremes. Private equity is equity, and carries with it substantial systematic risk; but it trades in an illiquid and inefficient market, and can be concentrated in assets that are different from any public counterpart.

To understand both the equity and private components of private equity requires a framework that can bring together all the relevant sources of information from public markets and private assets, as shown in the table below. The MSCI Private Equity Factor Model uses a factor model to pool information across many sources to assemble a coherent view of private equity alongside traditional asset classes.

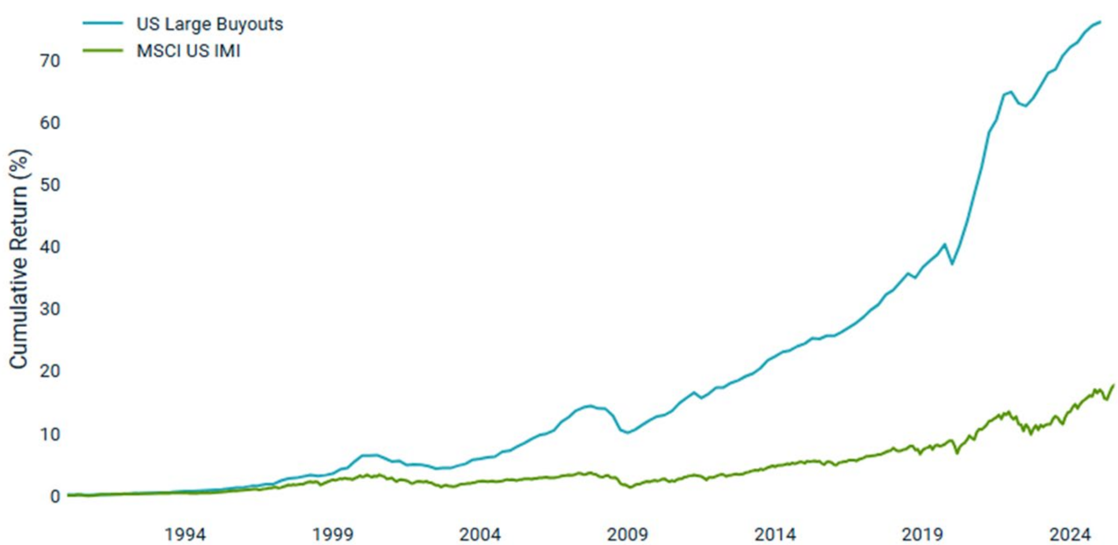
Pros and cons of private- and public-market information

Information source	Benefits	Drawbacks
Private equity valuations and fundamentals	Accurate in the long run "Faithful" to the investment	Subjective in the short run Lagged Scarce Smoothed
Public equity and debt	Timely Market-based	Misses liquidity premia Misses private market effects Mismatched assets

Both the “equity” and “private” nature of private equity are evident in the chart below. In the short run, the smooth valuations gradually rise and fall over many quarters, resulting in low volatility. The lag in the response to the public market also leads to low contemporaneous correlations with public assets. But in the long run, these artifacts of the valuations disappear. The low short-run *volatility* does not prevent large *risk* at a longer horizon.

In addition to the commonality between public and private equity at long horizons, the chart also shows at least one important difference: Whether due to manager skill or a liquidity premium, private equity has significantly outperformed its public counterpart.

Private equity has outperformed public equity



The cumulative returns of public and private equity (net of fees) show the striking outperformance of private equity. In the short run, the smooth valuations of private equity might suggest low risk. Similarly, the lag in returns leads to low contemporaneous correlations with public equity, at the quarterly horizon. However, these artifacts disappear at longer horizon, where large risk and high correlations are apparent. Source: MSCI

To capture these effects, we model the “true”[6] underlying value of private equity by decomposing the return of the private equity fund or holdings as

$$True\ Return = (Beta \cdot Public\ Factor\ + Pure\ Private + Asset\ Specific) \times Leverage$$

Different data sources are used to inform each component.

The public factor and asset-specific components are constructed by feeding *bottom-up* information from the private assets into the Barra equity and fixed income models, while the Beta and Pure Private

[6] The “true” value of private assets is the (abstract) value for which the asset could be bought or sold in an orderly transaction. It differs from the valuation, which is known not to reflect all available information, and should also be distinguished from the depressed price an asset might fetch in a quick sale on the secondary market.

factor components incorporate *top-down* information at the sub-asset-class level that distinguishes private equity from the public counterpart. Our desmoothing techniques are used to overcome the distortions in the private equity valuations.

Pure private factors

The charts “Rolling annual returns (US buyouts)” and “Rolling annual returns (US ventures)” below show the broad behavior of ventures and buyouts in the United States compared with public proxies[7] constructed to match the fundamentals of these assets. Although significant commonality is apparent, it is also clear that the public proxy is insufficient to capture the entirety of private equity’s behavior. The differences between the private and public counterparts represent large *pure private* components, which are important sources of risk and return. The pure private components provide an intermediate level of diversification between market risk and idiosyncratic risk.

There are many reasons to expect a pure private component of return that cannot be captured by a public proxy:

- **Liquidity premia.** Literature abounds (see Ang (2013) e.g.) attempting to calibrate the premium investors would demand in equilibrium to hold illiquid private assets, though the uncertainty in these models is very wide, and significant risk remains in the liquidity premium.
- **Differences in company characteristics.** Ventures are typically younger, smaller and more speculative than stocks that have made it to an IPO, for example.
- **Market inefficiencies.** Harris (2013) showed that long-run returns tend to suffer in vintage years with high capital commitments, perhaps because capital crowds the opportunity set in these years.

These sources of return can systematically drive a wedge between the performance of private equity and public equity. The MSCI Private Equity Factor Model captures these important return sources with factors estimated from the desmoothed returns of different strategies and regions, as shown in the table below.[8] As discussed in the “Market insights” section, these pure private factors represent a large source of risk and return to private equity.

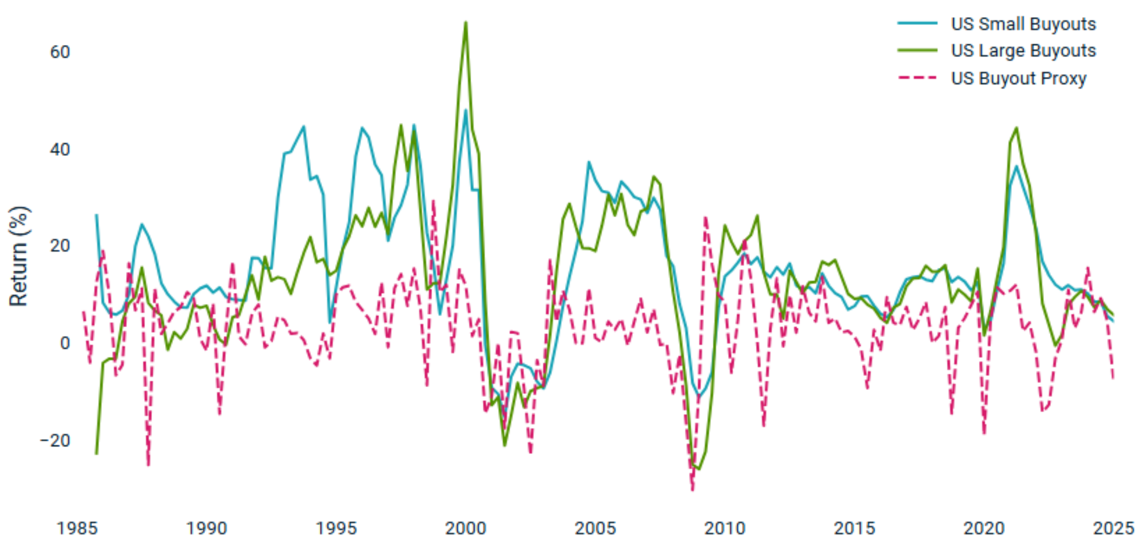
[7] See the “Methodology overview: Public proxies” section.

MSCI Private Equity Factor Model strategies by region

	Factor
1	US Early Stage Ventures
2	US Late Stage Ventures
3	US Small Buyouts
4	US Large Buyouts
5	US Mezzanine
6	US Distressed
7	Asia Early Stage Ventures
8	Asia Late Stage Ventures
9	Asia Small Buyouts
10	Asia Large Buyouts
11	Asia Distressed
12	Europe Early Stage Ventures
13	Europe Late Stage Ventures
14	Europe Small Buyouts
15	Europe Large Buyouts
16	Europe Mezzanine
17	Europe Distressed

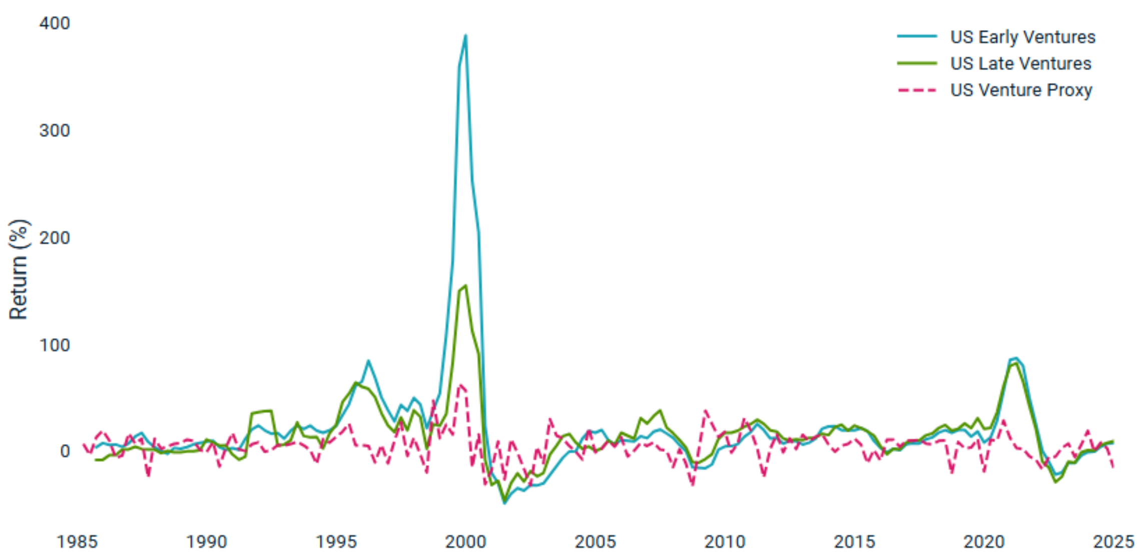
The MSCI Private Equity Factor Model covers six private equity strategies in Europe and the United States, and five strategies in Asia.

Rolling annual returns (US buyouts)



The rolling annual returns of U.S. buyouts have significant commonality with a public proxy portfolio, but also significant differences, which cannot be attributed to differences in leverage or beta. The difference in return is the "pure private" return.

Rolling annual returns (US ventures) have a large 'pure private' component



Private equity betas

Private equity also differs from public equity in its sensitivity to market returns. The greater leverage of buyouts and the cyclical nature of ventures typically lead to high levels of market beta. The MSCI Private

Equity Factor Model distinguishes private equity beta coefficients along the same segments (e.g., U.S. Early Stage Ventures) as the pure private factors, reflecting differences among strategies and regions.

Estimating these betas accurately requires both high-quality private asset data, and methodology to overcome the distortions from its inherent smoothness. The MSCI data (formerly Burgiss) used to estimate our model is widely regarded[9] as superior to other sources. One benefit of MSCI's data is that it is sourced from limited partners, making it much less prone to the selection and survivorship biases that have plagued some private equity studies.

To estimate risk from the private asset data, we make use of our Bayesian desmoothing methodology introduced in Shepard (2014). In addition to the desmoothing discussed in the next section and in the Appendix, the methodology uses Bayesian techniques to synthesize public market returns, private asset valuations, peer group behavior and subjective economic relationships.

The subjective component takes the form of "priors": What would be our best guess of betas prior to looking at any data? How confident are we in these prior estimates? The Bayesian approach uses these priors to reduce noise by combining raw data with some economic intuition.

For buyouts, this intuition is based primarily on buyouts' use of leverage.[10] There is significant debate about the beta of buyouts in the literature, but broader agreement that the leverage of buyouts tends to be larger than that of the typical public firm. Roughly, younger buyouts often have about twice the leverage of publicly held companies, but this leverage declines toward parity at the time of exit. It has been argued,[11] however, that buyout firms tend to be low beta to begin with, and that carried interest tends to reduce upside beta by 20%, weakening the effect of higher leverage.

For ventures, the prior expectation for beta is based on their typical lack of leverage, and the beta of small cap companies. Although small-cap stocks tend to have large volatility, the size effect on beta is small. In the United States, for example, the Barra Size factor has a negligible equity market beta.[12]

The betas for mezzanine and distressed debt are subject to additional uncertainty. Compared with their public proxies,[13] differences in leverage, duration and credit quality can all result in variations in the private-public beta, resulting in larger relative uncertainty compared with ventures or buyouts.

[9] See Harris (2013) for a discussion of different sources of private equity data.

[10] Differences in leverage from one fund to another are applied individually, but the average leverage is needed to calibrate the overall "buyout effect" in each market.

[11] See Harris (2013).

[12] Because the Size factor measures the behavior of large cap relative to small cap stocks, the negligible factor beta indicates large cap stocks tend to be slightly more cyclical, while small cap stocks on average have a beta slightly less than 1.

[13] The Merrill Lynch Global Emerging Market Credit Asia, Merrill Lynch US High Yield Index, and Merrill Lynch Europe High Yield index are used as public proxies for debt.

These considerations result in the priors listed in the table below. These priors happen to be roughly consistent with the range of beta estimates in the literature, but those estimates are typically formed *after* looking at the data, and should not form the basis of a prior.

In general, the purpose of the Bayesian methodology is to synthesize information and reduce noise, not to impose a strong view on the data. In the U.S. market, for example, the forecast betas of ventures are larger than those of buyouts, despite priors to the contrary.[14]

Ultimately, we don't expect the priors to be correct; they're simply our best guess before looking at any private asset data. But, as explored in "Appendix: Methodology details," the use of priors can significantly reduce noise, perhaps cutting the estimation error by a factor of two or more.

[14] See the chart "Forecast betas to public proxy" for an overview of forecast betas. Similarly, "Sensitivity of risk estimates to priors" shows the weak sensitivity of beta to the particular choice of prior, and "Desmoothing and risk estimation simulation" shows that even an incorrect prior can lead to more accurate forecasts than a correct but timid prior taken with unnecessarily wide uncertainty. These tables can be found in the Appendix.

Priors for beta to public proxy

Region	Strategy	Beta prior	Prior uncertainty
Asia	Large Buyouts	0.91	0.50
	Small Buyouts	0.91	0.50
	Distressed	1.00	0.50
	Early Ventures	0.84	0.25
	Late Ventures	0.84	0.25
Europe	Large Buyouts	0.95	0.50
	Small Buyouts	0.95	0.50
	Distressed	1.00	0.50
	Mezzanine	1.00	0.50
	Early Ventures	0.79	0.25
	Late Ventures	0.79	0.25
US	Large Buyouts	1.16	0.50
	Small Buyouts	1.16	0.50
	Distressed	1.00	0.50
	Mezzanine	1.00	0.50
	Early Ventures	0.83	0.25
	Late Ventures	0.83	0.25

The priors used in estimates of the beta to the public proxy. Before looking at any data, the beta would be expected to fall within the stated range of the prior with about 65% confidence. Beta priors were updated in May 2020.

Smoothing

The smoothness of private equity returns poses a difficult challenge for risk-forecasting. While the scarcity of private asset data typically blurs the picture, the smoothness can systematically distort the apparent risk, understating both the stand-alone risk and the systematic correlations of private equity.

The chart below provides a demonstration of the distortions caused by valuation smoothing. The figure compares the volatility and beta of private equity valuations to other asset classes, over a range of return horizons.

In contrast with public equity, the low short-term risk rises steeply as the horizon increases: the valuations are flawed in the short term, but *eventually they converge* to the fair value, and reveal much higher risk levels.

The disconnect between the short-term and long-term behavior of the valuations is the source of the problem, but it also points to a solution. The long run convergence of valuation and true value implies that accurate information is embedded in the valuations, but we must work harder to extract it from beneath the layers of smoothing.

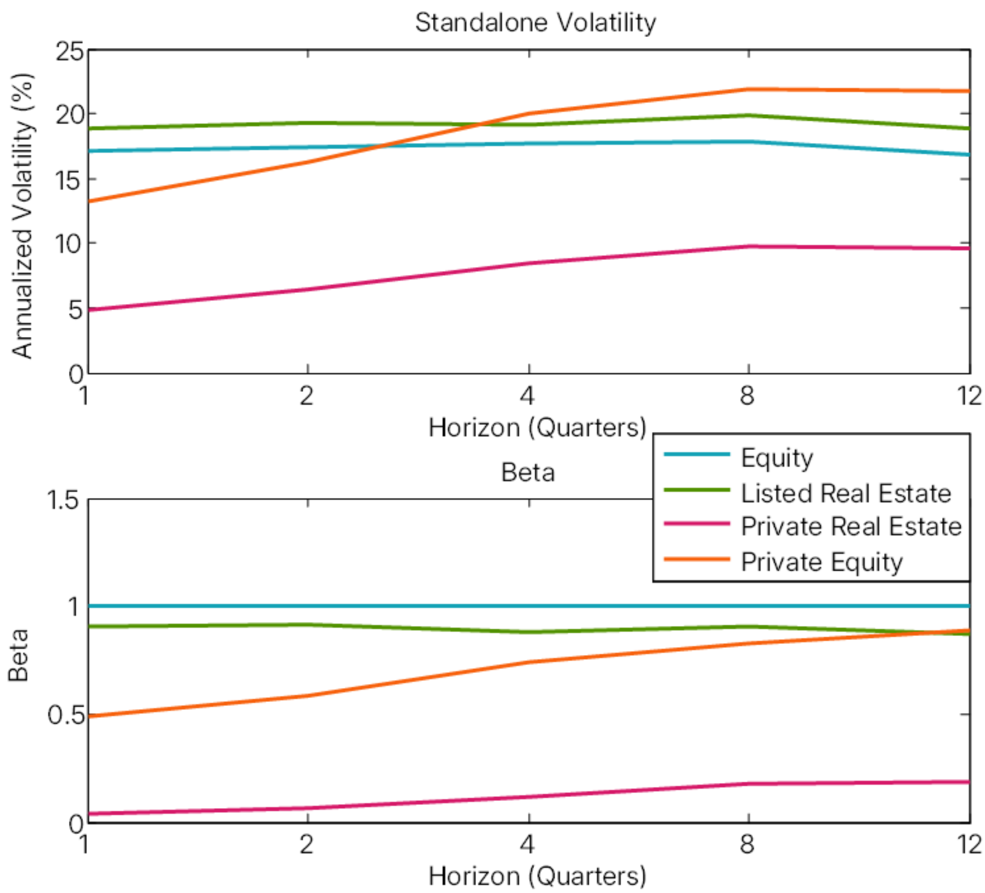
Geltner (1993) pioneered a *desmoothing* technique that makes this possible. By looking at differences between subsequent returns, Geltner showed how to back out the underlying "true" return to private assets. The *Methodology details* appendix reviews our Bayesian desmoothing methodology which extends Geltner's approach to include more sources of information, and reduce noise.

Although desmoothing can mitigate the distortions of the smoothed appraisals, it effectively results in only annual return observations.[15] Annual observations are useful for understanding the long-run risk of private equity, but on their own would lead to noisy and unresponsive estimates of risk. After the financial crisis of October 2008, for example, private equity might have looked like a low-risk safe haven for many months, until the first new data points arrived and demonstrated private equity to be closely linked to other risky assets.

Alone, neither the short- or long-term view of private equity is sufficient to understand its risks. But when used together, the different sources of information can produce a single, coherent view. The MSCI Private Equity Factor Model synthesizes desmoothed private equity valuations with the Barra Equity model view of the public markets to produce better insights into private equity risk than any one perspective provides.

[15] This is true even if quarterly valuations are available, because of the seasonality of many appraisals. For more details, see the section "Methodology details: Bayesian desmoothing."

Volatility and beta of private vs. public assets across horizons



The annualized volatility and beta of private assets rise significantly with the return horizon, in contrast with equity and listed real estate. A simple $\sqrt{T}$ scaling to annualize the volatility leaves the public equity and REIT curves basically flat. However, the upward slopes in the private asset curves indicate that the short horizon returns are smoothed, and do not capture the true long-run risk of these asset classes. Note that Private Real Estate is unlevered, while the other time-series include leverage: The slope may be consistently compared across asset classes, but not the level of the curves. Source: MSCI, NCREIF

Public proxies

Valuations at the fund level or holdings level exhibit an even higher degree of smoothing than indexes. Holdings are often valued at cost or at an assumed rate of growth for much of the investment period, with the actual value only reported at the fund exit in a final large return. As a result, the time-series of returns at the holdings level are generally not useful for estimating risk.

However, many limited partners receive granular fund- or holdings-level information about their investments, often including the investment type, country and sector. This information can be used to construct granular exposures to the factors in the public models, and to connect to the public models' specific risk forecasts. For example, a large buyout in the health care sector in the United States can be represented with factor exposures in the US Equity Factor Model, and receive a specific risk forecast based on comparable public health care companies.

Using the public factor models to construct the public proxy has many advantages over using individual stocks as proxies. Finding a public proxy to match a set of fundamental characteristics can be a tedious exercise, and often there simply does not exist a public stock that matches all the desired characteristics of the private equity investment. In either case, a proxy stock brings with it the noise of the particular firm selected.

Alternatively, employing a broad index, such as one covering the health care sector, avoids the challenges of individual stocks, but indexes fail to account for known asset characteristics, and also miss the asset-specific risk that is an important source of risk.

Factor models make it possible to simply tune the characteristics of the proxy based on what is known about the private equity investment. An asset's sector, country, size and other characteristics can be directly captured with fundamental factor exposures. Compared with standard public proxies, the factor model proxy is likely to be easier to construct, and a cleaner representation of the asset characteristics.

The public proxy provides the primary link between private equity and the public markets, and allows consistently responsive estimates of changing private equity risk from the behavior of the public markets.

Specific risk

The final piece of private equity return is the asset-specific return, which reflects the large idiosyncratic component of private equity. The valuations of individual funds or holding companies are much too smooth (and fat-tailed) to be used to robustly estimate specific risk. It is also not practical to wait many years to gather data from each asset before estimating its specific risk, which would be necessary to construct statistically robust estimates.

We avoid these problems by using the specific risk models of the Barra public equity and bond models. These models estimate specific risk based on stock and bond characteristics, in addition to their historical residual returns. We apply these to model private equity specific risk as:

$$Specific\ Risk\ =\ Average\ Public\ Specific\ Risk\ \times\ Relative\ Leverage\ \times Fund\ Size\ Coefficient$$

The first component takes the average specific risk over public assets in the same sector, such as U.S. health care stocks. By linking to the public models, the specific risk forecasts provide robust measures of diversification as the public model responds to changes in the markets.

Because many investors cannot see through to the leverage applied to individual portfolio companies, specific risk is scaled by the leverage relative to the average. Investors without detailed leverage information can neglect this component, or activate it only in cases of abnormal leverage.

Modeling specific risk and leverage at the holdings level provides more detail, but the fund-size coefficient can be used to model private equity at the fund level, if holdings-level information is unavailable. The coefficient reduces specific risk to reflect the diversification[16] that occurs within the fund's portfolio.

For individual companies, the specific return can drive 50% of the total return. Analyzing this data is important for assessing individual deals. But the importance of other components increases as we move from the company, to fund, portfolio and to the total plan levels. As companies are aggregated into a portfolio, the asset-specific components diversify away and decline in importance, while less diversifiable factors take on greater importance.

[16] The fund size coefficient is $1/\sqrt{N_{eff}}$, where the effective number of assets $N_{eff} = 1/\sum h_i^2$ is determined by the fund's weights h_i in each holding. If the fund holds equal weight in N companies, then $N_{eff} = N$. The product of the fund size coefficient and the relative leverage ratio define the specific risk scalar of each asset or fund.

Market insights

The MSCI Private Equity Factor Model sheds light on the four central questions for understanding private equity in the context of total risk and asset allocation:

- How large is the diversification effect of private equity?
- How much of private equity’s performance is due to a liquidity premium and manager skill, versus high market beta?
- How much opportunity for active risk is there in private versus public equity?
- How does global private equity differ from domestic?

As discussed in the “Methodology overview” section, the typical “private” and “equity” views of private equity both miss important components of the answers, while the MSCI model view represents a middle ground, which shows private equity to be both a source of systematic risk and a source of diversification.

The table below compares these views in the context of a broad pension fund portfolio, in which private equity and private real estate together comprise 16% of the total weight. The top panel shows the “equity” view, in which the private assets are represented with public proxies. The bottom panel shows the “private” view, based on raw private asset valuations without accounting for smoothing.

In the middle panel of the table, the Barra Integrated Model shows alternatives contributing about 30% of the total diversification.[17] In the portfolios of many pension funds, alternatives contribute an even larger portion of the overall active risk.

If only public proxies are used to model private equity, the stand-alone risk forecasts are somewhat lower than those of the model, but the correlations appear to be much higher, and the diversification (and active risk) are much lower. This result reflects a significant weakness in the public proxy: Diversification and active risk are among the main attractions to alternative investments, but the public proxy is largely blind to them.

The bottom panel shows the other extreme, based on raw private asset valuations. In this view, alternatives appear to offer much lower risk, and much lower correlation to other asset classes.[18]

The MSCI model view represents a middle ground to the starkly different (and surprisingly common) extremes of the public proxy and raw returns, showing alternative investments to generate both systematic risk and diversification.

[17] Diversification is defined as the difference between the total risk and the sum of the stand-alone risks. Diversification can be attributed to individual return sources with a generalization of the Correlated Risk Attribution (or X-sigma-rho) methodology (Menchero (2010)).

[18] The diversification effect from real estate also appears low in this perspective, for the same reason that cash is not a diversifier. Cash reduces risk because it is low risk on a stand-alone basis, not because it is uncorrelated.

Risk-modeling approaches for private assets

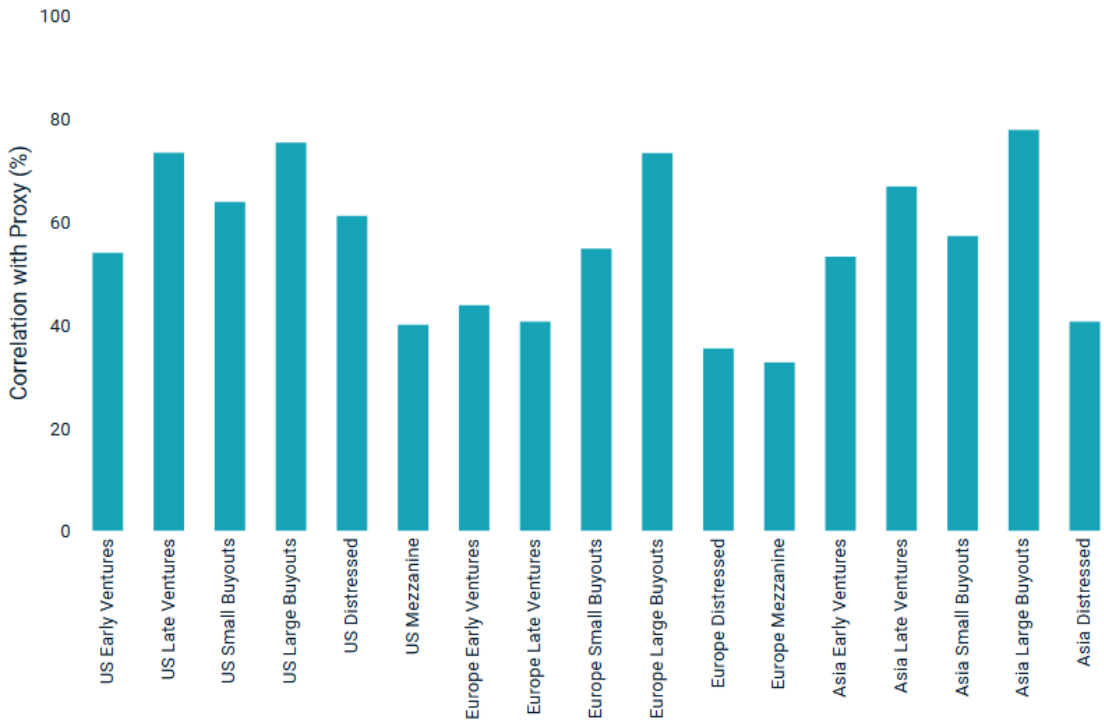
Public proxies	**Weight**	**Total risk**	**Correlation**	**Risk contribution**	**Diversification**
Total	100	8.0	1.0	8.0	1.6
Bonds	40	3.8	0.2	0.3	1.2
Public equity	40	14.5	1.0	5.7	0.1
Private equity	**6**	**17.2**	**0.9**	**0.9**	**0.1**
US real estate	**10**	**10.8**	**0.9**	**1.0**	**0.1**
Hedge funds	4	4.8	0.8	0.2	0.0
MSCI model					
Total	100	7.8	1.0	7.8	2.0
Bonds	40	3.8	0.2	0.3	1.2
Public equity	40	14.5	1.0	5.6	0.2
Private equity	**6**	**22.0**	**0.7**	**1.0**	**0.3**
US real estate	**10**	**10.2**	**0.7**	**0.8**	**0.3**
Hedge funds	4	4.8	0.8	0.2	0.0
Raw valuations					
Total	100	6.5	1.0	6.5	2.0
Bonds	40	3.8	0.2	0.4	1.2
Public equity	40	14.5	1.0	5.6	0.2
Private equity	**6**	**10.2**	**0.4**	**0.2**	**0.4**
US real estate	**10**	**3.1**	**0.3**	**0.1**	**0.2**
Hedge funds	4	4.8	0.8	0.2	0.0

Three approaches to modeling the risk of alternatives have starkly different implications. The top panel shows the contributions to risk of a typical pension fund portfolio using public proxies for alternatives; this view shows alternatives providing little diversification. In the other extreme, the bottom panel shows risk for the same portfolio, based on raw private asset valuations without accounting for smoothing; this view shows alternatives offering very low correlation and risk. The middle panel shows a balance between these two extremes; the risk according to the full Barra Integrated Model demonstrates that alternatives are a source of both diversification and systematic risk.

The diversification effects of private equity in the table above stem from the pure private factors of private equity returns. The long-run correlations between public and private equity are significantly higher than suggested by the raw valuations. However, the pure private components of return have driven different behavior in even a broadly diversified private equity portfolio relative to the public counterpart, as explored further in the chart below.

The correlations in the chart are typically somewhat lower than the corresponding correlations in private real estate.[19] While listed and unlisted real estate significantly converge in the long run, the long-run behavior of private equity returns has a larger component without any public counterpart.

Public-private correlation



The current forecast correlations between broad segments of private equity and their public proxies are much higher than the raw valuations suggest, but strong "pure private" components of return result in significant diversification from private equity, even in the long run.

The table below gives another indication of the diversification benefits of private equity, showing correlations among broad portfolios of public and private equity. The high correlations among the public equity portfolios reflect the high degree of convergence of the global equity markets: Global equity no longer is the diversifier it once was. In contrast, the lower correlations among the private equity portfolios

[19] Please see Figure 4 of Shepard (2014).

show that global private equity is a far more fragmented market: Significant diversification effects remain in private assets.

Correlations of public vs. private equity portfolios

Portfolio correlations	US equity	Europe equity	Asia equity	US private equity	Europe private equity	Asia private equity
US equity	1.00	0.79	0.74	0.65	0.44	0.46
Europe equity	0.79	1.00	0.79	0.53	0.50	0.48
Asia equity	0.74	0.79	1.00	0.49	0.39	0.60
US private equity	0.65	0.53	0.49	1.00	0.35	0.22
Europe private equity	0.44	0.50	0.39	0.35	1.00	0.24
Asia private equity	0.46	0.48	0.60	0.22	0.24	1.00

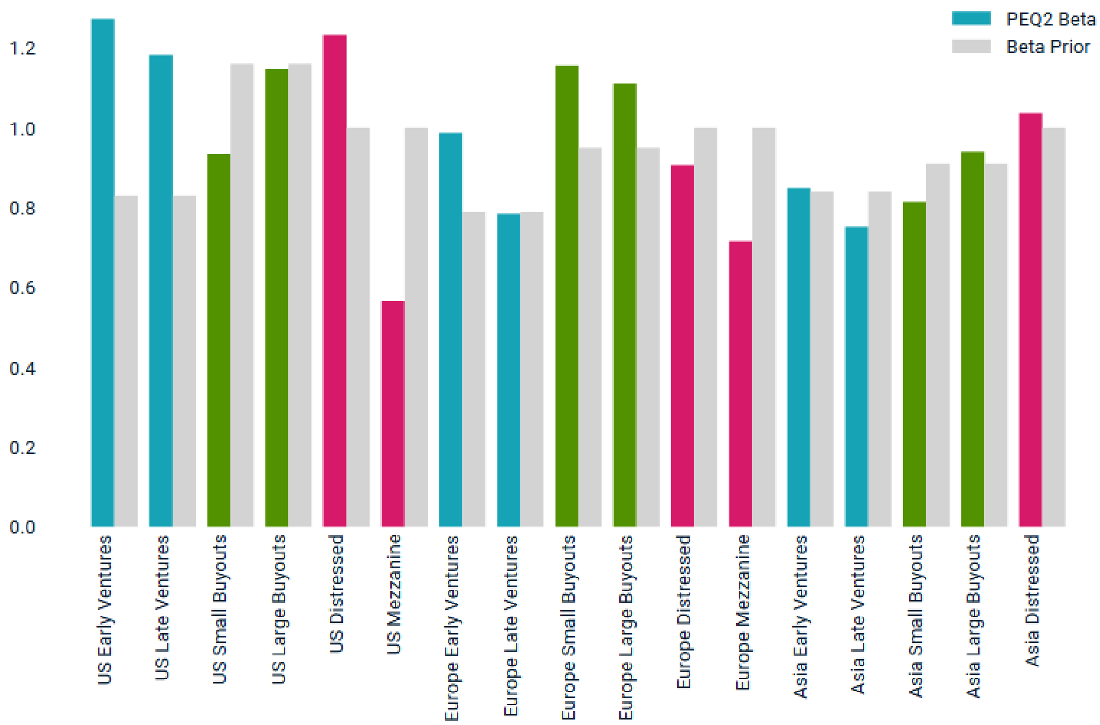
The current model correlations among broad portfolios of public and private equity show much greater opportunity for diversification in global private equity than public equity.

Despite the lower correlations, the chart below shows private equity to be a significant source of systematic risk. The beta of many segments of the market relative to each segment's public proxy is seen to be well above one in most cases.

The chart also compares the forecast betas to the priors used by the Bayesian methodology, to reduce noise. It is notable that the beta estimates for ventures and buyouts in the United States are in the opposite order from the priors, discussed further in the "Methodology overview: Beta" section. Although the priors play a central role in reducing noise, they do not determine the outcome to the exclusion of the data, as explored further in the chart "Sensitivity of risk estimates to priors" and in "Appendix: Methodology details."

The different betas for mezzanine and distressed debt are consistent with broader priors for these segments. Distressed debt is expected to have higher credit spreads than mezzanine debt, but uncertainty in the duration (relative to the Merrill Lynch High Yield proxy) can shift the beta significantly.

Forecast betas to public proxy

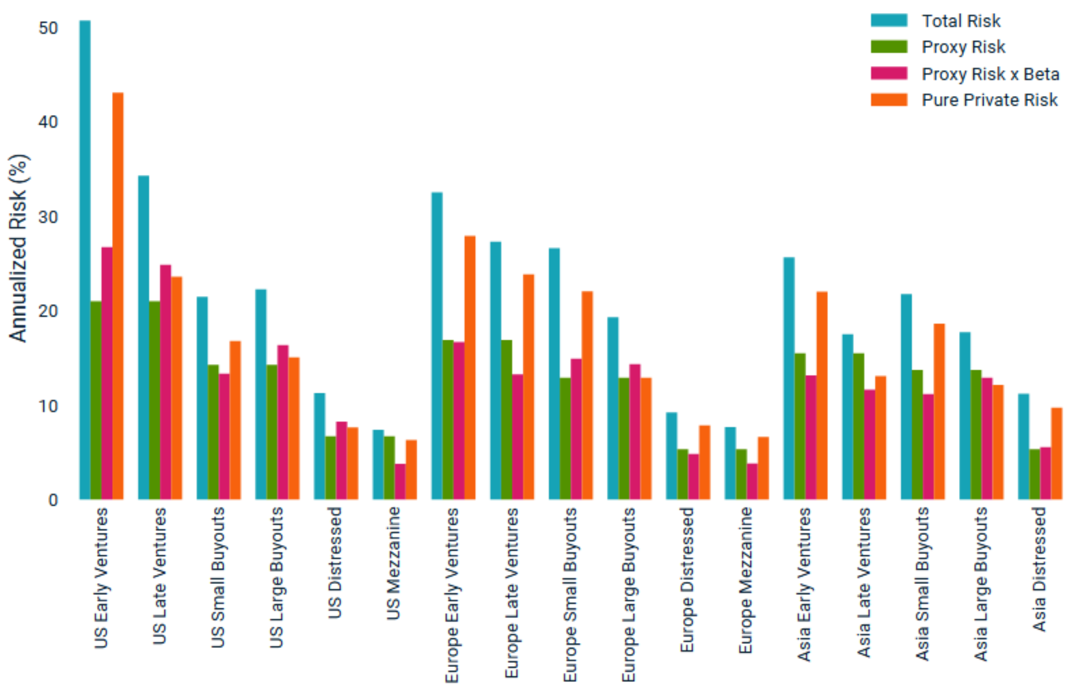


Forecasts of the beta of private equity (as of the end of 2024) to the public proxy of each segment. The high betas show private equity to be a source of significant systematic risk, despite the lower correlations in the chart "Public-private correlation." The significant differences with the priors indicate that the priors influence, but do not dictate, the overall estimates.

The combination of high betas and lower correlations suggests private equity has been significantly riskier than its public counterparts. The chart below compares the risk of private equity to the risk of the public proxy, and the proxy risk scaled by beta. For many segments of private equity, the total risk includes a large contribution from the pure private component. The pure private component of early stage ventures is especially large, but it is consistent with other estimates (see Cochrane (2005), e.g.).

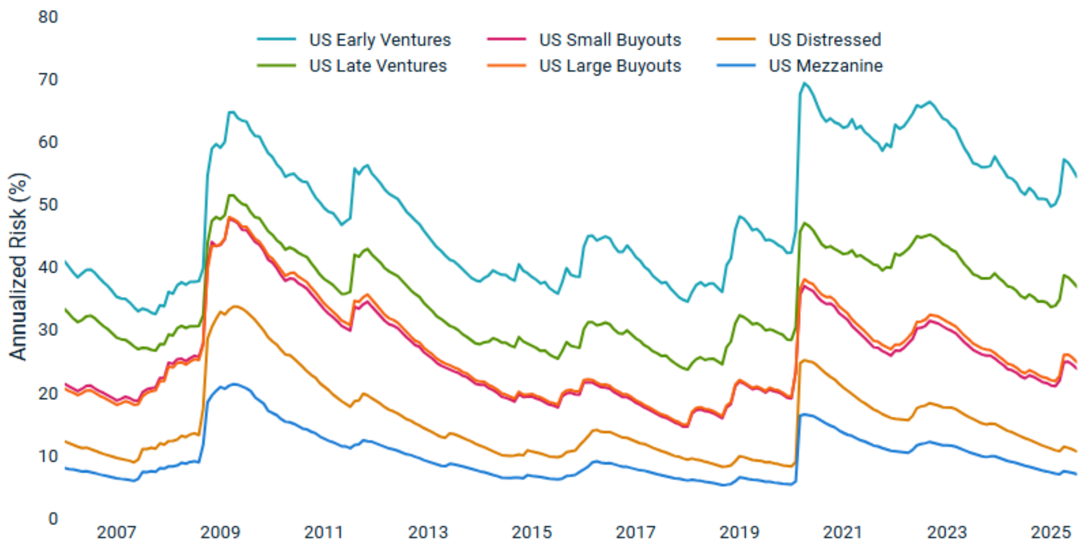
The chart "Historical CAPM alpha" shows how these risk forecasts have changed over time. Differences in risk are driven by the private asset returns in the long run, but in the short term the model responds to changing market conditions through the public proxies. The consistency with the public market risk is especially important during times of crisis, when private assets could otherwise appear to be safe havens until the first data points arrive, perhaps months later.

Private equity risk



The total risk of private equity is typically larger than the public proxy, even on a beta-adjusted basis. The additional risk is due to the pure private component, which can be as large as half the total risk in some cases, such as early stage ventures.

Trends in total private risk



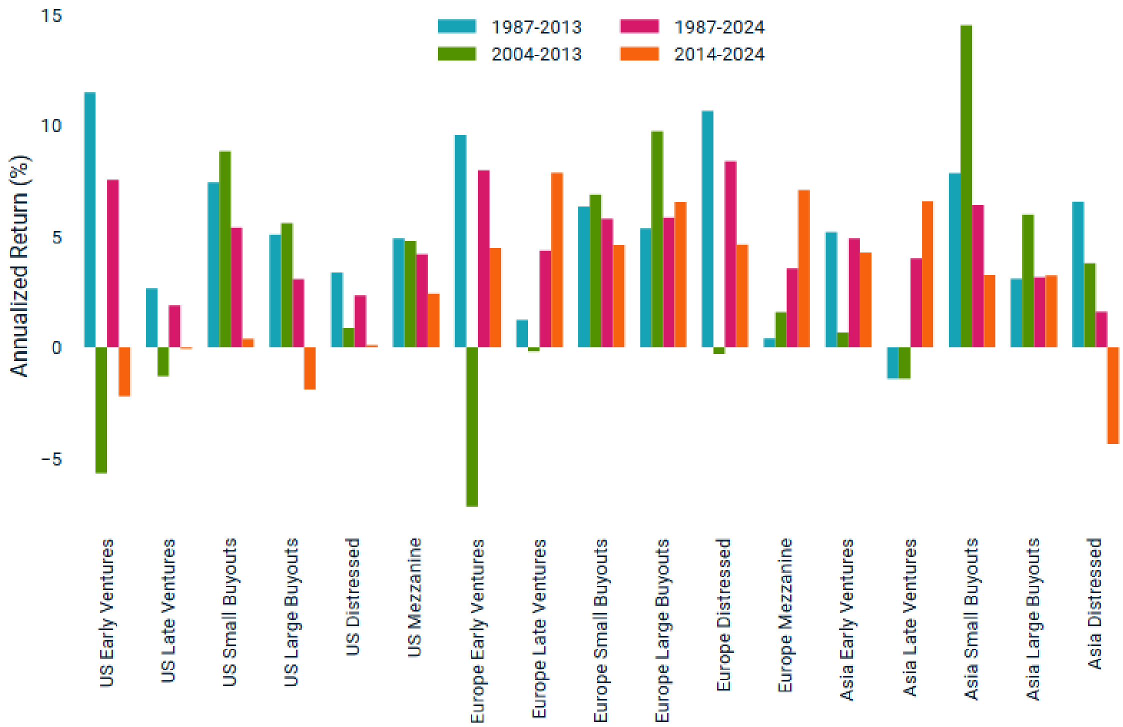
The stand-alone risk forecasts of private equity in the U.S. are notable both for their magnitude and their responsiveness. By linking private equity to the markets, our model responds to shocks in the public markets, maintaining a consistent view of risk between public and private assets.

Estimates of beta have important implications for assessing the historical performance of private equity. Some research (Sorensen (2013)) has suggested private equity has a very high beta, and the higher returns are merely a reflection of higher market return and risk premium. Do the betas greater than one explain the historic outperformance of private equity in terms of a simple risk premium, or is there evidence of skill or a liquidity premium, even net of the higher beta?

The chart below provides mixed evidence. Over the long run, every segment of private equity has at least matched its public counterpart, on a risk-adjusted basis, with many segments posting remarkable returns. A six percentage point annual outperformance over 25 years led to a cumulative outperformance of 350%, for example.

Are the large returns a thing of the past, since widespread adoption of the Yale model (Swenson (2009)) drew many more pension fund and endowment assets into private equity and other private assets? Or is the higher return a liquidity premium, which might be expected to persist even in equilibrium? Over the last 10 years, many private equity segments have performed far below their longer term average, with ventures performing especially below average.

Historical CAPM alpha



Net of the higher betas (and net of fees), most segments of private equity have significantly outperformed the market over the long run.

Conclusion

Many investors have been attracted to private equity's historic high returns, diversification effects, and opportunities for active risk. Long-horizon investors able to manage private equity's illiquidity hope to earn a liquidity premium and benefit from taking active risk in a less efficient market.

But the illiquidity and lack of transparency also make understanding private equity challenging. The smoothness of private equity valuations from one quarter to the next can give the illusion that private equity is a low risk "free lunch," in contrast to the large drawdowns experienced over longer horizons.

To avoid the distortions of smooth valuations, some investors use public proxies to model private equity. However, while avoiding the appearance of a "free lunch," these proxies are also blind to the benefits that draw investors to private equity in the first place.

The MSCI Private Equity Factor Model provides a middle ground between these extremes. Combining private equity data with factor models and desmoothing methodology, the model makes it possible to robustly gauge the central questions of asset allocation and risk management of private equity.

Appendix: Methodology details[20]

Bayesian desmoothing

The challenges of private asset data require innovative econometric techniques to avoid drawing the wrong conclusions from smooth valuations and to reduce the noise of small data sets. The MSCI Private Equity Factor Model adopts a broad Bayesian desmoothing framework to address these issues.

The underlying mathematical and computational details can be challenging to work out, but the idea is simple: *What is our best estimate for risk, given what we know?* Bayesian statistics provide a framework to answer this basic question.

Bayesian techniques are often thought of as "shrinking to a prior," but there is much more to them. In addition to standard Bayesian shrinkage, Bayesian techniques can be used to derive a variety of statistical methods:

- *Thin factor corrections* remove biases from small sample sizes.
- *Non-point estimates* address problems in combining noisy parameters.
- *Induced priors* account for peer group behavior.

Each of these is explained further in sections below.

In general, the purpose of the Bayesian methodology is to synthesize information and reduce noise, not to impose a strong view on the data. For example, the high correlations between public and private equity are the result of properly accounting for the effects of smoothing, not a prior inserted by hand. In the U.S. market, for example, the estimated betas of ventures have been larger than buyouts, despite priors with the opposite ordering, as shown in the chart "Forecast betas to public proxy." Similarly, the table "Sensitivity of risk estimates to priors" shows the weak sensitivity of beta to the particular choice of prior.

A Bayesian approach can significantly reduce noise, even in the absence of a strong prior. Knowing only that the smoothing parameter must be between 0 and 1 can significantly improve the accuracy of its estimates, for example. Knowing that the estimated smoothing parameters are only estimates — with some inherent uncertainty — also has a significant effect on the risk forecasts based on desmoothed returns. Incorporating the behavior of a peer group can yield an effective prior. All of these are benefits of a Bayesian approach that go beyond shrinkage toward prior information.

Often, the Bayesian approach leads to intuitive corrections, some of which have been applied by hand before. But rather than being ad hoc adjustments justified by intuition, they are instead derived from an explicit set of assumptions.

[20] Portions of this section are adapted from Shepard (2014).

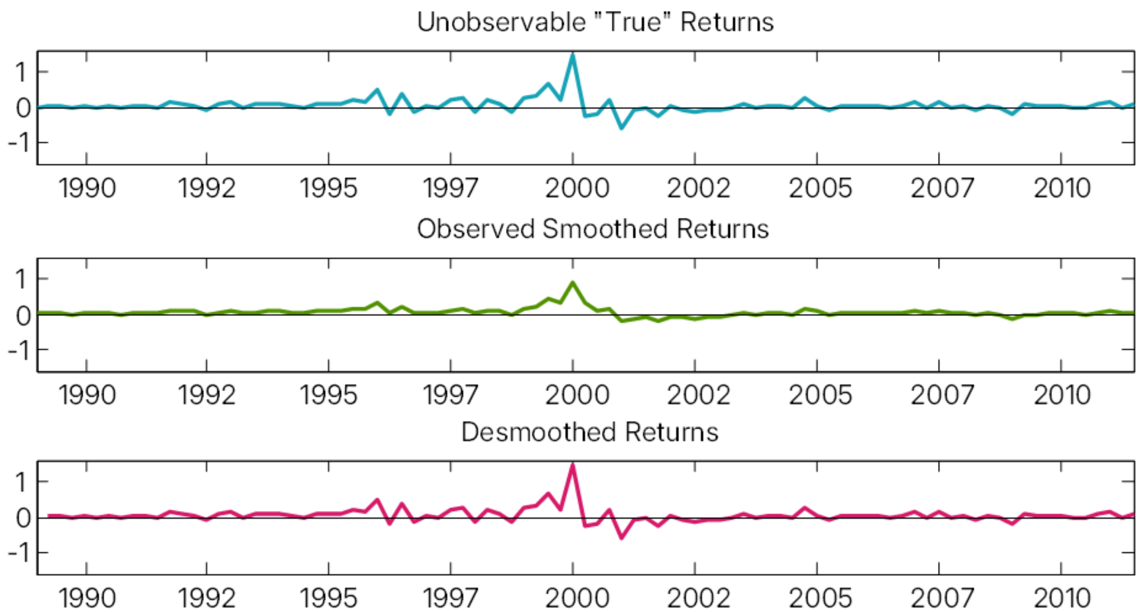
Desmoothing

The smooth behavior of private equity returns, as demonstrated in the chart "Volatility and beta of private vs. public assets across horizons," is a consequence of market inefficiency. If it were possible to transact at the reported valuations, a consistent profit could be made by buying after positive returns and riding the wave of similar subsequent returns.

However, private equity does not trade in an efficient market. The *valuation* of private equity may differ from the *value* that investors would actually pay for it. Since no one is obliged to transact at the price of such valuations, there is no arbitrage opportunity to close the mispricing.

To understand private equity, we must therefore distinguish the *observed return*, which is based on changes in valuations, from the *true return* reflecting changes in the actual value.

Recovering true returns via desmoothing



Desmoothing can uncover the true returns from only observations of the smoothed returns. For this demonstration, the middle "observed" returns are simulated from a known "true" return, top. From observations of only the smoothed returns, the desmoothed returns almost exactly reproduce the true returns. In the real world, we have access only to smoothed returns, and cannot directly observe the true returns.

This difference between the observed and "true" returns makes it difficult to understand private equity. However, because the valuations eventually converge to the true value, the information must be in there somewhere. Geltner (1993) recognized that it is possible to uncover this information. He pioneered *desmoothing*[21] as a way to back out the return of the underlying true returns from observations of the smoothed valuations.

In the simplest case, the smoothing of valuations is governed by what is called an AR(1) process.[22] In this process, the valuation P does not track the "true" value V, but instead follows it in a game of catch-up. Each period, the valuation moves from its previous value P_{t-1} a fraction $(1-\lambda)$ toward the new "true" value V_t:

$$P_t = P_{t-1} + (1-\lambda)(V_t - P_{t-1})$$

The parameter λ is called the smoothing parameter. If the smoothing parameter is zero, then $P_t = V_t$: there is no smoothing. In the other extreme, if $\lambda = 1$, then $P_t = P_{t-1} = P_{t-2}$ ...: the valuation never moves from its initial level, regardless of the true value.

Even though we cannot observe the true value V_t, Geltner recognized that it's possible to reconstruct the true return r_t from the observed, smoothed returns s_t. To a good approximation,[23] the above behavior of the valuations leads to a corresponding relationship between the "true" returns and the observed, smoothed returns:

$$s_t = (1-\lambda)r_t + \lambda s_{t-1}$$

This can be simply rearranged to find the true returns in terms of the observed returns:

$$r_t = \frac{s_t - \lambda s_{t-1}}{1-\lambda}$$

Unfortunately, there are two significant obstacles to applying this simple form of desmoothing:

- Seasonality: The smoothing parameter is likely not a single number, but varies seasonally with differences in valuation rates each quarter.
- Noise: The smoothing parameter is not directly observable, but must be estimated.

Geltner desmoothing assumes the smoothing parameter is *constant*, and assumes it is *known*. The smoothing of a portfolio or index is likely to vary from quarter to quarter due to the seasonality of valuations. Many assets are only valued in the fourth quarter, for example, resulting in higher levels of smoothing in the first three quarters' returns. Applying the basic desmoothing relation to quarterly returns would introduce distortions, and fail to reproduce the true quarterly returns.

The simple AR(1) smoothing process has been generalized to account for additional lags in the relationship between true and smoothed returns, with AR(4) or MA(4) processes,[24] for example.

[21] A note on terminology: To avoid ambiguity, we use the term "desmoothed" rather than "unsmoothed," which elsewhere is sometimes used to refer to the raw, "not desmoothed" returns, and sometimes the opposite.

[22] AR(1) refers to an Auto-Regressive process with 1-lag.

[23] This approximation is corrected by higher-order terms in the returns, $\mathcal{O}(r^2)$...

[24] The AR(4) process generalizes the AR(1) process to 4 lags, $s_t = (1-\sum_{l=1}^{4}\lambda_l)r_t + \sum_{l=1}^{4}\lambda_l s_{t-l}$. MA(4) refers to a Moving Average process with 4 lags, $s_t = (1-\sum_{l=1}^{4}\lambda_l)r_t + \sum_{l=1}^{4}\lambda_l r_{t-l}$.

However, our model does not adopt these more complex multi-lag approaches, as even they are not well-suited to seasonality in smoothing.[25] Instead, the model bases desmoothing on annual returns, even when quarterly returns are available. To take advantage of the additional data of quarterly returns, we use four different versions of the year: one year starting in Q1, one starting in Q2, and so forth.

The use of annual returns in this way leads to more robust risk estimates. The details of the short-horizon behavior get diversified away[26] at longer horizons. It is not necessary (or advantageous) to model the complexity of smoothing at the quarterly horizon, nor risk the errors that can arise from estimating many more model parameters.

The uncertainty in the smoothing parameter can be an important source of estimation error when using desmoothed returns to estimate risk. Because it enters in the denominator of the desmoothing relationship, uncertainty in the smoothing parameter can significantly skew the risk forecasts. For example, if a true smoothing parameter of .5 is misestimated as .75, the estimated risk will average more than twice the “true” risk.[27]

These issues highlight the importance of *robustness* of the methodology to the details of the underlying smoothing process. A desmoothing technique that would be optimal for one particular smoothing process can give skewed forecasts if the underlying smoothing is different. Rather than apply the desmoothing that is appropriate for the details of any one particular smoothing process, it is better to use an approach that remains accurate over a range of possibilities.

In the context of an uncertain smoothing process, we find that it is better to directly incorporate desmoothing into the risk-estimation methodology, rather than first building desmoothed returns, then applying standard risk techniques to the desmoothed time-series.

For example, a *single-step* regression estimates the smoothing parameters and beta coefficients together, in a single regression. In contrast, *two-step* desmoothing first applies desmoothing to the private returns, and then subsequently estimates risk from the desmoothed returns. The chart “Desmoothing and risk estimation simulation” shows the results of a simulation that explores these and other approaches to estimating risk from smoothed returns.

Intuitively, it is not surprising that single-step desmoothing can do better than the two-step approach. Looking only at the time-series of private asset valuations, it can be difficult to resolve an ambiguity: Do periods of successive positive returns reflect smoothing in the valuations, or a bull run of positive true returns to the true value of private equity? Either interpretation could be consistent with the private asset returns, resulting in wide uncertainty in the smoothing parameters.

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25 These processes assume the lagged coefficients λ_l vary by number of lags l, but are constant over time. With seasonal smoothing, the contribution from a given lag would vary with both the time of year and the number of lags. A model that accounts for both many lags and seasonality would require fitting many parameters, making it less robust.

26 In a phenomenon closely analogous to the Central Limit Theorem, long-horizon returns converge to an AR(1) process over a wide range of processes and lagging structures at shorter horizons.

27 On average, the estimated variance would be greater than the true variance by a factor of 4.3.

In single-step desmoothing, the returns of public equity can help resolve this ambiguity. Does public equity exhibit a similar run of positive returns? If so, the run of observed returns should not be interpreted as smoothing of valuations, but as the actual — if lucky — performance of the "true" value.

Bayesian shrinkage

Many applications of Bayesian statistics take the form of *shrinkage*. An estimate based on observations is blended with a *prior,*[28] the best guess we had before we made the observations:

$$w \cdot Observation + (1 - w) \cdot Prior$$

The two points of view are blended with weights w determined by the relative size of the estimation error versus the uncertainty in the prior.

Bayesian statistics instruct us how to combine the information in the observations with the information we had before we made an observation. If the prior is tight, then we essentially knew the answer before making the observations, so the weight on the observations is small. Similarly, if the observations are noisy, they provide less information to move us from the prior. On the other hand, if the prior is only a rough guess, we put more weight on the observations, and more weight still as we gather additional information.

A basic example of Bayesian shrinkage is the Vasicek beta (see Vasicek (1973)). The standard estimate of a stock's market beta is by Ordinary Least Squares (OLS) regression of the stock's returns versus the market returns. The noise in this estimate is low if a long, stable history of returns is available.

If only a short return history is available, however, the OLS estimate can become very noisy, driven by coincidences in the returns in the sample, and producing beta estimates that defy common sense. The average beta of all stocks is 1, with most stocks falling in a relatively narrow range around the average. A beta of .6 or 1.4 is reasonable, but few stocks should have beta less than zero or greater than two.

The Vasicek beta incorporates this "common sense," by blending the OLS estimate with the *prior* for beta that we would guess if we had no data to look at:

$$\beta_{Vasicek} = w \cdot \ \beta_{OLS} + (1 - w) \ \cdot \ \beta_{Prior}$$

Here the prior $\beta_{Prior} = 1$, and the weight w depends[29] on the noise in β_{OLS} relative to the tightness of our prior. If the estimation error is low, or we don't have much conviction in the prior, then $w \rightarrow 1$, and we put

[28] The prior for a quantity x is a full probability distribution $P(x)$, not just a single value. However, sometimes "prior" is used as shorthand for the prior expected value, $E(x) = \int xP(x)dx$, which is the meaning here.

[29] The weight is given by $w = \frac{\sigma_{Prior}^2}{\sigma_{Prior}^2 + \sigma_{OLS}^2}$, where σ_{Prior} is the width of the prior, and $\sigma_{\mathrm{OLS}}$is the noise in the OLS estimate, which is large if only a short data sample is available.

all the weight on the standard OLS estimate. However, if noise is large, the return observations provide limited information, and the best estimate puts much more weight on the prior.[30]

The Vasicek beta may seem intuitive, but perhaps *ad hoc*. Is that particular choice of weights the only good choice? In fact, the Vasicek beta is not ad hoc, but can be derived[31] from a Bayesian analysis: *What is the best estimate of beta, given what we know*? Bayesian statistics provide answers to such questions in the form of *conditional expected values*:

What is the *best estimate*...	→	**What is the *expected value*...**
... *given* what we know?	→	***conditioned on* the data and priors?**

The shrinkage of the Vasicek beta is just one form that Bayesian expected values can take. In other cases, the Bayesian expected value cannot be solved so simply, and more advanced analytic and computational methods are required in practice.

Estimation of smoothing parameters is such a case. The form of the estimator is not the simple shrinkage of the Vasicek beta, but the idea is the same. In its simplest form, we ask: *What is the best estimate of the smoothing parameter, given return observations, and a uniform prior between 0 and .95.*[32]

The chart below shows the result of a simulation study of this simple Bayesian estimator, comparing the error in estimates of the smoothing parameter with the standard OLS estimator. The reduction in noise is very large for short histories, and still significant for data histories longer than 20 years. The improvement is extended significantly when additional effects are taken into account: induced priors, thin factor corrections, and the single-step regression with public equity.

Although the use of Bayesian techniques and priors has a very significant effect on the behavior of the model, the particular choice of priors is much less important. For example, the table "Sensitivity of risk estimates to priors" shows the posterior beta estimate of U.S. Large Buyouts over a broad range of possible priors. The model's original prior of 1.5 ± .5 results in a beta estimate of 1.27, but a broad range of other priors would yield very similar estimates. For example, a prior of 2 with the same relative error (± .7) would lead to a nearly identical beta of 1.30.

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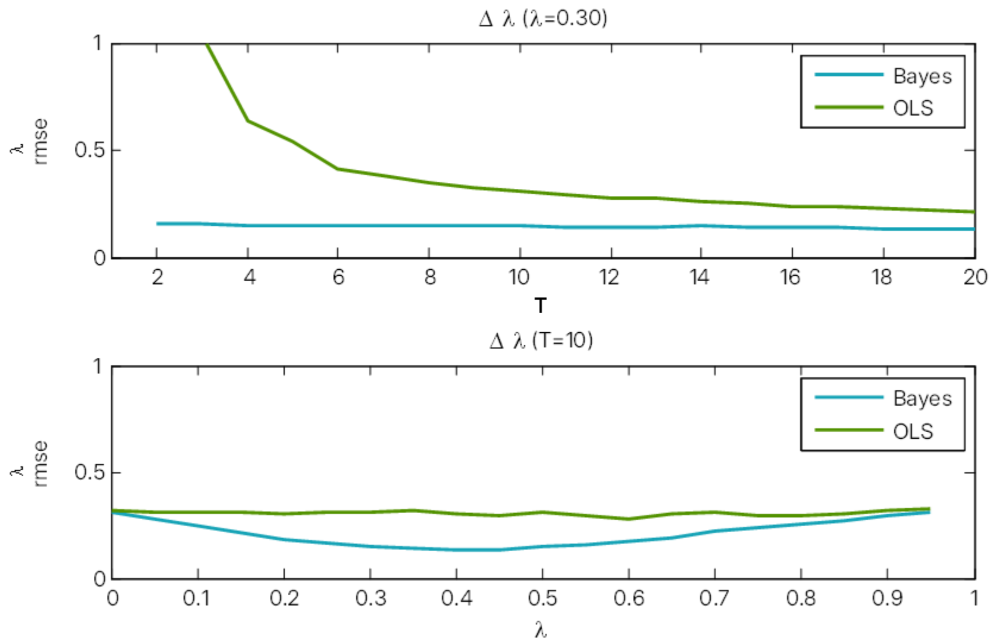
[30] If more information about a stock is known — such as industry membership, size, or other characteristics — the Vasicek prior can be refined considerably: this is one interpretation of factor models and "Barra betas."

[31] The Vasicek beta is the conditional expected value of beta given stock returns, Market returns, and a Gaussian prior: $E(\beta|r, R, prior)$.

[32] A smoothing parameter above .95 would be very extreme. The timescale for the valuation to absorb a change in the true value is roughly $1/(1-\lambda)$. A smoothing parameter over .95 implies a timescale of more than 20 years. A more realistic upper bound is lower, perhaps .8, corresponding to a 5-year time scale for valuations to approach true values.

Estimation error in Bayesian vs. OLS



The estimation error for the smoothing parameter is significantly reduced in a Bayesian approach, even in the absence of a strong prior, relative to the standard Ordinary Least Squares (OLS) estimator. In this simulation study, estimation error is measured as the Root Mean Squared Error (RMSE) between the estimate and the true value. The top panel looks at estimation error as the number of return observations is varied up to 20 years, and the bottom panel looks at different "true" values of the smoothing. Even though the prior is very weak — the smoothing parameter is assumed to be anywhere between 0 and .95 — the benefit is large.

Sensitivity of risk estimates to priors

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.7	0.78	0.92	1.02	1.08	1.13	1.15	1.17	1.19	1.20	1.21
0.8	0.87	0.97	1.06	1.11	1.14	1.17	1.18	1.20	1.20	1.21
0.9	0.95	1.03	1.09	1.13	1.16	1.18	1.19	1.20	1.21	1.22
1.0	1.03	1.09	1.13	1.16	1.18	1.19	1.20	1.21	1.22	1.22
1.1	1.12	1.15	1.17	1.19	1.20	1.21	1.21	1.22	1.22	1.23
1.2	1.21	1.21	1.21	1.21	1.22	1.22	1.22	1.23	1.23	1.23
1.3	1.30	1.27	1.25	1.24	1.23	1.23	1.23	1.23	1.23	1.23
1.4	1.39	1.34	1.29	1.27	1.25	1.25	1.24	1.24	1.24	1.24
1.5	1.48	1.41	1.34	1.29	1.27	1.26	1.25	1.25	1.25	1.24
1.6	1.58	1.49	1.38	1.32	1.29	1.27	1.26	1.26	1.25	1.25
1.7	1.67	1.57	1.43	1.35	1.31	1.29	1.27	1.26	1.26	1.25
1.8	1.77	1.65	1.49	1.38	1.33	1.30	1.28	1.27	1.26	1.26
1.9	1.87	1.74	1.54	1.41	1.34	1.31	1.29	1.28	1.27	1.26
2.0	1.97	1.83	1.60	1.44	1.36	1.32	1.30	1.28	1.27	1.27
2.1	2.07	1.93	1.66	1.47	1.38	1.34	1.31	1.29	1.28	1.27
2.2	2.17	2.03	1.73	1.51	1.40	1.35	1.32	1.30	1.29	1.28
2.3	2.27	2.13	1.80	1.54	1.42	1.36	1.33	1.30	1.29	1.28
2.4	2.37	2.24	1.88	1.58	1.44	1.37	1.34	1.31	1.30	1.28
2.5	2.47	2.34	1.97	1.62	1.46	1.38	1.34	1.32	1.30	1.29

Priors play a crucial role in reducing noise, but the posterior risk estimates are only weakly sensitive to the particular choice of prior. Below, the beta of US Large Buyouts is shown as a function of the prior expected beta (rows), and the uncertainty in the prior (columns). The model's original prior of 1.5 ± .5 results in a beta estimate of 1.27. A broad range of other choices of prior results in very similar estimates, with ranges of ±.05 and ±.1 highlighted in teal and light pink.

Non-point estimates and thin-factor corrections

A common approach to estimating risk from smoothed time series proceeds in steps:

I. Estimate the smoothing parameter.

II. Estimate the desmoothed returns.

III. Estimate risk from the desmoothed returns.

While each step involves an uncertain *estimate*, it is passed on to the next step as a single number — a "point estimate" — which is used as if it were *known*. Errors in the smoothing parameter can have a large effect on risk forecasts, and these errors are skewed: errors in smoothing can lead to arbitrarily large risk forecasts, but risk cannot be less than zero. Does this lead to a tendency to over-forecasting risk? Is there any way to improve the risk forecasts, taking into account the uncertainty of the ingredients along the way?

Bayesian statistics allow us to account for this uncertainty naturally. Rather than the three steps above, there is a single step:

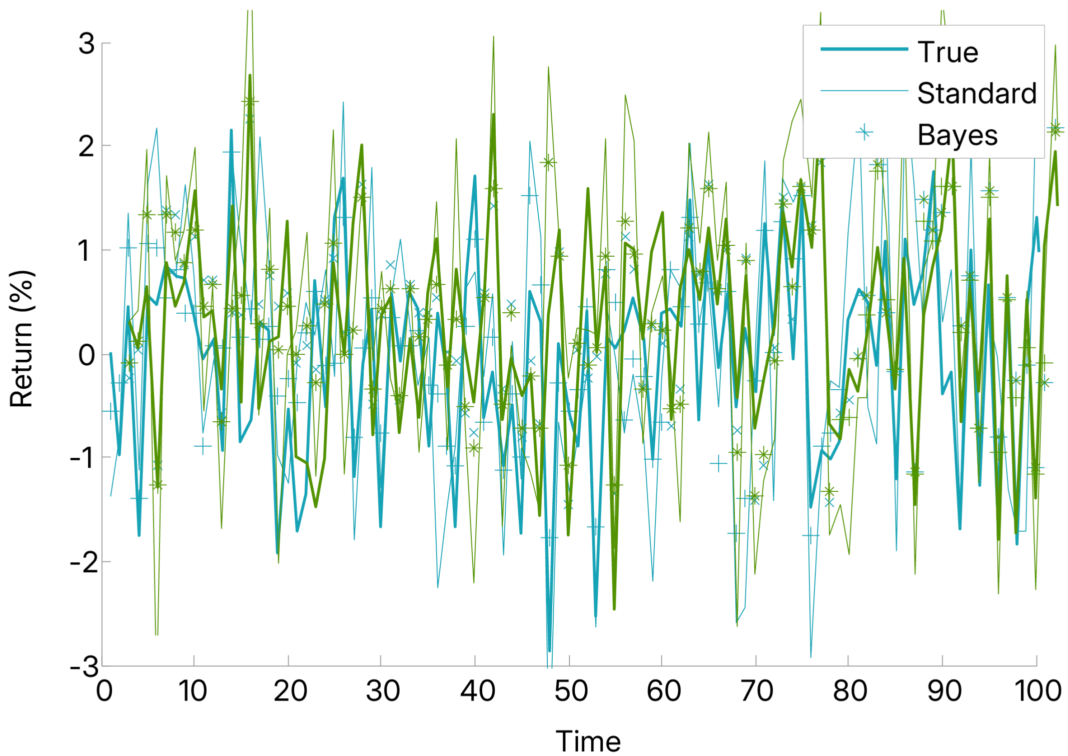
I. Estimate risk from the smoothed returns, given priors on the smoothing parameters.[33]

[33] An example of a Bayesian estimate of beta that accounts for uncertainty in the smoothing parameter is

$$E(\beta|s, R, priors) = \int d\beta d\lambda dr \cdot \beta \cdot P(\beta, \lambda, r|s, R, priors)$$

Here the joint probability $P(\beta, \lambda, r|s, R, priors)$ incorporates the posterior uncertainty in beta, the smoothing parameter λ and the "true" returns r, conditional on the observed smooth returns s, market returns R, and any priors, such as $0 \leq \lambda \leq .8$. Evaluating the right hand side is computationally difficult, but conceptually straightforward.

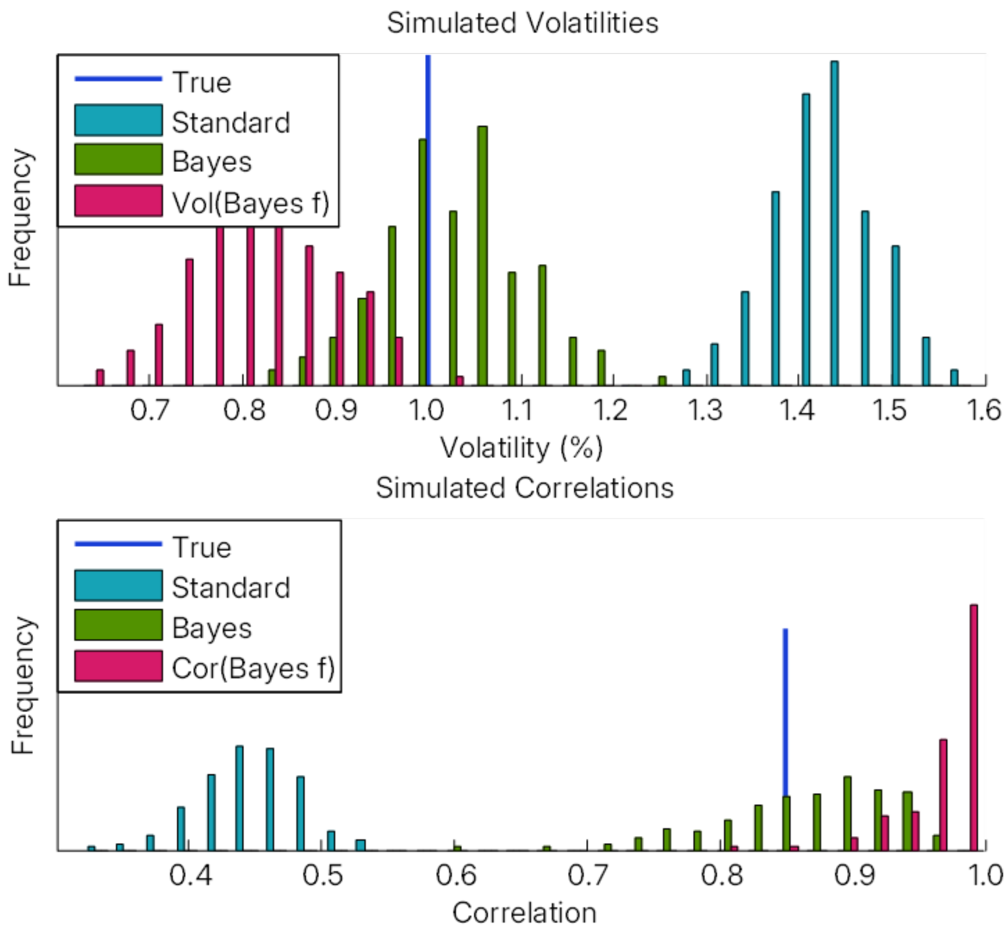
1



A simulation of “thin factor” returns: The two solid curves show the true returns of two highly correlated factors. The dashed curves show standard Ordinary Least Squares (OLS) estimates of these returns taken from small samples of assets. The noise in the small sample inflates the volatility of the factors, and significantly reduces their correlations. A Bayesian estimate of the factor returns uses only the same information as the OLS estimate, but is much more accurate. However, even with the improved estimates of the returns, the best estimate of risk is not taken from this point-estimate of returns, but includes further modifications.

Another important example of non-point estimates arises with *thin factors*, which are estimated from a relatively small number of assets. The small sample size can contribute to large noise in the factor return estimates. Many factors have a small number of funds over much of their history, making this source of noise an important issue. If ignored, it can lead to exaggerated risk forecasts, and under-forecast correlations.

2



Estimates of risk for the simulation of “Simulation of thin factor returns: OLS vs. Bayesian estimates.” The small sample size not only adds noise, but introduces large biases in the standard OLS estimates. Even applying standard risk estimates (volatility and correlation) to the improved Bayesian estimated factor returns (Bayes f) does not eliminate the biases. The best estimates of risk use not only the best estimate of returns, but also incorporate the uncertainty in the return estimates (as in the Bayes risk forecasts). The Bayes forecasts do not eliminate noise altogether, but they are far more accurate than the alternatives.

For example, consider estimating the risk and correlation of two broad markets, but imagine that we were restricted to looking at only a handful of stocks in each market. How does the risk estimated from the small sample compare with the true risk in the broad markets? The chart “Simulation of thin factor returns: OLS vs. Bayesian estimates” shows a simulation study where the true broad market returns are generated with a high degree of correlation, but they are estimated from a small subset of the market.

The Ordinary Least Squares (OLS) estimated returns are much more volatile, and much less correlated than the true returns. Using this estimate could give the impression that the two markets have much more idiosyncratic return than they actually do. In contrast, a Bayesian estimate of the factor returns, based on the same information as the OLS estimate, is much more accurate, even without any prior.

However, even with the improved estimates of the returns, the best estimate of risk does not just take the covariance of the point estimates of returns:

$$Estimate(Risk) \neq Risk(Estimate)$$

Rather, the Bayesian risk estimate produces further thin-factor corrections. The result of these corrections is shown in the chart "Risk estimates from OLS vs. Bayesian forecasts," which compares the standard risk forecast to partial and full Bayesian approaches. In the standard approach, noise in the return estimates exaggerates the volatility and understates the correlations. A partial Bayesian approach, which builds standard risk estimates from the improved estimates of return of "Simulation of thin factor returns: OLS vs. Bayesian estimates," is also somewhat biased, but toward lower volatility.

The full Bayesian risk estimates[34] use not only the best estimate of returns, but also incorporate the uncertainty in these return estimates. This non-point estimate of risk does not eliminate noise altogether, but is far more accurate than the alternatives.

In the context of private equity risk, the effect of small sample size can be large. Individual fund returns are very idiosyncratic, which can introduce large noise levels when market returns are estimated from smaller samples of funds, e.g. the early history of the Asian private equity data set.

The induced prior

In some cases, we don't have strong priors on a quantity of interest, but we can construct an effective prior from the distribution of a peer group of related observations. Before looking at *any* data, for example, we may not have a strong opinion about the smoothing parameters of U.S. private equity. But once we have looked at returns for some segments of the U.S. market, we would not expect the smoothing parameter of the next segment to be vastly different.

In "Appendix: Methodology details," we saw shrinkage estimators that primarily depend on the prior mean and variance: What is our best guess before looking at the data? How confident are we? In the absence of a formal prior, intuition suggests we can build an effective prior from the mean and variance[35] of observations of related quantities. Bayesian statistics can be used to give mathematical precision[36] to these intuitive ideas.

[34] This takes the form $\tilde{F} = E(F|r)$, where F is the factor covariance matrix, and the asset returns r are governed by a factor model $r = Xf + u$. The factor returns f are not directly observable, but instead must be estimated from the asset returns, typically as $\hat{f} = (X'X)^{-1}X'r$. However, the sample covariance of $\hat{f}$ is not the best estimate of the true covariance matrix, and is biased by the noise in $\hat{f}$. The Bayesian estimator $\tilde{F} = E(F|r)$ is unbiased and significantly less noisy.

[35] This is almost right, but with one wrinkle. The observed dispersion of observations is broadened from the distribution of "true" values by the sampling error in the observations. Roughly, the observed variance is the true variance plus the sampling error.

[36] For noisy observations $\hat{x}_i$ of quantities x_i in a peer group, we can take the conditional expected value of a particular value i relative to observations over the whole peer group, $E(x_i|\hat{\boldsymbol{x}})$, not just $E(x_i|\hat{x}_i)$. If the noise is independent across observations, then the conditional probability $p(x_i|\hat{\boldsymbol{x}})$ can be written as $p(\hat{x}_i|x_i)\tilde{p}(x_i; \hat{x}_{j\neq i})/p(\hat{x}_i)$.The *induced prior*

The MSCI Private Equity Factor Model uses induced priors to reduce noise in estimates of smoothing and risk parameters. For each parameter estimated in the model, peer groups are defined over related parameters. For example, the smoothing parameter of each factor is assigned a peer group of similar factors.

Simulation studies

Simulations are a useful tool for evaluating the effectiveness of risk methodology. Private asset data is too scarce and too smooth for standard back-testing. It is often difficult to build even a single risk forecast, let alone a series long enough to test against many independent return observations. Even if the number of return observations is not itself small, the high degree of smoothing leads to a much smaller effective number of observations. For example, a sample of four years of quarterly returns has 16 data points, which would normally be considered marginally acceptable for statistical estimates. However, because the smooth data points are not independent, the effective number of observations is far fewer than 16.

Simulations also make it possible to lift back the curtain, and compare various risk estimates with the true values underlying the simulations. The chart below shows some results of a broad simulation study of desmoothing techniques, and underlying smoothing processes. In all cases, 40 quarters of public and true private asset returns are simulated[37] with a correlation of 70%, followed by a variety of smoothing processes:

- AR(1) Constant Smoothing: The most benign form of smoothing
- AR(1) Seasonal Smoothing: Four different smoothing parameters are applied
- AR(1) Random Smoothing: A random smoothing parameter is used each period
- AR(4) Constant Smoothing: A multi-lag generalization of standard smoothing
- MA(4) Constant Smoothing: A multi-lag generalization of standard smoothing

See Footnote 24 for a definition of the AR(4) and MA(4) processes.

The private-public beta is estimated from the simulated series using a range of estimators:

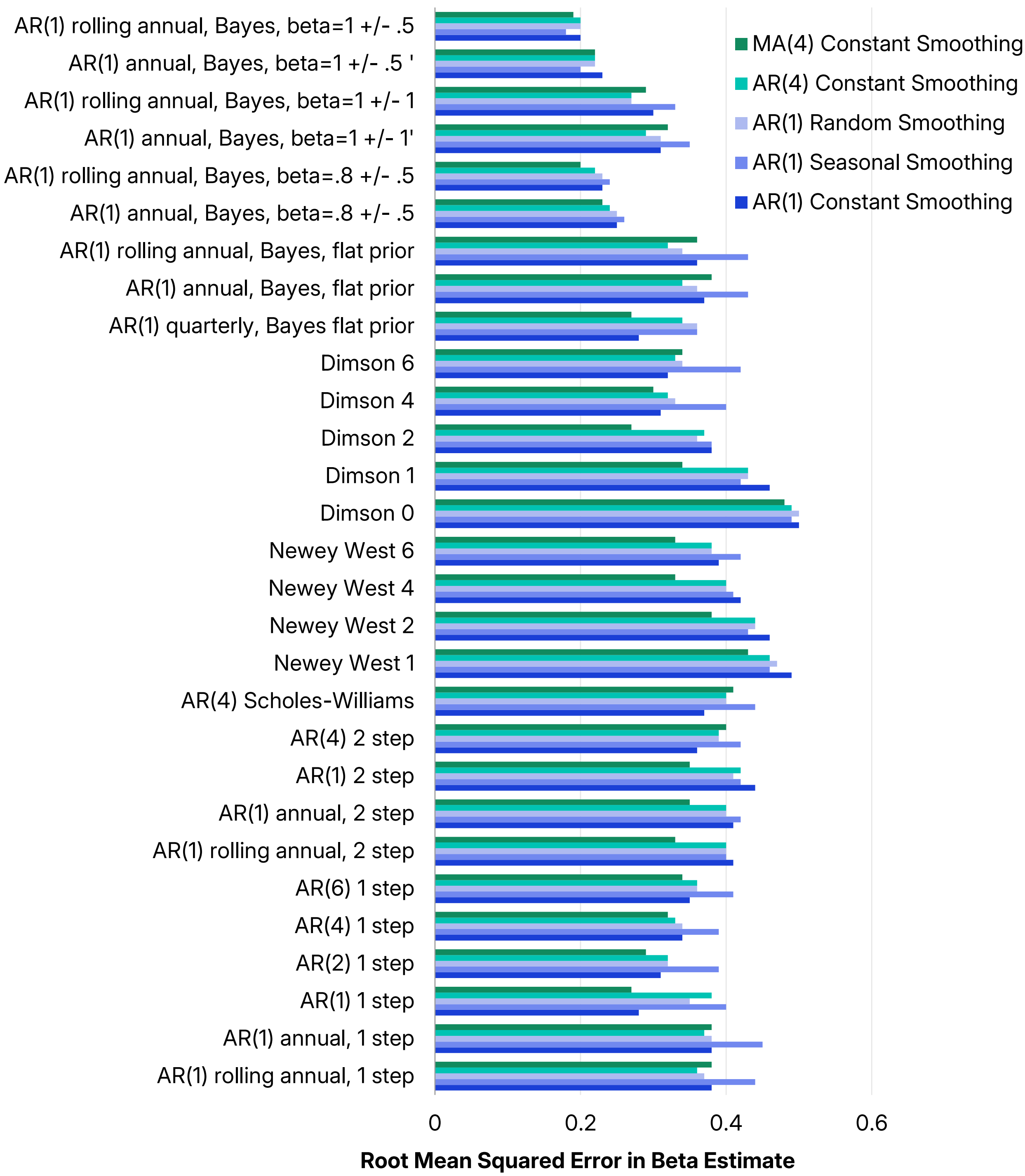
- One-step versus two-step estimators: As discussed in the *Desmoothing* section, two-step estimators first apply desmoothing to the private time series, then estimate beta from the desmoothed returns.
- Bayesian: The class of estimators used in the MSCI Private Equity Factor Model, generalizing the AR(1) (Rolling) Annual one-step estimators. A range of priors for beta is studied.
- AR(n) One-step: Applies n lags of desmoothing, and two lags of serial correlation with the public returns.
- Annual estimators: Use only annual returns.

$\tilde{p}(x_i; \hat{x}_{j\neq i})$ acts as a prior $p(x_i)$. Roughly, the induced prior for observation i takes the shape of the empirical distribution of the other observations $\hat{x}_{j\neq i}$.

[37] The true private returns are simulated as public return + pure private return, with a beta chosen arbitrarily to be 1. The relative degree of noise depends on the public-private correlation, chosen to be 70% for this simulation, but not on the value of beta, or the volatility of the time series.

- Rolling Annual Estimators: Average beta estimated from four versions of annual returns, starting in Q1, Q2, and so forth.
- Dimson: A type of one-step estimator (see Dimson (1979) and Dimson (1985)).
- Scholes-Williams: A type of two-step estimator (see Scholes (1977)).
- Newey-West: Constructs beta from estimates of a covariance matrix accounting for serial correlation (see Newey (1987)).

Desmoothing and risk estimation simulation



A simulation study tests a variety of desmoothing and risk estimation methods over a range of underlying smoothing processes. The root mean squared error measures the average deviation of an estimate of beta from the true value of 1: lower values are better. The MSCI Private Equity Factor Model uses Bayesian estimators with priors of moderate confidence (like the ±.5 here), which do significantly better than other approaches.

The results of the simulation shown in the chart above reveal a wide range of forecast accuracy, and point to a number of conclusions for estimating risk of private assets.

The **robustness** of estimators varies significantly. For example, the AR(1) one-step estimator performs well if the smoothing follows the process it assumes, but is much less accurate if other processes are at work.

The possible **seasonality** of valuations poses a challenge to many estimators. Unfortunately, the seasonal smoothing process may come closest to the actual form of smoothing in the markets.

The performance of **single step** estimators is significantly better than the two-step estimators. As discussed in "Appendix: Methodology details," the single-step estimators take advantage of the public returns to better distinguish smoothed returns from coincidental runs of similar returns.

The Newey-West estimators perform poorly in this regime of very high autocorrelation. In order to capture enough "echoes" of the large inherent autocorrelation, the Newey-West estimator must use many lags, introducing noise.

The benefits of quarterly data are small compared with **annual data**. For all but the simplest smoothing process, the benefits of more data points are offset by the need to estimate more parameters to capture the smoothing at the quarterly horizon, and the lack of independence between smoothed quarterly returns.

Higher **complexity** does not benefit the estimation. Even for returns following the AR(4) process for which it is designed, the AR(4) one-step estimator is less accurate than the AR(2) one-step estimator. Although the AR(4) estimator is "correct" in this context, the additional parameters introduce noise.

Most striking of all, however, are the significant gains in accuracy that come from the use of the **Bayesian estimators**. Even in the absence of any priors, the Bayesian approach tends to do better than standard approaches. Once priors are incorporated, the gains are remarkable.

It is notable that even an **incorrect prior does better** than a correct prior imposed without sufficient conviction. The prior of .8 ± .5 for beta is smaller than the true beta of 1, but the tighter confidence intervals result in more accurate forecasts than the accurate but wide prior of 1 ± 1. It is best to have an accurate prior, but it can be even more important to avoid being too conservative with the prior.

We cannot know the true processes at work in the markets, nor the accuracy of our priors, but the Bayesian estimators used in the MSCI Private Equity Factor Model are seen to be far more accurate than other approaches, over a wide range of possible scenarios.

The results shown in the chart above are a sample from a broader simulation study, which looked at a wide range of parameter values, and studied more complicated effects and estimators, such as the thin-factor corrections discussed in "Appendix: Methodology details."

Another sample from this study shows the average values of risk parameters produced with various estimators, in the table below. The naïve estimates based on raw returns are far from the true values, giving an appearance of lower risk across all risk measures. Standard desmoothing gives significantly higher estimates of risk and correlation than the naïve estimates, and relatively unbiased estimates of beta. Overall, standard desmoothing is seen to provide significantly more accurate estimates of risk than the raw returns.

However, the estimates of the residual risk are biased upward, a bias which can be as large as a factor of five for some choices of parameters. The bias gives the impression that private assets have lower correlation, and provide greater diversification, than the true parameters of the simulation.

Correcting this bias with the Bayesian estimator leads to a further increase in the correlation estimates. The bias depends on the details of the parameters chosen, but in this fairly typical example the average of 81% is much closer to the true 80% than either the raw 57%, or standard 73%.

The economic interpretation of this is significant: **Private equity is equity**. The returns of public and private equity are influenced by a variety of important factors, such as differing liquidity premia, as discussed in the "Methodology overview" section. However, the long-run returns of these assets are seen to have far more commonality than the standard approaches to smoothing have shown.

Average risk parameters from simulation study

	Total risk	Beta	Correlation	Residual risk
True	12.5%	1.00	80.0%	7.5%
Raw returns	5.3%	0.30	57.5%	4.3%
Standard desmoothing	7.1%	0.51	72.8%	4.8%
Bayesian desmoothing	12.6%	1.01	81.2%	7.3%

Average values of risk parameters in a simulation study of different risk estimators. The true values of these parameters are set in the simulation with the values in the top row. In this simulation, a relatively mild set of parameters is chosen: The smoothing process is a simple AR(1) with constant coefficient, and 30 years of quarterly returns are used.

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